MT414: Numerical Analysis
Homework 2
Due September 29, 2006

1. Let $a = 0.96$ and $b = 0.99$.

   (a) Using two-digit rounding arithmetic, compute $\dfrac{a+b}{2}$.

   (b) Using two-digit rounding arithmetic, compute $a + \dfrac{b-a}{2}$.

   (c) Which of these two values is a better approximation to the actual value of $\dfrac{a+b}{2}$?

2. Find the rates of convergence of the following functions as $n \to \infty$:

a. $\displaystyle\lim_{n\to\infty} \sin\frac{1}{n} = 0$

b. $\displaystyle\lim_{n\to\infty} \sin\frac{1}{n^2} = 0$

c. $\displaystyle\lim_{n\to\infty} \left(\sin\frac{1}{n}\right)^2 = 0$

d. $\displaystyle\lim_{n\to\infty} \log(n+1) - \log(n) = 0$

3. Find the rates of convergence of the following functions as $h \to 0$:

a. $\displaystyle\lim_{h\to 0} \frac{\sin h}{h} = 1$

b. $\displaystyle\lim_{h\to 0} \frac{1-\cos h}{h} = 0$

c. $\displaystyle\lim_{h\to 0} \frac{\sin h - h\cos h}{h} = 0$

d. $\displaystyle\lim_{h\to 0} \frac{1-e^h}{h} = -1$

4. Suppose that $0 < q < p$ and $\alpha_n = \alpha + O(n^{-p})$. Show that $\alpha_n = \alpha + O(n^{-q})$.

5. Suppose that $0 < q < p$ and $F(h) = L + O(h^p)$. Show that $F(h) = L + O(h^q)$.

6. Use the bisection method to find a solution accurate to within 0.01 for the equation $x^4 - 2x^3 - 4x^2 + 4x + 4 = 0$ on the interval $[-1, 4]$.

7. Let $f(x) = x^4 + 2x^2 - x - 3$. Use algebraic manipulations to show that each of the following functions has a fixed point at p if and only if $f(p) = 0$:

a. $g_1(x) = (3 + x - 2x^2)^{1/4}$

b. $g_2(x) = \left(\dfrac{x+3-x^4}{2}\right)^{1/2}$

c. $g_3(x) = \left(\dfrac{x+3}{x^2+2}\right)^{1/2}$

d. $g_4(x) = \dfrac{3x^4+2x^2+3}{4x^3+4x-1}$

8. Use the functions $g_k(x)$ in the previous problem and perform four iterations (if possible, without dividing by 0 or taking the square root of a negative number), starting with $p_0 = 1$ and $g_k(p_n) = p_{n+1}$. Which of the four functions seems to give the best approximation to a solution of the equation $f(x) = 0$?