MATH1180
Homework 4
Due Friday, February 24

Please (re)read Section 4.1 (Experiments & Sample Spaces), Section 4.2 (Events, Operations and Probability) and Section 4.3 (Analysis of Diagnostic Tests) of the Baglivo textbook.

Please submit solutions to the following problems.

When submitting homework, please remember the following:
- Show all work leading to each solution.
- *Staple* all sheets together. A paper clip is not acceptable.
- Do not submit crossed-out or sloppy work.
- Do not submit ripped or torn pages.
- Be sure to submit your own work.

PROBLEM 1 (BIOMETRIKA 42:412–416). Between 1943 and 1946, the eyesight of the more than seven thousand women aged 30–39 employed in the British weapons industry was tested. For each woman, the unaided distance vision of the right eye and the unaided distance vision of the left eye were recorded using the four-point scale 1, 2, 3, and 4, where 1 is the highest grade and 4 is the lowest grade.

The following table gives the probability that a randomly chosen women from the population of women studied by the British had each possible score in her right and left eyes:

	Left Eye Score 1	*Left Eye Score* 2	*Left Eye Score* 3	*Left Eye Score* 4
Right Eye Score 1	0.203	0.036	0.017	0.009
Right Eye Score 2	0.031	0.202	0.058	0.010
Right Eye Score 3	0.016	0.048	0.237	0.027
Right Eye Score 4	0.005	0.011	0.024	0.066

For this population of women, find the probability of each possible difference in scores, where the difference is defined as follows:

$$\text{Difference} = \text{Right Eye Score Minus Left Eye Score.}$$

(The possible differences are $-3, -2, -1, 0, 1, 2, 3$.) Display your probabilities in a chart. Use your chart to describe the distribution of differences; be sure to relate your answer to the eyesight study.

PROBLEM 2 (AF). A correction in the February 14–21, 2005 issue of *The New Yorker* magazine stated that the January 3 issue that year

> "contained the incorrect statement that four-fifths of Bush voters identified moral values as the most important factor in their decision. In fact, four-fifths of those identifying moral values as the most important factor in their decision were Bush voters."

(a) Identify two events, A and B, such that *The New Yorker* reported the result as $P(A|B)$ when it should have reported $P(B|A)$.
(b) If $P(A) = 0.15$, $P(B) = 0.51$, and $P(B|A) = 4/5$, find $P(A \cap B)$.
(c) Using the information in (b), find $P(A|B)$. (This shows that the two conditional probabilities can be quite different in value.)

PROBLEM 3 (OGD). As part of a marketing study commissioned by the local cable company, researchers looked at the viewing habits of all individuals living in an adult community. The cable company was interested in knowing if an individual watched news regularly (event A), or watched sports regularly (event B). They were also interested in knowing if an individual was college educated (event C).

- Among individuals who were college educated:
  - 72% regularly watched news,
  - 50% regularly watched sports, and
  - 40% regularly watched both news and sports.
- Among individuals who did not have college degrees,
  - 62% regularly watched news,
  - 85% regularly watched sports, and
  - 55% regularly watched both news and sports.

Finally, note that 40% of residents of the community are college-educated.

(a) Consider an experiment in which an individual's name is chosen from the population of individuals living in the adult community, and assume that each choice is equally likely. Use the information above to develop a contingency table of probabilities for this experiment:

COLLEGE-EDUCATED (C):

	Regular Sports (B)	*Not Regular Sports* (B^c)
Regular News (A)	$P(A \cap B \cap C)$	$P(A \cap B^c \cap C)$
Not Regular News (A^c)	$P(A^c \cap B \cap C)$	$P(A^c \cap B^c \cap C)$

NOT COLLEGE-EDUCATED (C^c):

	Regular Sports (B)	*Not Regular Sports* (B^c)
Regular News (A)	$P(A \cap B \cap C^c)$	$P(A \cap B^c \cap C^c)$
Not Regular News (A^c)	$P(A^c \cap B \cap C^c)$	$P(A^c \cap B^c \cap C^c)$

(Your job is to find the appropriate probability to put in each cell in the table. Note that the sum of the 4 numbers in the first table will be $P(C)$, the sum of the 4 numbers in the second table will be $P(C^c)$, and the sum of the 8 numbers in both tables will be exactly equal to 1.)

(b) Based on the distribution given in part (a), find $P(A)$, $P(B)$, and $P(A \cap B)$.

PROBLEM 4 (TC). Consider an experiment in which one individual is chosen from those living in an adult community, and assume that each choice is equally likely. Let F be the event that the individual played varsity sports in college. Which of the following probabilities is greater than the other:

(*a*) P(The individual works in a bank and is an avid sports fan | F).

(*b*) P(The individual works in a bank | F).

Give a reason for your choice. You may assume that at least one member of the community played varsity sports in college.

PROBLEM 5 (AF). For genetic reasons, color blindness is more common among men than women: 5 in 100 men and 25 in 10,000 women suffer from color blindness.

(*a*) By defining events, identify these proportions as conditional probabilities.

(*b*) If the population is half men and half women, what proportion of the population is color blind?

(*c*) If the population is 40% men and 60% women, what proportion of the population is color blind?

PROBLEM 6 (PG). The *National Center for Health Statistics* routinely publishes information on principal sources of payment for hospital discharges in the United States. The following table gives the probabilities that a randomly selected individual from those who were discharged in a recent year used each of several sources of payment:

Private Insurance	*Medicare*	*Medicaid*	*Other Govt. Program*	*Self-Payment*	*Other Method No Charge*	*Method Not Stated*
0.387	0.345	0.116	0.033	0.058	0.028	0.033

(*a*) What is the probability that the principal source of payment was Medicare, Medicaid or some other government program?

(*b*) Given that the principal source of payment was one of the government programs, what is the probability that it was

(*i*) Medicare?

(*ii*) Medicaid?

(*iii*) Some other government program?

PROBLEM 7 (TC). Based on recent information collected by emergency department staff at a large suburban hospital:

- 65% of people seeking emergency care are seen immediately.
- Among those who are not seen immediately, 40% are seen within 30 minutes of arriving.
- Among those who must wait more than 30 minutes, 60% are seen within one hour.

(*a*) Find the probability that a patient waits between 0 and 30 minutes to be seen.

(*b*) Find the probability that a patient waits between 0 and 60 minutes to be seen.

(*c*) Find the probability that a patient waits more than 60 minutes before being seen.

PROBLEM 8 (TC). A sleep study among college students investigated the impact of staying awake all night before an exam to study. The following table lists probabilities that a randomly chosen student from the population "pulled an all-nighter" before the most recent exam (event A) or not, and whether the student passed the exam (event B) or not:

	Pass (B)	*Fail* (B^c)
All-Nighter (A)	0.10	0.05
Not All-Nighter (A^c)	0.71	0.14

(*a*) Are the events "pulled an all-nighter" and "passed the exam" independent, positively associated, or negatively associated? Why?

(*b*) Are the events "pulled an all-nighter" and "failed the exam" independent, positively associated, or negatively associated? Why?

PROBLEM 9 (OGD). Two hospitals (A and B) are comparing their recovery rates. Both hospitals treat three types of patients: those with minor illnesses, those with severe illnesses, and those with life-threatening illnesses.

(*a*) For Hospital A, recovery rates (as percents) for each type of patient are as follows:

Minor Illness	*Severe Illness*	*Life-Threatening Illness*
60%	30%	8%

The patient population seen at Hospital A contains 30% who have minor problems, 40% who have severe problems, and 30% who have life-threatening problems. Find the probability that a randomly chosen patient from the population of Hospital A recovers from their illness.

(*b*) For Hospital B, recovery rates (as percents) for each type of patient are as follows:

Minor Illness	*Severe Illness*	*Life-Threatening Illness*
50%	25%	5%

The patient population seen at Hospital B contains 50% who have minor problems, 30% who have severe problems, and 20% who have life-threatening problems. Find the probability that a randomly chosen patient from the population of Hospital B recovers from their illness.

(*c*) Although the conditional recovery rates by severity of illness are all higher for Hospital A than for Hospital B, the recovery probability you computed in part (*b*) was greater than the probability you computed in part (*a*). Clearly explain why this is true.

(*d*) What proportion of all patients who recovered from illness at Hospital A had a life-threatening problem? What proportion of all patients who recovered from illness at Hospital B had a life-threatening problem?