MT305.01: Advanced Calculus for Science Majors
Examination 2
March 2, 2012

Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. No credit will be given for answers without explanations.

Cheating will result in a failing grade.

Calculators may not be used during this examination.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

1. (*20 points*) Solve
$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 2\sin 2t.$$

2. (*20 points*) Solve
$$\frac{d^2y}{dt^2} + 9y = \begin{cases} 0 & 0 \le t < 1 \\ 9(t-1) & 1 \le t < 2 \\ 9 & 2 \le t \end{cases} \qquad y(0) = 0, \quad y'(0) = 0.$$

3. (*20 points*) Solve
$$\frac{d^2y}{dt^2} + y = \sec t.$$

4. (*20 points*) Use Laplace transforms to show that
$$(\cos t) * \left(\frac{1}{k}\sin(kt)\right) = (\sin t) * (\cos kt).$$
You may assume that if $\mathscr{L}(f) = \mathscr{L}(g)$ and f and g are continuous, then $f = g$.

5. (*20 points*) Solve
$$t^2\frac{d^2x}{dt^2} - 5t\frac{dx}{dt} + 11x = 0.$$

Variation of Parameters

If $y_h = C_1y_1 + C_2y_2$ is the general solution to $y'' + P(x)y' + Q(x)y = 0$, and $y_p = u_1y_1 + u_2y_2 + y_h$ solves $y'' + P(x)y' + Q(x)y = f(x)$, then
$$y_1u_1' + y_2u_2' = 0$$
$$y_1'u_1' + y_2'u_2' = f(x)$$

Table of Laplace Transforms

$f(t)$	$F(s) = \mathscr{L}(f) = \int_0^\infty e^{-st} f(t)\,dt$
$f'(t)$	$s\mathscr{L}(f) - f(0)$
$f''(t)$	$s^2\mathscr{L}(f) - sf(0) - f'(0)$
$\int_0^t f(\tau)\,d\tau$	$\frac{1}{s}F(s)$
$\int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$
$e^{at}f(t)$	$F(s-a)$
$\mathscr{U}(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)\mathscr{U}(t-a)$	$e^{-as}F(s)$
$\delta(t-a)$	e^{-as}
$tf(t)$	$-F'(s)$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
te^{at}	$\frac{1}{(s-a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}\sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$e^{at}\cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$t\sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$t\cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$\sin \omega t + \omega t\cos \omega t$	$\frac{2\omega s^2}{(s^2+\omega^2)^2}$
$\sin \omega t - \omega t\cos \omega t$	$\frac{2\omega^3}{(s^2+\omega^2)^2}$