ELECTOOL. Some Tools for the Analysis of Electoral Systems*

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This document describes electoral formulas and indicators used by the `electool` package for *Stata*[1]. The package contains two different programs: `v2seats` and `electind`. This document is intended to be used only in conjunction with the `electool` package and only describes the procedures used by `v2seats` and `electind`. For assistance with syntax, use `man v2seats` and `man electind`, after installation, as usual.

1 Electoral formulas (v2seats)

This section describes electoral formulas used by the `v2seats` command. It only provides information about how each formula works. For assistance with syntax, use `man v2seats`, after installation, as usual. Most of the electoral formulas used by `v2seats` are well known and have been documented in the literature. Please, refer to academic as well as official sources for further explanation and details about the foundations and mathematical properties for each formula. See for instance, Benoit (2000), Blais and Massicotte (1997), Gallagher (1992), and Golder (2005) for a concise review.

Electoral formulas are designed to allocate seats to parties. Many formulas have been proposed in the literature and there are many different implementations of each formula in real electoral systems. This document only describes formulas used by `v2seats`, which covers most of the formulas in use around the world. There are two main different types of electoral formulas: majority and proportional methods. Majority methods are intended to select one single winner, while proportional methods allocate seats proportionally to the number of votes for each party. Proportional formulas can be classified into two broad families: largest reminder methods and highest average methods.

*Provided as additional documentation for the `electool` package, available at the Statistical Software Components Archive (SSC) from the Boston College Department of Economics. Please refer to help files for assistance with syntax. Comments and questions are very welcome.

[1] *Stata* is a registered *trademark* of StataCorp.

1.1 Majority methods

Majority formulas are designed to select one single winner. They are mostly used in single-member districts, but they can also be used in multi-member districts. The following formulas are available:

1. **Majority**. To get elected one candidate needs at least half of the vote. If no candidate is selected, not allocated seats can be allocated at later stages using different procedures.

2. **Plurality**. It is also known as FPTP (first past the post). The most voted candidate gets elected. In multiple-member districts the most voted party wins all the seats. In this case, plurality formula is also known as BV (block-vote).

3. **Ordered plurality**. The first n candidates are elected. This method can only be used in multi-member districts.

1.2 Largest remainder methods

Largest remainder methods allocate seats proportionally to the number of votes. The number of votes for each party is divided by a quota, which is the result of dividing the total number of valid votes (v) by some integer close to the number of seats to be allocated (s). At the first stage, the integer part of the resulting quotient is allocated to each party. At the second stage, not allocated seats are successively awarded to parties having the largest remainder. Each quota can be defined by its denominator, as it is shown in the next table:

Table 1: Largest remainder methods

Name	Quota
Hare	v/s
Hagenbach-Bischoff	$v/(s+1)$
Droop	$\lfloor (v/(s+1)) + 1 \rfloor$
Imperiali	$v/(s+2)$
Reinforced Imperiali	$v/(s+3)$

Source: Adapted from Gallagher (1992).

1.3 Highest average methods

Highest average methods allocate seats proportionally to the number of votes. They allocate seats in a way that assures the highest quotient or average by seat for each party. The number of votes for each party is successively divided by a sequence of divisors. Each formula can be defined by its n_{th} divisor, as it is shown in the next table:

Table 2: Highest average methods

Name	n_{th} divisor	Sequence (first five divisors)
D'Hondt	n	$1, 2, 3, 4, 5, ...$
St. Lagüe	$2n - 1$	$1, 3, 5, 7, 9, ...$
Modified St. Lagüe	$2n - 1^{\dagger}$	$1.4, 3, 5, 7, 9, ...$
Hungarian St. Lagüe[2]	$2n - 1^{\dagger}$	$1.5, 3, 5, 7, 9, ...$
Danish method	$3n - 2$	$1, 4, 7, 10, ...$
Imperiali divisors[3]	$(n + 1)/2$	$1, 1.5, 2, 2.5, 3, ...$

†Except for the first divisor.
Source: Adapted from Gallagher (1992).

1.4 Other methods

Two other methods are available, though they are not typically used to allocate seats to parties[4]. Huntington and Adams methods are highest average formulas, with the specificity of their first divisor being 0. That guarantees that all the parties above the electoral threshold (if any) will get one seat before the most voted party gets a second seat, given that $v_i/0$ is assumed to be equal to ∞ for practical purposes. These formulas are shown in the next table:

Table 3: Other methods

Name	n_{th} divisor	Sequence (first five divisors)
Huntington	$\sqrt{n(n-1)}$	$0, 1.41, 2.45, 3.46, 4.47, ...$
Adams	$n - 1$	$0, 1, 2, 3, 4, ...$

Source: Adapted from Gallagher (1992).

1.5 Mixed methods

Some electoral systems combine different electoral formulas at once. Most of these systems can be simulated successively running `v2seats`. However, some combinations of highest average and largest remainder methods are implemented in one

[2]See Benoit (2000).
[3]Do not get confused with Imperali quota.
[4]They have been typically used to allocate seats to districts.

step. All these methods use one of the quotas previously presented (largest remainder methods) at the first stage. At the second stage, remainders are divided by the number of seats awarded at the first step plus one ($r_i/(s_i+1)$). Then, not allocated seats are awarded to parties having the highest average remainders. Note that this is equivalent to use a largest remainder method at the first stage, and then any highest average method at the second stage.

1.6 Customized methods

Most allocation methods in use can be implemented using the formulas available in `v2seats`. Nevertheless, `v2seats` can also be used to easily build customized electoral formulas. User can define majority, largest remainder and highest average methods using `maj(exp)`, `lrm(exp)`, `hav(exp) [first(numlist)]` and `seqv(matname)` options. Only one customized formula at a time is allowed. Note also that these options are not compatible with `formula(method)`.

When using `maj(exp)`, `exp` can be the right hand side of any formula describing the majority rule. Use only v to set up the formula, where v is the total number of votes. For example, using `maj(v/2)` is equivalent to use the custom majority rule (half of the vote) setting `formula(major)`, while `maj(2*v/3)` is equivalent to set the majority rule at 2/3. Setting the majority rule below $v/2$ means that more than one candidate can be elected at a time, even in single-member districts. This is useful to simulate run-off methods in which candidates need to get a minimum share of the vote in the first round to get access to the second round.

When using `lrm(exp)`, `exp` can be the right hand side of any formula describing the quota. Use only v and s to set up the formula, where v is the total number of votes and s is the number of seats to be allocated. For example, using `lrm(v/(s+1))` is equivalent to use the Hagenbach-Bischoff quota setting `formula(hagb)`, while `lrm(int((v/(s+1))+1))` gives the Droop quota which is equivalent to set `formula(droop)`.

When using `hav(exp)`, `exp` can be the right hand side of any formula describing the sequence of divisors. Use only n to set up the formula, where n is the n_{th} divisor. `[first(numlist)]` also allows to se up a customized sequence for the first n divisors if needed. The first n divisors in `<numlist>` will override the sequence defined by `hav(exp)` up to the n_{th} divisor. However, only the first one needs to be changed for most methods. For example, using `hav(2*n-1)` is equivalent to use the original St. Lagüe method setting `formula(stlague)`, while `hav(2*n-1) first(1.4)` gives the modified St. Lagüe method, which is equivalent to set `formula(stlm)`.

Another way to define a customized highest average method is to use a customized vector of divisors. Option `seqv(matname)` is needed to tell `v2seats` which vector contains the sequence of divisors. User has to previously define this vector using appropriate commands in Stata. Only column vectors are allowed. If the vector contains not enough divisors an error message will be issued. This option is not compatible with `hav(exp)` or any other allocation method.

Use customized formulas with caution and check the syntax carefully. It is strongly advised to inspect details after running `v2seats` using customized formulas.

2 Electoral indicators (electind)

This section describes electoral indicators computed by the `electind` command. This command can be used after `v2seats` command or as a stand-alone command if you already have votes and seats by party in your data set. Note also that some indicators do not require to have information about seats by party.

This section is only intended to provide information about the formula for each indicator. For assistance with syntax, use `man electind`, after installation, as usual. Most of the electoral indicators computed by `electind` are well known and have been documented in the literature, though some of them are relatively new. Please, refer to original sources for further explanation and details about the foundations and mathematical properties for each indicator.

For the sake of clarity, the most conventional notation has been used whenever possible. Let n denote the number of parties that compete for representation in a particular election at time t. Each of these parties can be defined by some set of m attributes, being X_i^k the k attribute for party i. Let V_i denote the fraction of votes for party i, and R_i the fraction of representatives for party i. Let also assume that parties can be classified into s groups according to their ideological preferences. Thus, we can define the following electoral indicators:

1. **Disproportionality:**

   (a) Rae index (Rae, 1971):

   $$PR = \frac{\sum\limits_{i=1,V_i>0.5}^{n} |V_i - R_i|}{n}$$

   (b) Loosemore-Hanby index (Loosemore and Hanby, 1971):

   $$LH = \frac{1}{2}\sum_{i=1}^{n} |V_i - R_i|$$

   (c) Mackie-Rose index (Mackie and Rose, 1991):

   $$MR = 100 - LH$$

   (d) Grofman index (Grofman and Lijphart, 1986):

   $$G = \frac{1}{ENP_e}\sum_{i=1}^{n} |V_i - R_i|$$

   (e) Advantage ratio (Taagepera and Laakso, 1980):

   $$AR = \max\left\{\left(\frac{R_i}{V_i}\right) : i = 1, ..., n\right\}$$

(f) Gallagher index (Gallagher, 1991):

$$GLS = \sqrt{\frac{\sum_{i=1}^{n} (V_i - R_i)^2}{2}}$$

(g) Lijphart index (Lijphart, 1994):

$$LLS = \sqrt{\frac{\sum_{i=1, V_i > 0.5}^{n} (V_i - R_i)^2}{2}}$$

(h) Saint-Lagüe index (Lijphart and Gibberd, 1977):

$$SL = \sum_{i=1, V_i > 0}^{n} \frac{(V_i - R_i)^2}{V_i}$$

(i) Maximum deviation index (Gallagher, 1991):

$$MD = \max \{|V_i - R_i| : i = 1, ..., n\}$$

(j) Bias index (Cox and Shugart, 1991):

$$\begin{aligned} R_i &= a + bV_i + \epsilon : i = 1, ..., n \\ b &= \frac{\sum_{i=1}^{n} \left(V_i - \overline{V}\right)\left(R_i - \overline{R}\right)}{\sum_{i=1}^{n} \left(V_i - \overline{V}\right)^2} \end{aligned}$$

(k) Corrected bias index (Cox and Shugart, 1991):

$$\begin{aligned} R_{i, R_i > 0} &= a + bV_i + \epsilon : i = j, ..., n \\ b &= \frac{\sum_{i=j, R_i > 0}^{n} \left(V_i - \overline{V}\right)\left(R_i - \overline{R}\right)}{\sum_{i=j, R_i > 0}^{n} \left(V_i - \overline{V}\right)^2} \end{aligned}$$

2. **Fragmentation:**

(a) Rae's electoral F (Rae, 1971):

$$F_e = 1 - \sum_{i=1}^{n} V_i^2$$

(b) Rae's parliamentary F (Rae, 1971):

$$F_p = 1 - \sum_{i=1}^{n} R_i^2$$

(c) Electoral hyper fractionalization index (Kesselman, 1966; Wildgen, 1971):

$$H_e = \exp\left(-\sum_{i=1, V_i>0}^{n} V_i \ln(V_i)\right)$$

(d) Parliamentary hyper fractionalization index (Kesselman, 1966; Wildgen, 1971):

$$H_p = \exp\left(-\sum_{i=1, R_i>0}^{n} R_i \ln(R_i)\right)$$

(e) Electoral effective number of parties (ENP) (Laakso and Taagepera, 1979; Taagepera and Shugart, 1989):

$$ENP_e = \frac{1}{\sum\limits_{i=1}^{n} V_i^2}$$

(f) Parliamentary effective number of parties (ENP) (Laakso and Taagepera, 1979; Taagepera and Shugart, 1989):

$$ENP_p = \frac{1}{\sum\limits_{i=1}^{n} R_i^2}$$

(g) Electoral largest component (LC) index (Taagepera, 1999):

$$LC_e = \frac{1}{\max\{V_i : i = 1, ..., n\}}$$

(h) Parliamentary largest component (LC) index (Taagepera, 1999):

$$LC_p = \frac{1}{\max\{R_i : i = 1, ..., n\}}$$

(i) Electoral Dunleavy-Boucek index (Dunleavy and Boucek, 2003):

$$DB_e = \frac{ENP_e + LC_e}{2}$$

(j) Parliamentary Dunleavy-Boucek index (Dunleavy and Boucek, 2003):

$$DB_p = \frac{ENP_p + LC_p}{2}$$

(k) Molinar's electoral number of parties (Molinar, 1991):

$$MNP_e = 1 + \frac{1}{\sum_{i=1}^{n} V_i^2} \frac{\left(\sum_{i=1}^{n} V_i^2\right) - V_1^2}{\sum_{i=1}^{n} V_i^2}$$

(l) Molinar's parliamentary number of parties (Molinar, 1991):

$$MNP_p = 1 + \frac{1}{\sum_{i=1}^{n} R_i^2} \frac{\left(\sum_{i=1}^{n} R_i^2\right) - R_1^2}{\sum_{i=1}^{n} R_i^2}$$

3. **Concentration:**

(a) Electoral concentration:

$$CNC_e = V_1 + V_2$$

(b) Parliamentary concentration:

$$CNC_p = R_1 + R_2$$

4. **Competitiveness:**

(a) Electoral margin of victory:

$$CMP_e = V_1 - V_2$$

(b) Parliamentary margin of victory:

$$CMP_p = R_1 - R_2$$

(c) Competition ratio (Pérez-Liñán, 2001):

$$CR = \frac{V_2\,(R_2)}{V_1\,(R_1)} * 100$$

5. **Polarization:**

(a) Polarization index (Euclidean distance) (Klingemann, 1995; Sani and Sartori, 1983):

$$POL = \sum_{i=1}^{n} V_i \sum_{k=1}^{m} \left(X_i^k - \sum_{j=1}^{n} V_j X_j^k \right)^2$$

(b) Polarization index (absolute distance):

$$POL = \sum_{i=1}^{n} \sum_{j=1}^{n} V_i V_j \sum_{k=1}^{m} \left| X_i^k - X_j^k \right|$$

6. **Volatility:**

   (a) Total electoral volatility (Bartolini and Mair, 1990; Pedersen, 1983):

$$\begin{aligned} VT_e &= \frac{1}{2}\sum_{i=1}^{n} |\nabla V_i| \\ \nabla V_i &= V_{i,t+1} - V_{i,t} \end{aligned}$$

   (b) Total parliamentary volatility (Bartolini and Mair, 1990; Pedersen, 1983):

$$\begin{aligned} VT_p &= \frac{1}{2}\sum_{i=1}^{n} |\nabla R_i| \\ \nabla R_i &= R_{i,t+1} - R_{i,t} \end{aligned}$$

   (c) Inter-blocks electoral volatility (Bartolini and Mair, 1990; Pedersen, 1983):

$$VB_e = \frac{1}{2}\sum_{j=1}^{s} \left| \sum_{i} \nabla V_{ji} \right|$$

   (d) Inter-blocks parliamentary volatility (Bartolini and Mair, 1990; Pedersen, 1983):

$$VB_p = \frac{1}{2}\sum_{j=1}^{s} \left| \sum_{i} \nabla R_{ji} \right|$$

   (e) Intra-blocks electoral volatility (Bartolini and Mair, 1990; Pedersen, 1983):

$$VIB_e = VT_e - VB_e$$

   (f) Intra-blocks parliamentary volatility (Bartolini and Mair, 1990; Pedersen, 1983):

$$VIB_p = VT_p - VB_p$$

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