This is an open book examination. Please don’t spend all of your time re-reading the text, as you have only fifty-five minutes. You may also use your calculators.

Do all of your work in the blue booklets. Please work carefully and neatly. Please give reasons for all your answers. You will not get any credit if you guess at an answer, whether your guess is wrong or right.

1. (20 points) Let \( f(x) = x^2 - x - 1 \). Let \( g(x) = \sqrt{x + 1} \).
   (a) Show that if \( p \) is a fixed point of \( g \), then \( p \) is a root of \( f \).
   (b) Show that if \( p \) is a positive root of \( f \), then \( p \) is a fixed point of \( g \).
   (c) Let \( p_0 = 1.0 \), and define \( p_n = g(p_{n-1}) \). Compute \( p_1 \) and \( p_2 \).
   (d) Using the values of \( p_0, p_1, \) and \( p_2 \) that you just computed, compute \( \hat{p}_0 \) using the Aitken’s \( \Delta^2 \)-method.
   (e) Perform two iterations of Newton’s method with \( p_0 = 1 \).
   (f) Let \( p_0 = 1.0, p_1 = 1.1, \) and \( p_2 = 1.2 \). Using Müller’s Method to compute \( p_3 \).
   (g) Explain why Müller’s method gives a better answer than either of the other methods.

2. (20 points) Suppose that \( f_1(x) \) has a zero of multiplicity \( m_1 \) at \( a \), and \( f_2(x) \) has a zero of multiplicity \( m_2 \) at \( a \), with \( m_1 < m_2 \).
   (a) Let \( g(x) = f_1(x)f_2(x) \). Show that \( g(x) \) has a zero of multiplicity \( m_1 + m_2 \) at \( a \).
   (b) Let \( h(x) = f_1(x) + f_2(x) \). Show that \( h(x) \) has a zero of multiplicity \( m_1 \) at \( a \).

3. (20 points) Perform the following two computations:
   \[
   \left( 3 + \frac{1}{13} \right) + \frac{1}{19} \quad 3 + \left( \frac{1}{13} + \frac{1}{19} \right)
   \]
   (a) using 3-digit rounding arithmetic.
   (b) using 3-digit chopping arithmetic.

4. (20 points) Let \( f(x) = e^x \sin x \).
   (a) Compute \( P_2(x) \), the second Taylor polynomial for \( f(x) \), at \( x = 0 \).
   (b) Approximate \( \int_0^{0.2} e^x \sin x \, dx \) by computing \( \int_0^{0.2} P_2(x) \, dx \).
   (c) Use the error term \( R_2(x) \) to give a good upper bound for the error in the approximation in part (b).

5. (20 points) Let \( a \) be a fixed positive real number. Define the sequence \( x_n \) by
   \[
   x_0 = 1 \quad x_{n+1} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right), \quad n = 0, 1, 2, \ldots
   \]
   Show that the sequence \( \{x_n\} \) is the same sequence that arises by using Newton’s method on the function \( f(x) = x^2 - a \) with \( p_0 = 1 \).