Mathematics 310
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Homework 8
Due November 18, 2011

In all of these problems, F is a field.

1. Let $f = x^3 + 1$ and $g = 2x^4 + 3$ be polynomials in $\mathbf{F}_7$. Let $d = (f, g)$. Use the Euclidean algorithm to find d and to find polynomials a and b so that $af + bg = d$. *Note*: Remember that d is monic.

2. Suppose that $f, g, h \in F[x]$,with $(f, g) = 1$, $f|h$, and $g|h$. Prove that $fg|h$.

3. Let $I = (f)$ and $J = (g)$ be ideals in $F[x]$. Show that $I \subset J$ if and only if $g|f$.

4. Suppose that K and L are fields, with $K \subset L$ and $f, g \in K[x]$. Suppose that f and g are relatively prime as elements of $K[x]$. Prove that f and g remain relatively prime when considered as elements of $L[x]$.

5. Suppose that I and J are ideals in a commutative ring R. Define $I + J = \{i + j : i \in I, j \in J\}$.
(*a*) Show that $I + J$ is an ideal of R.
(*b*) Show that $I \cap J$ is an ideal of R.

6. Suppose that I and J are ideals of $\mathbf{Z}$, with $I = (m)$ and $J = (n)$.
(*a*) Let $r = [m, n]$, the least common multiple of m and n. Show that $I \cap J = (r)$.
(*b*) Let $d = (m, n)$. Show that $I + J = (d)$.