MATH1007
Homework 6
Answers

1. Suppose for a normally distributed data set, you are told that 95% of the data lies between 63.7 and 81.4. Find the mean μ, the median M, the standard deviation σ, and the two quartiles Q_1 and Q_3.

Answer: We have $\mu - 2\sigma = 63.7$ and $\mu + 2\sigma = 81.4$. Adding gives $2\mu = 145.10$ and $\mu = 72.55$. This in turn tells us that $\sigma = 4.425$. We know that in a normal distribution, $M = \mu = 72.55$. We now have $Q_1 \approx \mu - 0.675\sigma \approx 69.5631$ and $Q_3 \approx \mu + 0.675\sigma \approx 75.5369$.

2. Suppose that in a normally distributed data set, the median M is 43.2 and the 11th percentile is 23.8. What is the 89th percentile?

Answer: We can use the symmetry of the normal distribution to get this answer. We know that 11% of the data is less than 23.8, and 50% of the data is less than 43.2. Therefore, 39% of the data is between 23.8 and 43.2. By symmetry, 39% of the data is between 43.2 and 62.6. Therefore, 89% of the data is less than 62.6, and so 62.6 is the 89th percentile.

3. A fair coin is tossed 4000 times. (The phrase "fair coin" means that heads and tails are equally likely.) Let the random variable X record the number of heads.
(a) Find the mean μ and the standard deviation σ of X.
(b) Find numbers A and B so that the chances that X will be between A and B are 68%.
(c) Find numbers C and D so that the chances that X will be between C and D are 95%.
These answers are unlikely to be round numbers. Please work to 4 decimal places.

Answer: (a) The mean $\mu = np = 4000 \cdot 0.5 = 2000$. The standard deviation $\sigma = \sqrt{np(1-p)} = \sqrt{1000} \approx 31.6228$. ($b$) 68% of the data is between $\mu - \sigma \approx 1968.3772$ and $\mu + \sigma \approx 2031.6228$. ($c$) 95% of the data is between $\mu - 2\sigma \approx 1936.7544$ and $\mu + 2\sigma \approx 2063.2456$.

4. A fair die is rolled 200 times. (The phrase "fair die" means that each of the 6 sides of the die is equally likely to appear.) Let the random variable Y record the number of times that the number 4 appeared.
(a) Find the mean μ and the standard deviation σ of Y.
(b) Find numbers A and B so that the chances that Y will be between A and B are 68%.
(c) Find numbers C and D so that the chances that Y will be between C and D are 95%.
These answers are unlikely to be round numbers. Please work to 4 decimal places.

Answer: (a) The mean $\mu = np = 200 \cdot \frac{1}{6} \approx 33.3333$. The standard deviation $\sigma = \sqrt{200 \cdot \frac{1}{6} \cdot \frac{5}{6}} \approx 5.2705$. ($b$) 68% of the data is between $\mu - \sigma \approx 28.0629$ and $\mu + \sigma \approx 38.6038$. ($c$) 95% of the data is between $\mu - 2\sigma \approx 22.7924$ and $\mu + 2\sigma \approx 43.8743$.

5. Suppose that the probability that a Samsung telephone will explode is 0.10. Out of a shipment of 400 phones, find the probability that:
(a) at most 40 will explode.
(b) more than 52 will explode.

Answer: We compute $\mu = 400 \cdot 0.10 = 40$ and $\sigma = \sqrt{400 \cdot 0.10 \cdot 0.90} = 6$. ($a$) The median is 40 and so the probability that at most 40 will explode is 0.50 or 50%. (b) We compute $\mu + 2\sigma = 52$. Therefore, 97.5% of the data set is less than 52 and 2.5% is larger than 52. The probability that more than 52 will explode is 0.025 or 2.5%.

6. Recall this problem from an earlier homework:

I have a $350,000 mortgage with a 6.75% APR, compounded monthly, and a 25-year term.

(a) After I have made 12 payments (1 year of payments), how much money have I paid to the bank?
(b) How much of the money that I paid to the bank in that first year was interest, and how much was principal?

Answer: (*a*) We worked out last week that the monthly payment is 2418.19. Therefore, after 12 payments, I have paid $2418.19 \cdot 12 = \$29018.28$ to the bank.

(*b*) After 12 payments, the outstanding principal on the loan is 344436.64. Therefore, I have paid $350000 - 344436.64 = 5563.3554$ in principal, and the remainder, 23454.93, is interest.

We can verify this by computing the principal after each payment is made:

Month	Principal	Month	Principal	Month	Principal
1	349550.56	5	347727.38	9	345862.82
2	349098.59	6	347265.15	10	345390.11
3	348644.08	7	346800.33	11	344914.74
4	348187.01	8	346332.89	12	344436.70

The difference of \$0.06 between the two computations is due to round-off error.

7. *The New York Times* recently had a story about retirement savings. It emphasized that it is never too late to begin saving. Consider these examples:

(*a*) Suppose that a 51-year old person deposits \$30,000 each year in a bank account until she turns 65. For simplicity, assume 5% APR, compounded annually, with 15 deposits and 14 interest payments. How much is in the bank when she makes her 15th deposit as she turns 65? NOTE: You can actually do this calculation by hand, because you only need to compute 15 years worth of interest and deposits. I recommend instead that you do this by adding a geometric series, and checking by doing the computation year by year.

(*b*) Suppose instead that a 36-year old person deposits \$15,000 each year in a bank account until she turns 65. For simplicity, assume 5% APR, compounded annually, with 30 deposits and 29 interest payments. How much is in the bank when she makes her 30th deposit as she turns 65? NOTE: I recommend doing this by adding a geometric series.

Answer: Let P be the annual payment, r the annual rate, $R = 1 + r$, and n the number of interest payments. The amount in the bank at age 65 will be $PR^n + PR^{n-1} + \cdots + P = P(1 + R + \cdots + R^n) = P(R^{n+1} - 1)/(R - 1)$.

(*a*) In this case, we have $r = 0.05$, $R = 1.05$, $n = 14$, $P = 30000$, and the amount in the bank is 647356.91. We can check this:

Age	Balance	Age	Balance	Age	Balance
51	30000.00	56	204057.38	61	426203.61
52	61500.00	57	244260.25	62	477513.80
53	94575.00	58	286473.27	63	531389.49
54	129303.75	59	330796.93	64	587958.96
55	165768.94	60	377336.78	65	647356.91

(*b*) In this case, we have $r = 0.05$, $R = 1.05$, $n = 29$, $P = 15000$, and the amount in the bank is 996582.71. Notice that this is more than 50% greater than the amount if you wait until age 51 to start saving.