MT414: Numerical Analysis
Homework 4
Due October 27, 2006

1. Let $p_n = \dfrac{1}{n}$. Use the Aitken's Δ^2-method to compute $\hat{p}_n$.

2. A sequence $\{p_n\}$ is *superlinearly convergent* to the limit p if

$$\lim_{n\to\infty} \left| \frac{p_{n+1} - p}{p_n - p} \right| = 0.$$

(a) Show that if $p_n \to p$ of order $\alpha > 1$, then $\{p_n\}$ is superlinearly convergent to p.
(b) Let $p_n = \dfrac{1}{n^n}$. Show that p_n is superlinearly convergent to 0, but that p_n does not converge to 0 with any order $\alpha > 1$.

3. Suppose that we have the following values for a function $f(x)$:

x	$f(x)$
2.1	1.5602
2.2	1.4905
2.4	1.3833
2.5	1.3415

(a) Compute 2 different quadratic Lagrange interpolating polynomials using first the points 2.1, 2.2, and 2.4, and then using the points 2.2, 2.4, and 2.5.
(b) Compute the cubic Lagrange interpolating polynomial passing through all 4 of these points.
(c) Using each of those 3 polynomials, estimate the value of $f(2.3)$.

4. Suppose that we have the following values for a function $g(x)$:

x	$f(x)$
3.3	2.6834
3.4	2.9812
3.5	3.3234
3.7	4.1707

Use Neville's method to estimate $g(3.6)$.