MATH1007
Homework 3
Answers

1. Find a 5-term arithmetic sequence with common difference 2.1 which sums to 375.4. In other words, you need to find an arithmetic sequence P_0, P_1, ..., P_4 so that $P_1 - P_0 = P_2 - P_1 = \cdots = 2.1$, and $P_0 + \cdots + P_4 = 375.4$

Answer: The formula for the sum is $(P_0 + P_4)5/2$. We know that adds up to 375.4, so we have $P_0 + P_4 = 150.16$. We also know that $P_4 = P_0 + 4 \cdot 2.1 = P_0 + 8.4$. Therefore $2P_0 + 8.4 = 150.16$ and $P_0 = 70.88$. The sequence is 70.88, 72.98, 75.08, 77.18, and 79.28, and you can check that these 5 numbers sum to 375.4.

2. Find a 5-term geometric sequence with common ratio 1.1 which sums to 375.4. In other words, you need to find a geometric sequence P_0, P_1, ..., P_4 so that $P_1/P_0 = P_2/P_1 = \cdots = 1.1$, and $P_0 + \cdots + P_4 = 375.4$. Work to 4 decimal places.

Answer: The formula for the sum is $(P_5 - P_0)/(R - 1) = (P_5 - P_0)/0.1$. We are told that adds to 375.4, so $P_5 - P_0 = 37.54$. We also know that $P_5 = 1.1^5 P_0 \approx 1.6105 P_0$, and so the equation becomes $0.6105 P_0 = 37.54$. Dividing yields $P_0 \approx 61.4896$. The sequence is 61.4896, 67.6385, 74.4024, 81.8426, and 90.0269, and you can check that these numbers sum approximately to 375.4.

3. Over a period of one week, the NASDAQ index went up by 2.4% on Monday, went down by 5.8% on Tuesday, went down by 17.7% on Wednesday, went down another 17.7% on Thursday, and went up 14.2% on Friday. What is the percentage increase or decrease for the week?

Answer: An increase of 2.4% corresponds to multiplication by 1.024, a decrease of 5.8% corresponds to multiplication by $1 - 0.058 = 0.942$, a decrease of 17.7% corresponds to multiplication by 0.823, and an increase of 14.2% corresponds to multiplication by 1.142. We compute $(1.0240)(0.9420)(0.8230)(0.8230)(1.1420) \approx 0.7461$, corresponding to a decrease of 25.3866%. Notice that adding and subtracting percentages yields the incorrect answer of a 24.6% decrease.

4. Suppose that you deposit \$2340 in the bank. How much money will you have in 7 years with an APR of 2.5% compounded

(*a*) annually? (*b*) quarterly? (*c*) monthly? (*d*) continuously?

Answer: (*a*) We compute $(2340)(1.025)^7 \approx 2781.52$.
(*b*) We compute $(2340)(1 + \frac{0.025}{4})^{28} \approx 2786.00$.
(*c*) We compute $(2340)(1 + \frac{0.025}{12})^{84} \approx 2787.01$.
(*d*) We compute $(2340)e^{(0.025 \cdot 7)} \approx 2787.52$.

5. Find the effective annual percentage rate (the APY) if an annual percentage rate of 12.2% is compounded

(*a*) quarterly. (*b*) monthly. (*c*) daily. (*d*) continuously.

Work to 4 decimal places.

Answer: (*a*) We compute $(1 + \frac{0.122}{4})^4 \approx 1.1277$, corresponding to an APY of 12.7696%.
(*b*) We compute $(1 + \frac{0.122}{12})^{12} \approx 1.1291$, corresponding to an APY of 12.9058%.
(*c*) We compute $(1 + \frac{0.122}{365})^{365} \approx 1.1297$, corresponding to an APY of 12.9731%.
(*d*) We compute $e^{0.122} \approx 1.1298$, corresponding to an APY of 12.9754%.

6. You would like to buy a car. You have \$1900 for a down payment. The loan terms are 5.2% APR, compounded monthly, for 4 years (48 payments). You can afford a monthly payment of \$750. What is the maximum price of the car that you can afford?

Answer: The relevant formula is $M = \frac{Pp(1+p)^n}{(1+p)^n-1}$, where p is the monthly rate, P is the principal, M is the monthly payment, and n is the number of payments. Here, we have $p = 0.052/12 \approx 0.0043$, $M = 750$, and $n = 48$, resulting in the equation $750 = 0.0231P$, and so $P \approx 32439.44$. That is the loan amount. You also have 1900 for a down payment, and so you can afford to buy a car that costs 34339.44.

7. Repeat the previous problem if the loan is available for 5 years (60 payments) rather than 4.

Answer: We repeat the calculation with $n = 60$, and get $750 = 0.0190P$, and so $P \approx 39550.70$. Again, you need to add 1900, and you can afford a car that costs 41450.70.

8. Suppose that a young person with no initial savings decides to save 100 dollars per month at an APR of 3%. Assume that the investments are made monthly and that the return is compounded monthly at the end of each month. After 30 years, 360 deposits, and 360 interest payments, how much money is in the account?

Answer: The monthly percentage rate is $(1+\frac{0.03}{12})$. For simplicity, call this number R. The amount of money in the account is $100R^{360} + 100R^{359} + 100R^{358} + \cdots + 100R^2 + 100R$. This is a geometric series with ratio $1/R$. The formula for the sum is then $\frac{100-100R^{360}}{1/R-1} \approx 58419.37$.

If you prefer, you can rearrange the sum in increasing order: $100R + 100R^2 + \cdots + 100R^{360}$. The formula for the sum is then $\frac{100R^{361}-100R}{R-1} \approx 58419.37$, as before.

9. For this problem, assume that interest is collected monthly, and that payments are made monthly.

(a) A 3-year car loan for \$6000 has 3% annual interest. What is the monthly payment?

(b) A 6-year car loan for \$6000 has 3% annual interest. What is the monthly payment? How does your answer compare to your answer in part (a)?

(c) A 3-year car loan for \$6000 has 6% annual interest. What is the monthly payment? How does your answer compare to your answer in part (a)?

(d) A 3-year car loan for \$3000 has 3% annual interest. What is the monthly payment? How does your answer compare to your answer in part (a)?

Answer: The relevant formula is $M = \frac{Pp(1+p)^n}{(1+p)^n-1}$.

(a) Here $P = 6000$, $p = 0.03/12$, $n = 36$, and we get $M \approx 174.49$.

(b) Here $P = 6000$, $p = 0.03/12$, $n = 72$, and we get $M \approx 91.16$. Note that this is lower than in part (a), but more than half.

(c) Here $P = 6000$, $p = 0.06/12$, $n = 36$, and we get $M \approx 182.53$. This is a bit higher than the answer to (a), but much less than double.

(d) Here $P = 3000$, $p = 0.03/12$, $n = 36$, and we get $M \approx 87.24$. This is exactly half the payment in (a).

10. Suppose that you deposit \$750 in a bank account which compounds interest annually. After 5 interest payments, you have \$1000. What is the annual percentage rate for the account?

Answer: We know that $1000 = 750R^5$. That says that $R^5 \approx 1.3333$, and $R \approx 1.0592$. The APR is 5.92%.