MT414: Numerical Analysis
Homework 1
Due September 22, 2006

1. Let $f(x) = xe^{x^2}$.

(a) Find the fourth Taylor polynomial $P_4(x)$ for $f(x)$ about $x_0 = 0$.

(b) Find an upper bound for $|f(x) - P_4(x)|$ for $x \in [0, 0.4]$.

(c) Approximate $\int_0^{0.4} f(x)\,dx$ using $\int_0^{0.4} P_4(x)\,dx$.

(d) Find an upper bound for the error in the computation in part (c) by using your answer to part (b).

(e) Approximate $f'(0.2)$ by computing $P_4'(0.2)$. Use the correct answer for $f'(0.2)$ (to 5 decimal places) to compute the relative error in your computation.

2. Use the Intermediate Value Theorem and Rolle's Theorem to show that the equation $x^3 + 2x + k = 0$ has exactly one real solution, regardless of the value of the constant k.

3. Perform the following calculations

(i) exactly,

(ii) using three-digit chopping arithmetic, and

(iii) using three-digit rounding arithmetic.

(iv) Compute the relative errors in parts (ii) and (iii).

(a) $\frac{4}{5} + \frac{1}{3}$ **(b)** $\frac{4}{5} \cdot \frac{1}{3}$ **(c)** $\left(\frac{1}{3} - \frac{3}{11}\right) + \frac{3}{20}$ **(d)** $\left(\frac{1}{3} + \frac{3}{11}\right) - \frac{3}{20}$

4. Suppose that two points (x_0, y_0) and (x_1, y_1) are on a straight line with $y_1 \neq y_0$. Two formulas are available to compute the x-intercept of the line:

$$x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0} \qquad \text{and} \qquad x = x_0 - \frac{(x_1 - x_0) y_0}{y_1 - y_0}.$$

(a) Show that both formulas are algebraically correct.

(b) Suppose that $(x_0, y_0) = (1.31, 3.24)$ and $(x_1, y_1) = (1.93, 4.76)$. Use three-digit rounding arithmetic to compute the x-intercept using both of the formulas. Which method is better and why?

5. The Taylor polynomial of degree n for $f(x) = e^x$ is $\sum_{i=0}^{n} \frac{x^i}{i!}$. Use the Taylor polynomial of degree 9 and three-digit chopping arithmetic to find an approximation to e^{-5} using each of the following methods:

(a) $e^{-5} \approx \sum_{i=0}^{9} \frac{(-5)^i}{i!} = \sum_{i=0}^{9} \frac{(-1)^i 5^i}{i!}$.

(b) $e^{-5} = \frac{1}{e^5} \approx \frac{1}{\sum_{i=0}^{9} \frac{5^i}{i!}}$.

(c) Use your calculator to approximate e^{-5} to 8 places. Which formula, (a) or (b), gave the most accuracy, and why?

6. Suppose that $fl(y)$ is a k-digit rounding approximation to y. Show that

$$\left|\frac{y - fl(y)}{y}\right| \leq 0.5 \times 10^{-k+1}.$$