Mathematics 235
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Homework 10
Answers

1. **Joe Plutocrat** has been approached by 4 hedge funds with 4 different plans to minimize his taxes. The unknown state of nature is a combination of what the tax law will be next year, and how the economy will perform. Mr. Plutocrat estimates that there are 3 possible states of nature after considering the possibilities:

(*a*) Taxes will be levied at a high rate on all assets valued at more than $1,000,000 and the economy will grow because the deficit decreases.
(*b*) Tax rates will be lower, and the economy will stagnate.
(*c*) Tax rates will be lower, and the economy will soar.

Mr. Plutocrat computes that the net payoff, after expenses (in millions of dollars) from each of the four hedge funds in each of these three cases is:

Hedge	State of nature		
Fund	(*a*)	(*b*)	(*c*)
A	71	58	91
B	92	83	38
C	62	42	67
D	23	98	93

(*a*) Using the optimistic decision rule, what decision should Mr. Plutocrat make? Explain your answer.
(*b*) Using the conservative decision rule, what decision should Mr. Plutocrat make? Explain your answer.
(*c*) Use the minimax regret rule to help Mr. Plutocrat decide. Explain your answer.
(*d*) Mr. Plutocrat hires a consultant who says that he can predict the state of nature based on whether the stock market is heading up (U) or down (D). He says that $P(U) = 0.4050$ and $P(D) = 0.5950$. Moreover, he reports that $P(a|U) = 0.3951$, $P(b|U) = 0.4321$, and $P(c|U) = 0.1728$, and that $P(a|D) = 0.4034$, $P(b|D) = 0.1261$, and $P(c|U) = 0.4706$.

Using this information, what are $P(a)$, $P(b)$, and $P(c)$? In the absence of information about whether the market will head up or down, what does the Expected Value approach tell Mr. Plutocrat to do? What is the EVwoPI, the EVwPI, and the EVPI?

(*e*) What should he do if the stock market heads up? What should he do if the market heads down? What is the expected value of the strategy that uses the stock market's activity?

Answer: (*a*) The optimistic decision rule calls for picking hedge fund D, because the single largest gain in the table, 98, comes from that decision.

(*b*) The conservative decision rule leads to choosing hedge fund A, because that guarantees a gain of 58. Each of the other funds, in the worst case, has a gain of less than 58.

(*c*) Here's a table of regrets:

Hedge	State of nature			*Regrets*			
Fund	(*a*)	(*b*)	(*c*)	(*a*)	(*b*)	(*c*)	*Max*
A	71	58	91	21	40	2	40
B	92	83	38	0	15	55	55
C	62	42	67	30	56	26	56
D	23	98	93	69	0	0	69

The smallest of the maximum regrets is for hedge fund A, and therefore the minimax regret approach calls for picking hedge fund A.

(*d*) We have

$$P(a) = P(a|U)P(U) + P(a|D)P(D) = 0.4000$$
$$P(b) = P(b|U)P(U) + P(b|D)P(D) = 0.2500$$
$$P(c) = P(c|U)P(U) + P(c|D)P(D) = 0.3499$$

We then have

$$\text{EV(A)} = 74.7409$$
$$\text{EV(B)} = 70.8462$$
$$\text{EV(C)} = 58.7433$$
$$\text{EV(D)} = 66.2407$$

The expected value approach calls for picking hedge fund A, and EVwoPI is 74.7409. With perfect information, we would pick hedge fund B with state of nature (*a*), and hedge fund D with states of nature (*b*) and (*c*). The expected value is 93.8407, and so EVwPI is 93.8407. We can compute EVPI = EVwPI − EVwoPI = 19.0998.

(*e*) If the stock market heads up (*U*), the expected values of the 4 funds are:

$$\text{EV(A)} = 68.8387$$
$$\text{EV(B)} = 78.7799$$
$$\text{EV(C)} = 54.2220$$
$$\text{EV(D)} = 67.5035$$

Therefore, if the stock market heads up, Mr. Plutocrat should buy fund B, with an expected value of 78.7799.

If the stock market heads down (*D*), the expected values of the 4 funds are:

$$\text{EV(A)} = 78.7798$$
$$\text{EV(B)} = 65.4619$$
$$\text{EV(C)} = 61.8372$$
$$\text{EV(D)} = 65.4018$$

Therefore, if the stock market heads down, Mr. Plutocrat should buy fund A, with an expected value of 78.7798.

The expected value of this approach is $78.7799P(U) + 78.7798P(D) = 78.7798$. This is EVwSI, and therefore EVSI = EVwSI − EVwoSI = 78.7798 − 74.7409 = 4.0389. The efficiency of the strategy is $\frac{\text{EVSI}}{\text{EVPI}} = \frac{4.0389}{19.0998} \approx 0.2115 \approx 21\%$.

2. A theatrical producer earns an average of $1.6 million from a hit musical, and loses an average of $400,000 from a flop. In recent years, 25% of his productions have been hits, and 75% flops. If the producer knew in advance which shows would flop, he would decline to stage them.

A market research company offers to help the producer identify hit musicals. Their past history shows that if a show is actually going to be a hit, they will predict that it will be a hit 90% of the time. If a show will flop, they incorrectly predict that it will be a hit 25% of the time.

(*a*) What is the Expected Value without Perfect Information (EVwoPI)? What is the Expected Value with Perfect Information? What is the Expected Value of Perfect Information?
(*b*) How much money should the producer be willing to pay the market research firm? In other words, what is the Expected Value of Sample Information?
(*c*) What is the efficiency of the market research?

Answer: (*a*) Without perfect information, the expected value of producing a play is $0.25 \cdot 1.6 + 0.75 \cdot (-0.4) = 0.1$ million dollars. That is greater than the expected value of not producing a play, which is 0. Therefore, EVwoPI $= 0.1$. With perfect information, the producer would produce the musicals that are hits, and decline to produce the flops. We have EVwPI $= 0.25 \cdot 1.6 = 0.4$ million dollars. Therefore, EVPI $= 0.3$ million dollars.

(*b*) Here is a chart of joint probabilities:

	Market research prediction	
Actual	Hit	Flop
Hit	0.2250	0.0250
Flop	0.1875	0.5625
Sum	0.4125	0.5875

Therefore, $P(\text{actual hit}|\text{predicted hit}) = \frac{0.2250}{0.4125} \approx 0.5455$, and so $P(\text{actual flop}|\text{predicted hit}) \approx 1 - 0.5455 = 0.4545$. We can also compute $P(\text{actual hit}|\text{predicted flop}) = \frac{0.0250}{0.5875} \approx 0.0426$, and so $P(\text{actual flop}|\text{predicted flop}) \approx 1 - 0.0426 = 0.9574$.

Thus, if the market research company predicts a hit, we will produce the show. The expected value is $(0.5455 \cdot 1.6) + (0.4545 \cdot -0.4) \approx 0.6909$. If the company predicts a flop, we will not produce the show, with an expected value of 0. The expected value of each show is approximately $0.6909 \cdot 0.4125 \approx 0.2850$. This is EVwSI. Therefore, EVSI $=$ EVwSI $-$ EVwoSI ≈ 0.1850. The producer should be willing to pay the market research firm up to 0.1850 million dollars.

(*c*) The efficiency of the market research is $\frac{\text{EVSI}}{\text{EVPI}} = \frac{0.1850}{0.3} \approx 0.6167 \approx 62\%$.

3. A bank must decide whether to extend a \$100,000 line of credit to a company which has applied for a loan. Past experience with other firms in the same industry shows that 45% of these loans are high risk and lose \$60,000; 35% are moderate risk and make \$50,000; and 20% of the loans are low risk, and eventually result in a profit of \$100,000. If the bank does not extend the line of credit, there is neither a loss nor a gain.

(*a*) In the absence of any other information, what is the Expected Value of extending the line of credit?
(*b*) What is the Expected Value of Perfect Information?

A credit rating agency charges \$2,000 for a report which is not completely reliable. The agency's success rate is:

	Agency's Rating		
Actual rating	Low	Medium	High
Low	0.85	0.10	0.05
Medium	0.06	0.87	0.07
High	0.01	0.05	0.94

(*c*) If the credit agency's report is consulted, what strategy should be followed?
(*d*) What is the Expected Value of Sample Information, including the agency's fee?
(*e*) What is the efficiency of the survey? This computation should not include the agency's fee.

Answer: (*a*) Here is the payoff table, in thousands of dollars:

	Low risk	Medium risk	High risk
Loan	100	50	−60
No loan	0	0	0
Probabilities:	0.20	0.35	0.45

Therefore, the Expected Value of the loan is 10.5, and of course not extending any loans has an Expected Value of 0, so the bank should offer the loan.

(*b*) With perfect information, the bank would only offer loans to customers with low or moderate risk, and the expected value would now become 37.5, which is EVwPI. Therefore, the Expected Value of Perfect Information is $37.5 - 10.5 = 27$.

(*c*) To keep a clear distinction between the agency's rating and the actual risk, we use the notation AL, AM, and AH to refer to customers given a low, medium, or high rating by the agency, and use the letters L, M, and H to refer to the actual risk. We have

$$\begin{array}{llll} P(L) = 0.20 & P(AL|L) = 0.85 & P(AM|L) = 0.10 & P(AH|L) = 0.05 \\ P(M) = 0.35 & P(AL|M) = 0.06 & P(AM|M) = 0.87 & P(AH|M) = 0.07 \\ P(H) = 0.45 & P(AL|H) = 0.01 & P(AM|H) = 0.05 & P(AH|H) = 0.94 \end{array}$$

The table of joint probabilities is:

	AL	AM	AH
L	0.1700	0.0200	0.0100
M	0.0210	0.3045	0.0245
H	0.0045	0.0225	0.4230
Sum	0.1955	0.3470	0.4575

So we now have

$$\begin{array}{lll} P(L|AL) = 0.8696 & P(M|AL) = 0.1074 & P(H|AL) = 0.0230 \\ P(L|AM) = 0.0576 & P(M|AM) = 0.8775 & P(H|AM) = 0.0648 \\ P(L|AH) = 0.0219 & P(M|AH) = 0.0536 & P(H|AH) = 0.9246 \end{array}$$

Now, if the agency reports that a customer is low risk (AL), the loan will be extended, with an expected value of 90.9500. If the agency reports that a customer is medium risk, the loan will be extended, with an expected value of 45.7470. If the agency reports that a customer is high risk, the loan will not be extended.

(*d*) The expected value of this approach is $90.9500 \cdot 0.1955 + 45.7470 \cdot .3470 = 33.6549$. That is EVwSI. To compute EVSI, we first subtract EVwoSI, which is 10.5, yielding 23.1549, and then subtract the agency's fee of 2, so EVSI is 21.1549.

(*e*) The efficiency is $\frac{23.1549}{27} \approx 0.8576 \approx 86\%$.

4. To save on expenses, Rona and Jerry agreed to form a carpool for traveling to and from work. Rona preferred to use the somewhat longer but more consistent Queen City Avenue. Although Jerry preferred the quicker expressway, he agreed with Rona that they should take Queen City Avenue if the expressway had a traffic jam. The following payoff table provides the one-way time estimates in minutes for traveling to or from work:

	State of nature	
Decision Alternative	Expressway open (s_1)	Expressway jammed (s_2)
Queen City Avenue, d_1	30	30
Expressway, d_2	25	45

Based on their experience with traffic problems, Rona and Jerry agreed on a 0.15 probability that the expressway will be jammed.

In addition, they agreed that weather seemed to affect the traffic conditions on the expressway. Let

$$C = \text{clear}$$
$$O = \text{overcast}$$
$$R = \text{rain}$$

The following conditional probabilities apply:

$$P(C|s_1) = 0.8 \qquad P(O|s_1) = 0.2 \qquad P(R|s_1) = 0.0$$
$$P(C|s_2) = 0.1 \qquad P(O|s_2) = 0.3 \qquad P(R|s_2) = 0.6$$

(*a*) Use the Bayes' probability revision procedure to compute the probability of each weather condition and the conditional probability of the expressway open s_1 or closed s_2 given each weather condition.

(*b*) Show the decision tree for this problem.

(*c*) What is the optimal decision strategy and what is the expected travel time?

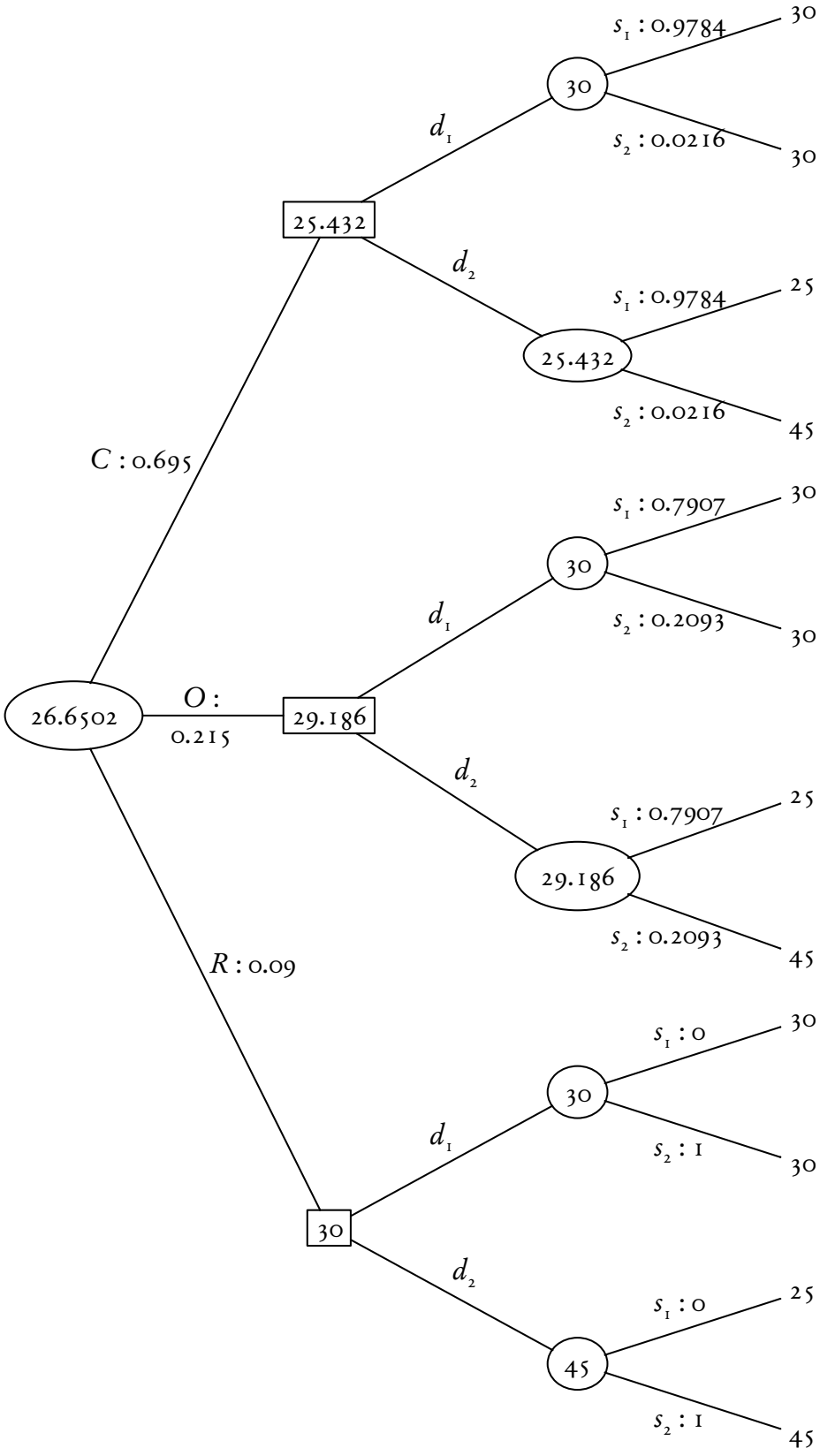
Answer: Note that without using the weather, the expected time on the expressway is $(0.85)(25) + (0.15)(45) = 28$, which is quicker than Queen City Avenue. If there were perfect information, the expected time would be $(0.85)(25) + (0.15)(30) = 25.75$. Therefore, the expected value of perfect information is 2.25.

We continue with a table of joint probabilities:

	C	O	R
s_1	0.680	0.170	0.000
s_2	0.015	0.045	0.090

Therefore, $P(C) = 0.695$, $P(O) = 0.215$, and $P(R) = 0.09$. That tells us that $P(s_1|C) = 0.68/0.695 \approx 0.9784$, $P(s_2|C) = 0.015/0.695 \approx 0.0216$, $P(s_1|O) = 0.17/0.215 \approx 0.7907$, $P(s_2|O) = 0.045/0.215 \approx 0.2093$, $P(s_1|R) = 0$, and $P(s_2|R) = 1$.

Using these in the decision tree, we see that the optimal strategy is to take the expressway if the weather is clear or overcast, and take Queen City Avenue if the weather is rainy. The expected time with this strategy is 26.6502, a saving of 1.3498, and the efficiency of this strategy is $1.3498/2.25 \approx 0.6$.



Decision Tree for Problem 4