Mathematics 414
Exam 1
Monday, October 16, 2006

This is an open book examination. Please don't spend all of your time re-reading the text, as you have only fifty-five minutes. You may also use your calculators.

Do all of your work in the blue booklets. Please work carefully and neatly. Please give reasons for all your answers. You will not get any credit if you guess at an answer, whether your guess is wrong or right.

1. *(20 points)* Let $f(x) = x^2 - x - 1$. Let $g(x) = \sqrt{x+1}$.
   (a) Show that if p is a fixed point of g, then p is a root of f.
   (b) Show that if p is a positive root of f, then p is a fixed point of g.
   (c) Let $p_0 = 1.0$, and define $p_n = g(p_{n-1})$. Compute p_1 and p_2.
   (d) Using the values of p_0, p_1, and p_2 that you just computed, compute $\hat{p}_0$ using the Aitken's Δ^2-method.
   (e) Perform two iterations of Newton's method with $p_0 = 1$.
   (f) Let $p_0 = 1.0$, $p_1 = 1.1$, and $p_2 = 1.2$. Using Müller's Method to compute p_3.
   (g) Explain why Müller's method gives a better answer than either of the other methods.

2. *(20 points)* Suppose that $f_1(x)$ has a zero of multiplicity m_1 at a, and $f_2(x)$ has a zero of multiplicity m_2 at a, with $m_1 < m_2$.
   (a) Let $g(x) = f_1(x)f_2(x)$. Show that $g(x)$ has a zero of multiplicity $m_1 + m_2$ at a.
   (b) Let $h(x) = f_1(x) + f_2(x)$. Show that $h(x)$ has a zero of multiplicity m_1 at a.

3. *(20 points)* Perform the following two computations:

$$\left(3 + \frac{1}{13}\right) + \frac{1}{19} \qquad 3 + \left(\frac{1}{13} + \frac{1}{19}\right)$$

(a) using 3-digit rounding arithmetic.
(b) using 3-digit chopping arithmetic.

4. *(20 points)* Let $f(x) = e^x \sin x$.
   (a) Compute $P_2(x)$, the second Taylor polynomial for $f(x)$, at $x = 0$.
   (b) Approximate $\int_0^{0.2} e^x \sin x \, dx$ by computing $\int_0^{0.2} P_2(x)\, dx$.
   (c) Use the error term $R_2(x)$ to give a good upper bound for the error in the approximation in part (b).

5. *(20 points)* Let a be a fixed positive real number. Define the sequence x_n by

$$x_0 = 1 \qquad x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right), \quad n = 0, 1, 2, \ldots$$

Show that the sequence $\{x_n\}$ is the same sequence that arises by using Newton's method on the function $f(x) = x^2 - a$ with $p_0 = 1$.