1. Suppose that you have a bank account which contains $13,500 on January 1. On March 15, you deposit $450, and on August 31, you deposit $750. Assume that there are 365 days in a year.

(a) Suppose that the account earns 7% APR compounded continuously. What is the balance on December 31?

(b) Suppose that the account earns 7% APR compounded daily. What is the balance on December 31?

Answer: Let's start by computing how many days there are in each period:

January 1	March 15	August 31
73	169	123

(a) Remember that the ratio for 73 days of continuously compounded 7% interest is $e^{0.07 \cdot 73/365}$. So we have

	January 1	March 15	August 31
Starting balance	13500.00	14140.33	15356.14
Ending balance	13690.33	14606.14	15722.68

The closing balance is $15,722.68.

(b) The ratio for 73 days of 7% annual interest compounded daily is $(1 + \frac{0.07}{365})^{73}$. We have

	January 1	March 15	August 31
Starting balance	13500.00	14140.31	15356.07
Ending balance	13690.31	14606.07	15722.58

The closing balance is $15,722.58.

2. Suppose that your credit card billing cycle runs from July 14 through August 13, a period of 31 days. Suppose that the balance on July 14 is $341.80. On July 19, you charge $32.50, on July 28 you make a payment of $300, and on August 10 you charge $89.75.

Suppose that the credit card has an annual interest rate of 17.5%. What is the interest fee for this billing period?

Answer: Again, we start by computing how many days there are in each period, and the balance for each:

	July 14	July 19	July 28	August 10
Days	5	9	13	4
Balance	341.80	374.30	74.30	164.05

The average daily balance is $216.12. The interest fee is $216.12 \cdot 0.175 \cdot \frac{31}{365} \approx \$3.21$.

3. Suppose that a young person with no initial savings decides to save k dollars per month at an APR of r%. Assume that the investments are made monthly and that the return is compounded monthly.

(a) Suppose that $r = 5\%$, and our young person wishes to have accumulated $1 million after 30 years. Assume 360 deposits and 360 interest payments. What is k?

(b) Suppose that k is $900/month, and our young person wishes to have accumulated $1 million after 30 years. What is r? (This is a hard computation, and you need to approximate r by trial and error.)

Answer: (a) Let $R = (1 + \frac{0.05}{12})$, the monthly ratio. As we worked out in class, the total amount of money in the bank is $kR^{360} + kR^{359} + \cdots + kR = k(R + R^2 + \cdots + R^{360}) = k(R^{361} - R)/(R - 1) \approx 835.73k$. We want this to be $1,000,000, and solving for k yields $k \approx \$1196.56$.

(b) Now R is unknown (as is r), and we have $k = 900$. Our equation is $900\frac{R^{361} - R}{R - 1} = 1000000$, or $\frac{R^{361} - R}{R - 1} = 1111.11$. Trial and error yields $R \approx 1.00541$, and then the annual rate is approximately 6.49%.

4. According to a *Philadelphia Inquirer* finance column,

Borrow $100000 with a 6% fixed-rate mortgage, and you'll pay nearly $116000 in interest over 30 years. Pay an extra $100 each month, and you'd pay just $76000—and be done with the mortgage nine years earlier.

This quotation comes from our textbook, which in turn explains that it was used by permission of the *Philadelphia Inquirer*, ©2012, all rights reserved.

(*a*) Verify the claim that you will pay about $116000 in interest.

(*b*) Verify that if the monthly payment is increased by $100, then the time needed to pay off the mortgage is about 21 years, and that the total interest paid is about $76000.

(*c*) How much should the monthly payment be increased to pay off the mortgage in 15 years? What is the total amount of interest paid?

Answer: (*a*) We start by computing the payment on a 30-year mortgage for 100000 at 6% APR, compounded monthly. We set $r = 0.06/12$, $R = 1+r$, $P = 100000$, and $M = P\frac{(R-1)R^{360}}{R^{360}-1}$. We get $M \approx 599.55$. We compute that 360 payments of 599.55 add up to 215838. Subtract the principal, and you get the interest: 115838. That is indeed nearly 116000.

(*b*) Now we change M to 699.55, leave $P = 100000$, $R = 1.005$ and solve $M = P\frac{(R-1)R^n}{R^n-1}$ for n. This is harder:

$$699.55 = 100000\left(\frac{0.005 \cdot 1.005^n}{1.005^n - 1}\right)$$
$$0.0070 = \frac{0.005 \cdot 1.005^n}{1.005^n - 1}$$
$$1.3991 = \frac{1.005^n}{1.005^n - 1}$$
$$1.3991 \cdot 1.005^n - 1.3991 = 1.005^n$$
$$0.3991 \cdot 1.005^n = 1.3991$$
$$1.005^n = 3.5056$$

Solving by trial and error (or logarithms) tells us that n is between 251 and 252 months (very nearly 251.5), and 252 months is 21 years.

Paying 699.55 for 21 years adds up to 176286.60. Subtract the principal and you are left with 76286.60. If you compute this using 251.5 payments, you get a total of 175936.83, and now the interest is 75936.83.

(*c*) This is much simpler. We start by computing the monthly payment if $n = 180$, and get 843.86. That is an increase of $843.86 - 599.55 = \$244.31$. The total payment is $843.86 \cdot 180 = 151894.80$, and that consists of the principal and 51894.80 interest.

5. Suppose that you have a large coin jar which contains nickels, dimes, and quarters. You would like to know roughly how much money is in the jar without taking the trouble to count all of the coins, so you try using two-sample estimation. You shake the jar, and pull out 75 coins: 32 nickels, 14 dimes, and 29 quarters. Put large black dots on each of the coins, put them back in the jar, shake well, and grab 55 coins. This time, you get 24 nickels, 10 dimes, and 21 quarters, and of these, there are dots on 8 nickels, 5 dimes, and 11 quarters. Estimate how much money is in the jar. Round off to a whole number of nickels, dimes, and quarters.

Answer: Let N be the number of nickels, D the number of dimes, and Q the number of quarters. We have $32/N = 8/24$, and $N = 96$. We have $14/D = 5/10$, and so $D = 28$. We have $29/Q = 11/21$, and $Q \approx 55.36$, which we round to 55. That's a total of $21.35.