Equations of the HOI code

The Six Steps to Build the Human Opportunity Index

1. Estimate a separable logistic model on whether child *i* had access to a given basic good or service as a function of his or her circumstances. For education, age was also used to predict the probability of completing each grade. The specification was chosen according to the needs of each circumstance: quadratic for years of education, logarithmic for real income, and categorical for age and the other dimensions. In all cases, the functions are linear in the parameters. From the estimation of this logistic regression, obtain coefficient estimates.

2. Given these coefficient estimates, obtain for each child in the sample the predicted probability of access to the basic good or service in consideration, $\hat{p}_i$ based on the predicted relationship, $\hat{\beta}_k$,and a vector of their circumstances x_{ki}.

$$\hat{p}_i = \frac{1}{1 + exp\left(\hat{\beta}_0 + \sum_{k=1}^{m} \hat{\beta}_k\right)}$$

3. Compute the overall coverage rate C,

$$C = \sum_{i=1}^{n} w_i \hat{p}_i$$

   where $w_i = \frac{1}{n}$ or some sampling weights.

4. Compute the Dissimilarity Index $\widehat{D}$

$$\widehat{D} = \frac{1}{2C} \sum_{i=1}^{n} w_i |\hat{p}_i - C|$$

5. Compute the penalty, $P = C * \widehat{D}$
6. Compute the $HOI = C - P$

Standard Error of the HOI

$$\widehat{\boldsymbol{V}}_{\boldsymbol{\theta}} = \frac{\boldsymbol{\partial}\widehat{\boldsymbol{\theta}}}{\boldsymbol{\partial}\boldsymbol{\beta}'}\widehat{\boldsymbol{V}}_{\boldsymbol{\beta}}\frac{\boldsymbol{\partial}\widehat{\boldsymbol{\theta}}}{\boldsymbol{\partial}\boldsymbol{\beta}}$$

$\widehat{\boldsymbol{V}}_{\boldsymbol{\beta}}$: is a consistent estimator of the matrix of variance of $\widehat{\boldsymbol{\beta}}$.

$$\frac{\boldsymbol{\partial}\widehat{\boldsymbol{\theta}}}{\boldsymbol{\partial}\boldsymbol{\beta}'} = \frac{1+\hat{\alpha}}{n}\sum_{i\epsilon L}\hat{p}_i(1-\hat{p}_i)\boldsymbol{x}_{\boldsymbol{i}}' + \frac{\hat{\alpha}}{n}\sum_{i\epsilon H}\hat{p}_i\left(1-\hat{p}_{i,n}\right)\boldsymbol{x}_{\boldsymbol{i}}'$$

$$\hat{\alpha} = \frac{\#H}{n}$$

$$n = \#H + \#L$$

$$H = \{i: \hat{p}_i \geq C\}$$

$$L = \{i: \hat{p}_i < C\}$$

Geometric version of the HOI

$$HOI_g = \left(\prod_{i=1}^{n} p_i^{w_i}\right)^{\frac{1}{\sum_{i=1}^{n} w_i}}$$

Decomposition Type 1

$$\Delta HOI_{A-B} = HOI_A - HOI_B$$

$$\Delta HOI_{A-B} = Scale_n + Distribution_n$$

$$Scale_n = C_A\left(1-\widehat{D}_B\right) - C_B\left(1-\widehat{D}_B\right)$$

$$Distribution_n = C_A\left(1-\widehat{D}_A\right) - C_A\left(1-\widehat{D}_B\right)$$

C_A: Coverage rate in A

C_B: Coverage rate in B

$\widehat{D}_A$: Dissimilarity Index in A

$\widehat{D}_B$: Dissimilarity Index in B

$Scale_n$: Scale effect (not adjusted by composition)

$Distribution_n$: Distribution effect (not adjusted by composition)

Decomposition Type 2

$$\Delta HOI_{A-B} = HOI_A - HOI_B$$

$$\Delta HOI_{A-B} = Scale + Equalization + Composition$$

$$Scale = C_A(1 - \widehat{D}_{mix}) - C_{mix}(1 - \widehat{D}_{mix})$$

$$Equalization = C_A(1 - \widehat{D}_A) - C_A(1 - \widehat{D}_{mix})$$

$$Composition = C_{mix}(1 - \widehat{D}_{mix}) - C_B(1 - \widehat{D}_B)$$

C_{mix}: Coverage rate in the mix (using the sample of A but the logit coefficients of B)

$\widehat{D}_{mix}$: Dissimilarity index in the mix (using the sample of A but the logit coefficients of B)

References

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