MT414: Numerical Analysis
Homework 6
Due November 13, 2006

1. Suppose that we have the following values for a function $f(x)$:

x	$f(x)$
2.1	1.5602
2.2	1.4905
2.3	1.4324
2.4	1.3833
2.5	1.3415
2.6	1.3055

Use formulas from the text and class to estimate as accurately as possible the values of $f'(x)$ for $x = 2.1, 2.2, \ldots, 2.6$.

2. Suppose that for some fixed values of x_0 and h, we know $f(x_0 - h)$, $f(x_0)$, $f(x_0 + h)$, and $f(x_0 + 2h)$. Derive a 4-point formula to estimate $f'(x_0)$ to $O(h^3)$.

3. Suppose that $N(h)$ is an approximation to a quantity M for every $h > 0$, and that

$$M = N(h) + K_1h^2 + K_2h^4 + K_3h^6 + \cdots,$$

for some constants $K_1, K_2, K_3, \ldots$. Use the values $N(h)$, $N(h/3)$, and $N(h/9)$ to produce an $O(h^6)$ approximation for M.

4. (a) Show that

$$\lim_{h \to 0} \left(\frac{2+h}{2-h}\right)^{1/h} = e.$$

(b) Compute approximations to e using the formula

$$N(h) = \left(\frac{2+h}{2-h}\right)^{1/h},$$

for $h = 0.4$, 0.2, and 0.1.

(c) Assuming that $e = N(h) + K_1h + K_2h^2 + K_3h^3 + \cdots$. Use extrapolation to compute an $O(h^3)$ approximation to e with $h = 0.4$.

(d) Show that $N(-h) = N(h)$.

(e) Use part (d) to show that $K_1 = K_3 = K_5 = \cdots = 0$ in the formula

$$e = N(h) + K_1h + K_2h^2 + K_3h^3 + K_4h^4 + K_5h^5 + \cdots,$$

so that the formula reduces to

$$e = N(h) + K_2h^2 + K_4h^4 + K_6h^6 + \cdots.$$

(f) Use the result of part (e) and an extrapolation to compute an $O(h^6)$ approximation to e with $h = 0.4$.

5. Show that

$$\int_a^b f(x)\,dx = 2hf(x_0) + \frac{h^3}{3}f''(\xi),$$

where $b - a = 2h$, $x_0 = a + h$, and $\xi \in (a, b)$.