Mathematics 310
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Homework 10
Due December 5, 2011

1. Show that there is no ring homomorphism $\phi : \mathbf{C} \to \mathbf{R} \oplus \mathbf{R}$.

2. Find a ring homomorphism $\phi : \mathbf{R} \oplus \mathbf{R} \to \mathbf{C}$.

3. Find the minimal polynomial in $\mathbf{Q}[x]$ for $\sqrt{2} + \sqrt[3]{7}$.

4. Suppose that E is a field containing q elements, and $E \subset F$. Suppose that F is a field, with $[F : E] = n$. Show that F contains q^n elements.

5. Suppose that $E \subset F$, where E and F are fields, and suppose as well that $[E : F] = p$, where p is a prime. Let a be any element of $F \setminus E$. Show that $F = E(a)$.

6. Degrees do not always behave the way that we would hope. Find two numbers a and b which are algebraic over $\mathbf{Q}$ with $[\mathbf{Q}(a) : \mathbf{Q}] = 2$, $[\mathbf{Q}(b) : \mathbf{Q}] = 3$, but the degree of the minimal polynomial for ab is *less* than 6.