

Mathematics 310  
Examination 2  
November 7, 2011

Please do all of your work in the blue booklets. Please work clearly and neatly, and label your answers. You do not need to do the problems in order. No credit will be given for answers without explanations.

Cheating will result in a failing grade.

No calculators may be used during this examination.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

All rings contain an identity element.

All ring homomorphisms  $\varphi : R_1 \rightarrow R_2$  must satisfy  $\varphi(1_{R_1}) = 1_{R_2}$ .

1. (20 points) Find the order of each of these elements of  $S_9$ , and identify each permutation as odd or even:

$$(a) \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 6 & 7 & 4 & 3 & 2 & 5 & 1 & 9 \end{pmatrix}$$

$$(b) \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 6 & 7 & 8 & 5 & 4 & 9 & 1 \end{pmatrix}$$

2. (20 points) Suppose that  $G_1$  and  $G_2$  are groups, and  $\varphi : G_1 \rightarrow G_2$  is a group homomorphism. Prove or give a counterexample to each of these two statements:

(i) If  $G_2$  is abelian and  $\varphi$  is surjective, then  $G_1$  is abelian.

(ii) If  $G_1$  is abelian and  $\varphi$  is injective, then  $G_2$  is abelian.

A counterexample requires you to find specific groups  $G_1$  and  $G_2$ , as well as the homomorphism  $\varphi$ .

3. (15 points) Find a non-trivial group homomorphism from  $S_3$  to  $\mathbf{Z}/6\mathbf{Z}$ , or show that no such homomorphism exists.

4. (15 points) Find a non-trivial group homomorphism from  $\mathbf{Z}/6\mathbf{Z}$  to  $S_3$ , or show that no such homomorphism exists.

5. (15 points) Find a non-trivial ring homomorphism from  $\mathbf{Z}/2\mathbf{Z}$  to  $\mathbf{Z}/4\mathbf{Z}$ , or show that no such homomorphism exists.

6. (15 points) Find a non-trivial ring homomorphism from  $\mathbf{Z}/8\mathbf{Z}$  to  $\mathbf{Z}/4\mathbf{Z}$ , or show that no such homomorphism exists.