MATH1007

Homework 1

Due Friday, September 9

When submitting homework, please remember the following:

- Show all work leading to each solution.
- You must use a staple (not paper clip) if your answers are longer than a single page.
- Do not submit crossed-out or sloppy work.
- Do not submit ripped or torn pages.
- Be sure to submit your own work.

1. Consider the sequence defined by the formula $A_N = \frac{2N+3}{N+11}$.

- (a) Find A_1 .
- (b) Find A_9 .
- 2. Consider the sequence defined by the recursive formula

$$\begin{array}{l} A_1=1\\ A_2=3\\ A_N=A_{N-1}+2A_{N-2}, \quad N\geqslant 3 \end{array}$$

List A_3 , A_4 , A_5 , and A_8 .

- 3. Suppose that we have a linear sequence in which $P_0 = 23$ and $P_1 = 24.1$.
- (a) List P_2 , P_3 , P_4 , and P_{23} .
- (b) Use the formula for an arithmetic sum to add up $P_0 + P_1 + \cdots + P_{23}$.

4. Suppose that we have an exponential sequence in which $P_0 = 23$ and $P_1 = 24.1$.

- (a) List P_2 , P_3 , P_4 , and P_{23} .
- (b) Use the formula for a geometric sum to add up $P_0 + P_1 + \cdots + P_{23}$.
- 5. Suppose that we have a linear sequence in which $P_0 = 24.1$ and $P_1 = 23$.
- (a) List P_2 , P_3 , P_4 , and P_{14} .
- (b) Use the formula for an arithmetic sum to add up $P_0 + P_1 + \cdots + P_{14}$.
- 6. Suppose that we have an exponential sequence in which $P_0 = 24.1$ and $P_1 = 23$. (a) List P_2 , P_3 , P_4 , and P_{14} .
- (b) Use the formula for a geometric sum to add up $P_0 + P_1 + \cdots + P_{14}$.

7. The city of Sylvania currently has 401 LED streetlights. The city council has decided to install 3 additional LED streetlights at the start of each week for the next 52 weeks. Each LED streetlight costs \$0.24 to operate for a week.

- (a) How many LED streetlights will Sylvania have at the end of 21 weeks?
- (b) What is the cost of operating the original 401 LED streetlights for 52 weeks?
- (c) What is the additional cost of the new LED streetlights at the end of 52 weeks?

8. This problem asks you to experiment with the logistic growth model

$$\mathbf{p}_{\mathbf{n}} = r\mathbf{p}_{\mathbf{n}-1}(1-\mathbf{p}_{\mathbf{n}-1})$$

for various values of p_0 and r. Do all of your work to at least 4 decimal places.

- (a) Suppose that r = 0.5 and $p_0 = 0.3$. Compute p_1 up to p_{10} .
- (b) Suppose that r = 1.5 and $p_0 = 0.3$. Compute enough terms of the sequence for you to observe a pattern.
- (c) Suppose that r = 2.5 and $p_0 = 0.3$. Compute enough terms of the sequence for you to observe a pattern.
- (d) Suppose that r = 3.2 and $p_0 = 0.3$. Compute enough terms of the sequence for you to observe a pattern.
- (e) Suppose that r = 3.5 and $p_0 = 0.3$. Compute enough terms of the sequence for you to observe a pattern.

9. Suppose that we consider the logistic equation with r = 3.4. What value of p_0 (other than 0) will produce a constant sequence in which $p_0 = p_1 = p_2$?