Mathematics 310
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Homework 6
Due October 28, 2011

1. Suppose that G is a finite group, H a subgroup, and $o(H) = o(G)/2$. Prove that $H \triangleleft G$. *Hint*: The best approach is to show that all left cosets are right cosets, and *vice versa*.

Hint: Remember that we proved that if $\phi : G_1 \to G_2$ is a homomorphism of finite groups G_1 to G_2, and $a \in G_1$, we proved that $o(\phi(a)) | o(a)$. That fact might be useful in the next few problems.

2. Show that the only homomorphism from $\mathbf{Z}/5\mathbf{Z}$ to $\mathbf{Z}/8\mathbf{Z}$ is the trivial homomorphism.

3. Show that the only homomorphism from S_3 to $\mathbf{Z}/3\mathbf{Z}$ is the trivial homomorphism.

4. Let G_1 be a finite group, and let $\phi : G_1 \to G_2$ be an injective homomorphism. Let a be any element of G_1. Show that $o(\phi(a)) = o(a)$.

5. Find a non-trivial homomorphism from $\mathbf{Z}/3\mathbf{Z}$ to S_3.

6. Find the cycle decomposition and order.

(*a*) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 1 & 4 & 2 & 7 & 9 & 6 & 8 & 5 \end{pmatrix}$.

(*b*) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 5 & 3 & 4 & 2 & 1 \end{pmatrix}$.

(*c*) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 5 & 3 & 4 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 5 & 6 & 7 & 4 \end{pmatrix}$.

Classify each of these 3 permutations as even or odd.