MATH1007
Homework 7
Answers

1. At Sylvania Tech, the probability that a student will default on his or her loans is 11%. There are 2500 students enrolled at Sylvania Tech. What is the probability that at 260 of them will default on their loans?

Answer: We model this using a normal distribution with $\mu = 0.11 \cdot 2500 = 275$ and $\sigma = \sqrt{2500 \cdot 0.11 \cdot 0.89} \approx 15.6445$. We get $\mu - \sigma \approx 259.3555$, and we know that 16% of the data set lies below $\mu - \sigma$. Therefore, the probability that at least 260 will default is 84%.

2. In American roulette, the probability of the ball landing on a red square is $p = \frac{18}{38}$. Suppose that I bet on red 10,000 consecutive times. Let Y be the number of times that I *lose*.
(a) What are the mean μ and standard deviation σ for the random variable Y?
(b) Approximately what is the likelihood that I will lose more than 5263 times?
(c) What is the approximate likelihood that I will lose between 5163 and 5363 times?
(d) What can you say about the likelihood of winning at least half of the 10,000 bets?

Answer: The probability of losing is $p = \frac{20}{38}$. (a) The mean $\mu = np = (10000)(\frac{20}{38}) \approx 5263.1579$ and the standard deviation $\sigma = \sqrt{np(1-p)} = \sqrt{10000 \cdot \frac{20}{38} \cdot \frac{18}{38}} \approx 49.9307$. ($b$) We see that $\mu \approx 5263$, so the likelihood of losing more than that many times is approximately 50%. (c) We can compute that $\mu - 2\sigma \approx 5163.2965$, and $\mu + 2\sigma \approx 5363.0193$. The approximate likelihood of losing between 5163 and 5363 times is 95%. (d) The question can be rephrased as asking for the probability of losing less than 5000 times. The probability of losing less than $\mu - 3\sigma \approx 5113$ times is 0.15%. We can compute $\mu - 5\sigma \approx 5013.5044$. The probability of a data point being less than $\mu - 5\sigma$ is not one that is covered in our text, and it is in fact less than 0.00001. In other words, it is essentially impossible to win more than half of 10000 consecutive bets on red.

3. The school newspaper at Sylvania Tech has polled a random sample of 800 students to determine how they will vote in the student government election. The newspaper's story, written by a statistics major, states:

> The election for student government president is less than a week away. Our most recent poll shows that Groucho leads Chico by 56% to 44%.

The usual practice is to use $p = 0.5$ for all close elections. Follow that practice below.
(a) Estimate the standard error for this poll.
(b) Compute a 95% confidence interval for the poll.
(c) Compute a 99.7% confidence interval for the poll.

Answer: (a) We first compute $\sigma = \sqrt{800 \cdot 0.5 \cdot 0.5} \approx 14.1421$. The standard error is $\frac{\sigma}{n} \approx 0.0177 \approx 1.8\%$. ($b$) We know that a 95% confidence interval is 2 standard errors, which in this case is 3.6%. This means that we believe with 95% confidence that Groucho's

actual percentage will be between $56 - 3.6 = 52.4\%$ and $56 + 3.54 = 59.6\%$, and Chico's actual percentage will be between $44 - 3.6 = 40.4\%$ and $44 + 3.6 = 47.6\%$. (c) We know that a 99.7% confidence interval is 3 standard errors. We believe with 99.7% confidence that Groucho's actual percentage is between $56 - 5.4 = 50.6\%$ and $56 + 5.4 = 61.4\%$, and Chico's actual percentage will be between $44 - 5.4 = 38.6\%$ and $44 + 5.4 = 49.4\%$.

4. Here is a preference schedule for three candidates A, B, and C:

	Number of voters			
	9	5	3	6
First choice	A	C	B	A
Second choice	C	A	A	B
Third choice	B	B	C	C

(a) How many people voted?
(b) Which candidate(s) would win using the plurality system?
(c) Which candidate(s) would win using the Borda count?
(d) Which candidate(s) would win using instant runoff voting?
(e) Suppose that candidate A withdraws because of an e-mail scandal. Which candidate wins the head-to-head competition between B and C?

Answer: (a) The number of voters is $9 + 5 + 3 + 6 = 23$. (b) Using the plurality system, A gets 15 votes, B gets 3 votes, and C gets 5 votes. The winner is A. (c) Using Borda count, A gets $15 \cdot 3 + 8 \cdot 2 = 61$ points, B gets $14 \cdot 1 + 3 \cdot 3 + 6 \cdot 2 = 35$ points, and C gets $9 \cdot 2 + 5 \cdot 3 + 9 \cdot 1 = 42$ points. Candidate A is the winner. (d) In the first round, A gets 15 votes, B gets 3 votes, and C gets 5 votes. Because A has a majority, she wins, and there is no need to proceed further. (e) If candidate A withdraws, the schedule becomes

	Number of voters			
	9	5	3	6
First choice	C	C	B	B
Second choice	B	B	C	C

Candidate C is the winner with 14 votes compared with 9 for candidate B.

5. Here is a preference schedule for 4 candidates A, B, C, and D:

	Number of voters			
	8	4	9	7
First choice	C	B	B	D
Second choice	B	D	A	A
Third choice	A	A	D	B
Fourth choice	D	C	C	C

(a) How many people voted?
(b) Which candidate(s) would win using the plurality method?
(c) Which candidate(s) would win using the Borda count?

(d) Which candidate(s) would win using instant runoff voting?

(e) Suppose that candidate A leaves the race.

     (i) Which candidate(s) would win using the plurality method?

     (ii) Which candidate(s) would win using the Borda count?

     (iii) Which candidate(s) would win using the instant runoff voting?

(f) Who wins the head-to-head competition between B and C?

Answer: (a) The number of voters is $8 + 4 + 9 + 7 = 28$. (b) Using the plurality method, A has 0 votes, B has 13 votes, C has 8 votes, and D has 7 votes. The winner is B. (c) Using the Borda count, A has $12 \cdot 2 + 16 \cdot 3 = 72$ points, B has $8 \cdot 3 + 13 \cdot 4 + 7 \cdot 2 = 90$ points, C has $8 \cdot 4 + 20 \cdot 1 = 52$ points, and D has $8 \cdot 1 + 4 \cdot 3 + 9 \cdot 2 + 7 \cdot 4 = 66$ points. The winner is B. The sum is 280, as it should be: 28 voters, and 10 points per ballot. (d) Using IRV, we have:

Round 1			Round 2	
Candidate	Votes		Candidate	Votes
A	0			
B	13		B	20
C	8		C	8
D	7			

The winner is B.

(e) If A leaves the race, the preference table becomes:

	Number of voters			
	8	**4**	**9**	**7**
First choice	C	B	B	D
Second choice	B	D	D	B
Third choice	D	C	C	C

You could combine the middle two columns if you prefer. (i) In plurality voting, B has 13 votes, C has 8 votes, and D has 7 votes, and B is the winner. (ii) The Borda counts are: B has 69 points, C has 44 points, and D has 55 points. The winner is B. The points total 168, which is indeed $6 \cdot 28$. (iii) Using IRV, we have exactly the same results as before, because A was the first candidate to be eliminated.

(f) In a head-to-head competition between B and C, B gets 20 votes and C gets 8 votes.

6. Here is a preference schedule for the 5 candidates A, B, C, D, and E:

	Number of voters					
	5	**5**	**3**	**3**	**3**	**2**
First choice	A	C	A	B	D	D
Second choice	B	E	D	E	C	C
Third choice	C	D	B	A	B	B
Fourth choice	D	A	C	C	E	A
Fifth choice	E	B	E	D	A	E

(*a*) How many people voted in this election?
(*b*) Which candidate(s) would win using the plurality method?
(*c*) Which candidate(s) would win using the Borda count?
(*d*) Which candidate(s) would win using instant runoff voting?
(*e*) Who wins in a head-to-head match-up between candidates B and E?
(*f*) Which candidate has the fewest last place votes?
(*g*) Suppose that candidate E decides not to run, so that there are only 4 candidates running.
    (*i*) Which candidate(s) would win under the plurality method?
    (*ii*) Which candidate(s) would win using the Borda count method?
    (*iii*) Which candidate(s) would win using the instant runoff voting?

Answer: (*a*) There are 21 voters. (*b*) Candidate A has 8 first-place votes, B has 3, C has 5, D has 5, and E has 0. The winner is A. (*c*) Using Borda count, A has 66 points, B has 64 points, C has 72 points, D has 65 points, and E has 48 points. The total is 315, which is $21 \cdot 15$. (*d*) Using IRV, we can eliminate E immediately, with no first-place votes. Then we have

Round 1			Round 2	
Candidate	Votes		Candidate	Votes
A	8		A	11
B	3			
C	5		C	5
D	5		D	5

We can stop, because A has a majority of the votes. (*e*) In a head-to-head matchup between B and E, B gets 16 votes and E gets 5 votes. (*f*) C has the fewest last-place votes. (*g*) If E is removed, the preference table is

	Number of voters					
	5	**5**	**3**	**3**	**3**	**2**
First choice	A	C	A	B	D	D
Second choice	B	D	D	A	C	C
Third choice	C	A	B	C	B	B
Fourth choice	D	B	C	D	A	A

You can combine the last two columns if you like. (*i*) The result using plurality does not change, because E had no first place votes. (*ii*) The Borda counts change: A has 56 points, B has 48 points, C has 54 points, and D has 52 points. A is still the winner. The points total 210, which is $21 \cdot 10$. (*iii*) The results are the same as before using IRV, because E was the first candidate eliminated.