MATH4410
Homework 8
Due April 16, 2021
Your Name Here

Your answers must be in the form of a typed PDF file, and must be e-mailed to me by 5PM EDT on April 16. Please name your file hw08-lastname-firstname.pdf. For example, my solution file is hw08-gross-robert.pdf. Please use hyphens and not underlines, please use all lower-case letters, and please do not leave any spaces in the name of your file. You will have 5 points added to your homework score if you follow these instructions.

I will try to acknowledge receipt of each e-mail.

1. Solve the initial value problem

$$\mathbf{x}' = \begin{bmatrix} -14 & 39 \\ -6 & 16 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

2. Solve the initial value problem

$$\mathbf{x}' = \begin{bmatrix} -7 & 4 \\ -6 & 7 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

3. Solve the initial value problem

$$\mathbf{x}' = \begin{bmatrix} 11 & -25 \\ 4 & -9 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

4. Suppose that $\mathscr{L}(f(t)) = F(s)$. Show that $\mathscr{L}(tf(t)) = -F'(s)$.

5. Remember that the convolution $f * g$ of two functions f and g is defined by the integral

$$f * g(t) = \int_0^t f(t - \tau)g(\tau)\, d\tau$$

Using the definition and properties of integrals, show that
   (a) $f * g = g * f$.
   (b) $f * (g + h) = (f * g) + (f * h)$.
   (c) $(f * g) * h = f * (g * h)$.

6. Suppose that a, b, and c are real constants, and $a \neq 0$. Let

$$w = \mathscr{L}^{-1} \left(\frac{1}{as^2 + bs + c} \right)$$

   (a) Show that w solves the initial value problem

$$aw'' + bw' + cw = 0, \qquad w(0) = 0, \quad w'(0) = \frac{1}{a}$$

   (b) Suppose that $f(t)$ is a continuous function, and define

$$h(t) = \int_0^t w(t - \tau)f(\tau)\, d\tau$$

   Show that $h(t)$ solves

$$ah'' + bh' + ch = f(t), \qquad h(0) = 0, \quad h'(0) = 0$$