

Mathematics 805
Homework 10
Due Friday, April 17, 1 PM

1. As before, let $B_n(x)$ be the Bernoulli polynomial of degree n . Show that $B_n(1-x) = (-1)^n B_n(x)$. What can you conclude about $B_{2n+1}(\frac{1}{2})$?

2. Start with the formula

$$\cot \pi x = \frac{1}{\pi x} + \frac{2x}{\pi} \sum_{n=1}^{\infty} \frac{1}{x^2 - n^2}$$

integrate term-by-term, exponentiate, and do a bit of algebra to get an infinite product formula for $\frac{\sin x}{x}$. Justifying the validity of integrating term-by-term is not easy, so you can omit the justification for once.

Hints for 8.7.38, from *A Course of Pure Mathematics*, by G.H. Hardy.

(1) Show that if $\alpha + 1 > 0$, then $\lim_{n \rightarrow \infty} \binom{\alpha}{n} = 0$.

(2) Show that

$$1 - \alpha + \binom{\alpha}{2} - \binom{\alpha}{3} + \binom{\alpha}{4} - \cdots + (-1)^n \binom{\alpha}{n} = (-1)^n \binom{\alpha - 1}{n}.$$