

MT414: Numerical Analysis
Homework 5
Due November 6, 2006

1. Suppose that we have the following values for a function $f(x)$:

x	$f(x)$
2.1	1.5602
2.2	1.4905
2.4	1.3833
2.5	1.3415

Compute the free cubic spline interpolation for $f(x)$, and use it to estimate the value of $f(2.3)$.

2. Suppose that we have the following values for a function $g(x)$:

x	$f(x)$
3.3	2.6834
3.4	2.9812
3.5	3.3234
3.7	4.1707

Compute the free cubic spline interpolation for $g(x)$, and use it to estimate the value of $g(3.6)$.

3. Consider the following table of values of the sine function:

x	$\sin(x)$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

Approximate π to at least 6 decimal places in the following computations.

- (a) Compute the quartic Lagrange polynomial $L_4(x)$ that passes through all 5 of these points.
- (b) Estimate $\sin \frac{\pi}{6}$ by computing $L_4(\frac{\pi}{6})$.
- (c) Compute the clamped cubic spline interpolant for these 5 points, using the obvious condition $S'(0) = 1$ and $S'(2\pi) = 1$. (We can compute those values because we know the derivative of the sine function.)
- (d) Estimate $\sin \frac{\pi}{6}$ by computing $S(\frac{\pi}{6})$.
- (e) Which approximation is more accurate?