I. Basic Functions

1. $\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1$
2. $\int \frac{1}{x} \, dx = \ln|x| + C$
3. $\int a^x \, dx = \frac{1}{\ln a} a^x + C$
4. $\int \ln x \, dx = x \ln x - x + C, \quad x > 0$
5. $\int \sin x \, dx = -\cos x + C$
6. $\int \cos x \, dx = \sin x + C$
7. $\int \tan x \, dx = -\ln|\cos x| + C$

II. Products of e^x, $\cos x$, and $\sin x$

8. $\int e^{ax} \sin(bx) \, dx = \frac{1}{a^2+b^2} e^{ax} \left[a \sin(bx) - b \cos(bx)\right] + C$
9. $\int e^{ax} \cos(bx) \, dx = \frac{1}{a^2+b^2} e^{ax} \left[a \cos(bx) + b \sin(bx)\right] + C$
10. $\int \sin(ax) \sin(bx) \, dx = \frac{1}{b^2-a^2} \left[a \cos(ax) \sin(bx) - b \sin(ax) \cos(bx)\right] + C, \quad a \neq b$
11. $\int \cos(ax) \cos(bx) \, dx = \frac{1}{b^2-a^2} \left[b \cos(ax) \sin(bx) - a \sin(ax) \cos(bx)\right] + C, \quad a \neq b$
12. $\int \sin(ax) \cos(bx) \, dx = \frac{1}{b^2-a^2} \left[b \sin(ax) \sin(bx) + a \cos(ax) \cos(bx)\right] + C, \quad a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x$, e^x, $\cos x$, $\sin x$

13. $\int x^n \ln x \, dx = \frac{1}{n+1} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C, \quad n \neq -1, \quad x > 0$
14. $\int p(x) e^{ax} \, dx = \frac{1}{a} p(x) e^{ax} - \frac{1}{a} \int p'(x) e^{ax} \, dx$

$$= \frac{1}{a} p(x) e^{ax} - \frac{1}{a^2} p'(x) e^{ax} + \frac{1}{a^3} p''(x) e^{ax} - \cdots$$

$(- + - + - \cdots)$ (signs alternate)

15. $\int p(x) \sin ax \, dx = -\frac{1}{a} p(x) \cos ax + \frac{1}{a} \int p'(x) \cos ax \, dx$

$$= -\frac{1}{a} p(x) \cos ax + \frac{1}{a^2} p'(x) \sin ax + \frac{1}{a^3} p''(x) \cos ax - \cdots$$

$(- + + - - + + \cdots)$ (signs alternate in pairs after first term)

16. $\int p(x) \cos ax \, dx = \frac{1}{a} p(x) \sin ax - \frac{1}{a} \int p'(x) \sin ax \, dx$

$$= \frac{1}{a} p(x) \sin ax + \frac{1}{a^2} p'(x) \cos ax - \frac{1}{a^3} p''(x) \sin ax - \cdots$$

$(+ + - - + + - - \cdots)$ (signs alternate in pairs)

IV. Integer Powers of $\sin x$ and $\cos x$

17. $\displaystyle\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx, \quad n \text{ positive}$

18. $\displaystyle\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx, \quad n \text{ positive}$

19. $\displaystyle\int \frac{1}{\sin^m x}\,dx = \frac{-1}{m-1}\frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1}\int \frac{1}{\sin^{m-2} x}\,dx, \quad m \neq 1, \quad m \text{ positive}$

20. $\displaystyle\int \frac{1}{\sin x}\,dx = \frac{1}{2}\ln\left|\frac{(\cos x)-1}{(\cos x)+1}\right| + C$

21. $\displaystyle\int \frac{1}{\cos^m x}\,dx = \frac{1}{m-1}\frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1}\int \frac{1}{\cos^{m-2} x}\,dx, \quad m \neq 1, \quad m \text{ positive}$

22. $\displaystyle\int \frac{1}{\cos x}\,dx = \frac{1}{2}\ln\left|\frac{(\sin x)+1}{(\sin x)-1}\right| + C$

23. $\displaystyle\int \sin^m x\cos^n x\,dx$: If m is odd, let $w = \cos x$. If n is odd, let $w = \sin x$. If both m and n are even and non-negative, convert all to $\sin x$ or $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18. If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21. The case in which both m and n are even and negative is omitted.

V. Quadratic in the Denominator

24. $\displaystyle\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C, \quad a \neq 0$

25. $\displaystyle\int \frac{bx+c}{x^2+a^2}\,dx = \frac{b}{2}\ln\left|x^2+a^2\right| + \frac{c}{a}\arctan\frac{x}{a} + C, \quad a \neq 0$

26. $\displaystyle\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{a-b}\left(\ln|x-a| - \ln|x-b|\right) + C, \quad a \neq b$

27. $\displaystyle\int \frac{cx+d}{(x-a)(x-b)}\,dx = \frac{1}{a-b}\left[(ac+d)\ln|x-a| - (bc+d)\ln|x-b|\right] + C, \quad a \neq b$

VI. Integrands Involving $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$, $a > 0$

28. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\frac{x}{a} + C$

29. $\displaystyle\int \frac{1}{\sqrt{x^2\pm a^2}}\,dx = \ln\left|x+\sqrt{x^2\pm a^2}\right| + C$

30. $\displaystyle\int \sqrt{a^2\pm x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2\pm x^2} + a^2\int \frac{1}{\sqrt{a^2\pm x^2}}\,dx\right) + C$

31. $\displaystyle\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2-a^2} - a^2\int \frac{1}{\sqrt{x^2-a^2}}\,dx\right) + C$