Mathematics 414
Exam 2
Monday, November 20, 2006

This is an open book examination. Please don't spend all of your time re-reading the text, as you have only fifty-five minutes. You may also use your calculators.

Do all of your work in the blue booklets. Please work carefully and neatly. Please give reasons for all your answers. You will not get any credit if you guess at an answer, whether your guess is wrong or right.

1. *(10 points)* Evaluate

$$\lim_{h\to 0}\left(\frac{3+h}{3-h}\right)^{1/h}.$$

2. *(50 points)* Estimate

$$\int_{-1}^{1} e^{-\sin x}\,dx$$

(a) Using Gaussian quadrature with $n = 3$.
(b) Using the composite trapezoidal rule with $n = 5$.
(c) Give an upper bound for the error in your estimate in part (b).
(d) Estimate the integral using the composite Simpson's rule, first with $n = 2$ and then with $n = 4$.
(e) Use the methods of Chapter 4, Section 6, to estimate the error that occurred in part (d) when you used $n = 4$.

3. *(10 points)* Suppose that you are given the values of $f(x_0)$, $f(x_0 + h)$, and $f(x_0 + 3h)$. Use all three values to estimate $f'(x_0)$, and give an error bound for your estimate.

4. *(30 points)* (a) Determine constants a, b, c, and d that will produce a quadrature formula

$$\int_{-1}^{1} f(x)\,dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

which has degree of precision 3. In other words, your formula should give the exact answer for $\displaystyle\int_{-1}^{1} x^n\,dx$ for $n = 0, \ldots, 3$.
(b) Use your formula to estimate

$$\int_{-1}^{1} e^{-\sin x}\,dx.$$