Mathematics 805
Examination 1
February 27, 2009

Please do all of your work in the blue booklets. Please work clearly and neatly, and label your answers. You do not need to do the problems in order. No credit will be given for answers without explanations.

The problems are not arranged in order of increasing difficulty, so you might want to read all of them before beginning.

1. *(20 points)* Suppose that A and B are subsets of a metric space. Prove or give a counterexample:
   (a) $(A \cap B)^\circ = A^\circ \cap B^\circ$.
   (b) $(A \cap B)^- = A^- \cap B^-$.

2. *(20 points)* (a) Define *compact.*
   (b) Using only your definition, prove that the union of 2 compact sets is compact.

3. *(20 points)* Suppose that A and B are subsets of a metric space. Prove or give a counterexample:
   (a) If $A \subset B$, then $A^- \subset B^-$.
   (b) If $A \subset B$, then $A^\circ \subset B^\circ$.

4. *(20 points)* (a) Define *connected* set.
   (b) Show that $\mathbf{Q}$ is not a connected set.

5. *(20 points)* (a) Let M_1 and M_2 be metric spaces, and let $f : M_1 \to M_2$. State the definition of *uniformly continuous.*

   (b) Suppose that D is a subset of a *compact* metric space M, and $f : D \to \mathbf{R}$ is uniformly continuous. Prove that $f(D)$ is bounded, preferably by applying several theorems.