MT305.01: Advanced Calculus for Science Majors
Examination 3
April 2, 2012

Do all of your work in the blue booklets. Please label your answers clearly, as I will not have time to perform extensive searches for answers. No credit will be given for answers without explanations.

Cheating will result in a failing grade.

Calculators may not be used during this examination.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

1. (*25 points*) Solve the differential equation

$$\frac{d^2y}{dt^2} + 4y = \begin{cases} 0 & 0 \le t < \pi \\ 1 & \pi \le t < 2\pi \\ 0 & 2\pi \le t \end{cases}$$

with initial conditions $y(0) = 1$ and $y'(0) = 1$.

2. (*25 points*) Suppose we write the solution to the differential equation $y'' + (x^2 + 1)y = 0$, with initial conditions $y(0) = 1$ and $y'(0) = 2$, in the form $y = \sum_{n \ge 0} c_n x^n$. Write out the first 5 non-zero terms in the series expansion for y.

3. (*25 points*) The two solutions of the differential equation $2xy'' + 5y' + xy = 0$ can both be written in the form $y = x^r \sum_{n \ge 0} c_n x^n$, for two different values of r, with $c_0 = 1$. Compute the two possible values of r.

4. (*25 points*) Write out the first 5 non-zero terms in the Fourier expansion of the function $f(x) = |\sin x|$ for $-\pi \le x \le \pi$.

Variation of Parameters

If $y_h = C_1 y_1 + C_2 y_2$ is the general solution to $y'' + P(x)y' + Q(x)y = 0$, and $y_p = u_1 y_1 + u_2 y_2 + y_h$ solves $y'' + P(x)y' + Q(x)y = f(x)$, then

$$y_1 u_1' + y_2 u_2' = 0$$
$$y_1' u_1' + y_2' u_2' = f(x)$$

Table of Laplace Transforms

$f(t)$	$F(s) = \mathscr{L}(f) = \int_0^\infty e^{-st} f(t)\,dt$
$f'(t)$	$s\mathscr{L}(f) - f(0)$
$f''(t)$	$s^2\mathscr{L}(f) - sf(0) - f'(0)$
$\int_0^t f(\tau)\,d\tau$	$\frac{1}{s}F(s)$
$\int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$
$e^{at}f(t)$	$F(s-a)$
$\mathscr{U}(t-a)$	$\dfrac{e^{-as}}{s}$
$f(t-a)\mathscr{U}(t-a)$	$e^{-as}F(s)$
$\delta(t-a)$	e^{-as}
$tf(t)$	$-F'(s)$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$
1	$\dfrac{1}{s}$
t	$\dfrac{1}{s^2}$
t^n	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\dfrac{1}{s-a}$
te^{at}	$\dfrac{1}{(s-a)^2}$
$\sin \omega t$	$\dfrac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\dfrac{s}{s^2+\omega^2}$
$\sinh at$	$\dfrac{a}{s^2-a^2}$
$\cosh at$	$\dfrac{s}{s^2-a^2}$
$e^{at}\sin \omega t$	$\dfrac{\omega}{(s-a)^2+\omega^2}$
$e^{at}\cos \omega t$	$\dfrac{s-a}{(s-a)^2+\omega^2}$
$t\sin \omega t$	$\dfrac{2\omega s}{(s^2+\omega^2)^2}$
$t\cos \omega t$	$\dfrac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$\sin \omega t + \omega t \cos \omega t$	$\dfrac{2\omega s^2}{(s^2+\omega^2)^2}$
$\sin \omega t - \omega t \cos \omega t$	$\dfrac{2\omega^3}{(s^2+\omega^2)^2}$