MT90301
Graduate Seminar: Concrete Mathematics
Examination 1
February 27, 2009

Please do all of your work in the blue books, and label your answers clearly. You must be explicit in discussing how you arrived at your solutions. Little or no credit will be given for solutions without explanation.

You should read the entire examination before starting. The problems are not arranged in order of increasing difficulty.

Cheating will be severely punished.

1. *(20 points)* Let m and n be positive integers. Show that

$$\left\lceil \frac{n}{m} \right\rceil = \left\lfloor \frac{n+m-1}{m} \right\rfloor.$$

2. *(20 points)* Prove using induction that

$$\sum_{0 \le k < n} H_k = nH_n - n.$$

3. *(20 points)* Try to sum the series

$$\sum_{1 \le k \le 2n} (-1)^k k^2$$

by using perturbation. What sum do you compute instead?

4. *(20 points)* Let m and n be positive integers. Prove that

$$\frac{x^{\underline{m}}}{(x-n)^{\underline{m}}} = \frac{x^{\underline{n}}}{(x-m)^{\underline{n}}},$$

provided that neither denominator is 0.

5. *(20 points)* Prove by rearranging terms that

$$\sum_{1 \le j < k \le n} (a_j b_k - a_k b_j)^2 = \left(\sum_{k=1}^{n} a_k^2 \right) \left(\sum_{k=1}^{n} b_k^2 \right) - \left(\sum_{k=1}^{n} a_k b_k \right)^2.$$