MT414: Numerical Analysis
Homework 3
Due October 6, 2006

1. On last week's homework, we used the bisection method to find a solution for the equation $x^4 - 2x^3 - 4x^2 + 4x + 4 = 0$ on the interval $[-1, 4]$.

(*a*) Perform 4 iterations of Newton's method to solve the same equation with $p_0 = -1$.

(*b*) Perform 4 iterations of Newton's method to solve the same equation with $p_0 = 4$.

(*c*) Perform 4 iterations of the secant method with $p_0 = -1$ and $p_1 = 4$ to solve the same equation.

(*d*) Perform 4 iterations of the secant method with $p_0 = 4$ and $p_1 = -1$ to solve the same equation.

(*e*) Perform 4 iterations of the method of false position with $p_0 = -1$ and $p_1 = 4$ to solve the same equation.

2. Let $f(x) = x \sin x$.

(*a*) Show that $f(x)$ has a double zero at $x = 0$.

(*b*) Let $p_0 = 1.5$, and perform 3 iterations of Newton's method to try to find the root.

(*c*) Let $\mu(x) = f(x)/f'(x)$. Perform 3 iterations of Newton's method using the function $\mu(x)$ to try to find the root. Is the convergence noticeably quicker than for $f(x)$?

3. The *ordinary annuity equation* is

$$A = \frac{P}{i}(1 - (1 + i)^{-n}),$$

where A is the amount of money to be borrowed, P is the amount of each payment, i is the interest rate per period, and there are n equally spaced payments. Suppose that a buyer needs a 30-year home mortgage of \$135,000, with payments of at most \$1,000 per month. (This means that there are 360 payments in all.) What is the maximal annual interest rate that the buyer can afford?

4. Suppose that $f(x)$ has m continuous derivatives (in our usual notation, f is C^m). Modify the proof of Theorem 2.10 in the text to show that f has a zero of multiplicity m at p if and only if $f(p) = f'(p) = f''(p) = \cdots = f^{(m-1)}(p) = 0$ and $f^{(m)}(p) \neq 0$.

5. Given a function $f(x)$ with continuous second derivative, let

$$g(x) = x - \frac{f(x)}{f'(x)} - \frac{f''(x)}{2f'(x)} \left(\frac{f(x)}{f'(x)}\right)^2 .$$

(*a*) Suppose that $f(p) = 0$. Show that $g'(p) = g''(p) = 0$. This means (you do not need to check this) that often the series $p_n = g(p_{n-1})$ will converge cubically.

(*b*) Let $f(x) = x^4 - 2x^3 - 4x^2 + 4x + 4$. Iterate $g(x)$ twice with a starting point of $p_0 = -1$. Is the result better than using the standard Newton's method?