MATH4410
Homework 1
Due February 12, 2021
Your Name Here

Your answers must be in the form of a typed PDF file, and must be e-mailed to me by 5PM EST on February 12. Please name your file hw01-lastname-firstname.pdf. For example, my solution file is hw01-gross-robert.pdf.

I will try to acknowledge receipt of each e-mail.

1. Remember that the differential equation

$$\frac{dv}{dt} = -0.4v + 9.8, \quad v(0) = 0$$

models the velocity of a falling object including air resistance.

(a) What is the solution of the differential equation?

(b) How long does it take for the falling object to reach 90% of terminal velocity?

(c) How far does the object go in that time?

2. Solve each of the following differential equations. In each equation, assume that y is a function of t.

(a) $y' - y = 2te^t$, $y(0) = 1$.

(b) $y' + 2y = te^{-t}$, $y(1) = 0$.

(c) $ty' + (t+1)y = t$, $y(1) = 1$.

(d) $t^3y' + 4t^2y = e^{-t}$, $y(1) = 0$.

3. Show that if a and λ are positive constants, and b is any real number, then every solution of

$$y' + ay = be^{-\lambda t}$$

has the property that $y \to 0$ as $t \to \infty$. You might need to treat as a special case the possibility that $a = \lambda$.

4. Consider the initial value problem

$$y' - y = 1 + 3\sin t, \quad y(0) = y_0.$$

Find the value(s) of y_0, if any, for which $y(t)$ does *not* tend to infinity as $t \to \infty$.

5. *Variation of Parameters* is another method for solving first-order linear differential equations. Write such an equation as

$$y' + p(t)y = g(t). \tag{1}$$

(a) Suppose first that $g(t) = 0$. Show that the solution is

$$y = A\exp\left[-\int p(t)\,dt\right]$$

for any constant A.

(b) Now suppose that $g(t)$ is not identically 0. Suppose that we try to find a solution of (1) in the form

$$y = A(t) \exp\left[-\int p(t)\, dt\right] \tag{2}$$

where now $A(t)$ is a *function* rather than a constant. Substitute expression (2) into the original differential equation (1) and conclude that $A(t)$ must satisfy

$$A'(t) = g(t) \exp\left[\int p(t)\, dt\right]. \tag{3}$$

Using the expression for $A'(t)$ in (3), it is possible to compute $A(t)$. (Don't forget the constant of integration!) It is then simple to substitute the function $A(t)$ back into (2) to compute y.

6. Use variation of parameters to solve:
   (a) $y' - 2y = t^2 e^{2t}$.

   (b) $y' + \dfrac{y}{t} = 3\cos 2t$.

   (c) $ty' + 2y = \sin t$.

   (d) $2y' + y = 3t^2$.