Professional Bettors, Odds-Arbitrage Competition, and Betting Market Equilibrium*

Brian R. Adams
Advanced Digital Information Corporation
Redmond, WA, USA

Frank W. Rusco
U.S. General Accounting Office
Washington, D.C., USA

W. David Walls†
Department of Economics
University of Calgary
Calgary, Alberta T2N 1N4
Canada

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†Corresponding author. E-mail: wdwalls@ucalgary.ca

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Abstract

A slew of empirical evidence on horse racetrack betting markets points to betting biases and market inefficiency. More recent empirical work has documented the absence of betting biases in racetrack betting markets characterized by a high volume of betting. This paper offers a competition-based model of betting behavior that is consistent with the pattern of betting biases reported in the literature. We postulate the existence of professional bettors who, being better informed and/or having different objectives than the general betting population, engage in odds arbitrage when doing so is profitable. We evaluate the case of a single odds-arbitraging bettor first in order to establish the fundamental properties of odds arbitrage. We then examine the effects of entry of professional bettors who play a Nash game in odds arbitrage; the results show that professionals' participation causes the final track odds to converge to the level implied by the horses' true win probabilities when there is a high volume of betting.

JEL Codes: D0, L1, D8

Key words: professional bettors, odds arbitrage, betting market equilibrium.

1 Introduction

Nearly all of the existing literature on horse racetrack betting points to market inefficiency. For example, the empirical results of Ali (1977), Hausch, Ziemba and Rubinstein (1981), Asch and Quandt (1982, 1987) and Asch, Malkiel and Quandt (1984) are all inconsistent with market efficiency.[1] Betting biases have been found by nearly all previous researchers with the exception of Busche and Hall (1988) and Busche (1994).[2] Betting biases in horse track gambling markets are known to be the most robust anomalous empirical regularity in studies of market efficiency (Thaler and Ziemba, 1988, p. 163).

Busche and Walls (2000) provide empirical evidence showing that market efficiency results across race tracks are systematically related to the volume of betting.[3] They argue that the linkage between market efficiency and betting volume is consistent with Smith and Walker's (1993ab) decision-cost hypothesis in that we only observe non-optimal (inefficient) betting behavior where the cost of non-optimal decisions is miniscule: "At race tracks with low bet volumes the potential gains to a professional bettor are proportionally small, so that deviations from the predictions of optimization theory—that

[1] See the recent survey article by Sauer (1998), the volume by Hausch, Lo and Ziemba (1994) and the references therein for a more complete listing of papers relating to the efficiency of racetrack betting markets.

[2] One particular betting bias—the favorite-longshot bias—where longshot horses are overbet relative to the favorites has been found by virtually all researchers. However, there is also some empirical evidence of favorites being overbet relative to longshots. The model developed in this paper offers an explanation of why betting biases of either type are not observed at horse tracks with a high volume of betting. Vaughan-Williams and Paton (1998) address the differences between the different types of betting biases.

[3] Within a given race, empirical evidence also shows that the volume of betting causes the track odds to converge toward the odds implied by optimal betting (Walls and Busche, 1996).

returns be equalized across horses—reflect the risk preferences of recreational bettors" (p. 487). In their analysis they postulate that professional bettors are attracted to the potential profits associated with large betting volumes, and that competition for these potential profits accounts for the market efficiency at these tracks.

In this paper we develop a model in which well-informed professional bettors engage in arbitrage when faced with sufficiently profitable betting opportunities. Our purpose is twofold. First, we explore the effects on observed final track odds of professional bettors under the assumption that the professionals are either better informed than the general betting public, or that the two types of participants have different preferences, and therefore, that there are opportunities for arbitrage to occur.[4] Second, we identify conditions under which races with initially biased odds present profitable opportunities for professionals.

We will demonstrate that both the conditions required to support the existence of professional bettors and the effect such bettors have on odds are consistent with the market inefficiency results reported by the authors cited above. The theoretical analysis in this paper proceeds by first examining the analytically tractable case of a single bettable horse and a single odds-arbitraging bettor to establish the fundamental properties of odds arbitrage. We then extend this simple model in two ways that are analytically intractable and that we examine through simulations: First, we allow for

[4]We do not attempt to explain the existence of favorite-longshot biases in some betting markets and longshot-favorite biases in others. Rather, we take these biases as given in order to isolate the impact of professionals on odds in races that otherwise would have such biases.

the possibility that the single bettor may be faced with profitable bets on more than one horse. Second, we model competition between multiple professional bettors and examine the effects of entry of professional bettors who play a Nash game in odds arbitrage. The findings show that the track odds converge toward the odds implied by horses' true win probabilities in the number of competitors and entry is attracted by potential profits that are proportional to the aggregate volume of betting.

2 A Single Professional Bettor with One Underbet Horse

Consider the highly stylized case of a single professional bettor who faces odds created by a large pool of casual (non-professional) bettors. The casual bettors are assumed to behave naively in the sense that they place bets based on their preferences and information but these bets are not necessarily consistent with economic efficiency and, having bet, they do not reassess and/or place new bets in response to subsequent changes in the odds.[5] The professional bettor is assumed to know the true win probabilities of each horse. We also assume that the professional bettor is risk neutral in the sense of maximizing expected profits given the initial odds. Finally, we assume that the professional bettor observes the final odds set by the casual bettors prior to placing bets.[6]

[5]In other words, consistent with the empirical literature cited above, we allow for the possibility that the betting equilibrium in the absence of professional bettors has systematically biased odds. We do not speculate about how such biased odds actually arise and persist because our goal is to examine how and when professional bettors might respond to such initially biased odds.

[6]With an added layer of complexity, this last assumption could be relaxed and the professional bettor could make bets based on the expectation of the final odds implied

The initial track odds are determined by the total amount bet (the win pool), the amount bet on individual horses, and the proportion of the win pool which is extracted by the track (the so-called track take). Let X_i be the number of dollars bet on horse i, $W = \sum_{i=1}^{H} X_i$ be the win pool, and β be the proportion $(1 - T)$ where T is the track take. The initial odds vector facing the bettor is

$$\mathbf{p} = (p_1, p_2, ..., p_H) \tag{1}$$

where $p_i = W\beta/X_i$. Potentially profitable arbitrage is possible for any horse for which initial odds are such that $p_i > 1/\pi_i$ where π_i is the bettor's estimate of the probability that horse i wins the race.[7]

Suppose the initial odds vector presents the bettor with an arbitrage opportunity for horse k.[8] Then the bettor's problem is to choose the optimal bet on horse k, b_k, to maximize

$$b_k \left(\pi_k \beta \left(\frac{W + b_k}{X_k + b_k} \right) - 1 \right) \tag{2}$$

subject to the constraint that b_k is non-negative. The first term in the brackets is simply the expected return for every dollar bet on horse k while the second term reflects the cost.[9] The objective function is strictly con-

by the casual bettors. Given the assumption of risk neutrality, this would not change the results qualitatively.

[7]We are restricting our analysis to simple win bets in which bettors place their money on a horse to win. From the perspective of a risk-neutral professional bettor, complex bets—exactas, quinellas, trifectas, etc.—are merely a computational problem. Analysis of complex bets adds greatly to the algebraic complexity of the analysis without adding anything fundamental to our understanding of the problem.

[8]We assume in this section that only one horse is sufficiently underbet by the amateur pool to allow arbitrage. We also assume that the bettor does not cause any other horses to become profitable betting prospects in the course of adding to the win pool by betting on the underbet horse. These assumptions are relaxed below after the basic comparative statics are derived for this simple case.

[9]We assume that the costs of physically placing the bet are small enough to ignore.

cave in b_k so if a non–negative bet is initially feasible, the optimal bet is characterized by the following first order condition:

$$b_k^* = X_k \left(\sqrt{1 + \frac{\pi_k}{1-\pi_k\beta}\left(\frac{W\beta}{X_k} - \frac{1}{\pi_k}\right)} - 1 \right) \tag{3}$$

The first order condition has an interesting interpretation: Holding the initial track odds constant, the optimal bet for the professional is proportional to the amount of money already bet on the horse. In other words, holding the initial track odds constant, an increase in the pool size causes a proportional increase in the size of the optimal bet. A related comparative static is that the optimal bet on a horse is increasing in the bettor's estimate of the probability that the horse wins (again, holding the initial track odds constant).[10] Finally, the optimal bet is decreasing in the size of the track take.[11]

Another way of interpreting the above comparative static results is in terms of the proportion of professional bets among all bets.[12] Rearranging

The costs of participating in the betting, which include forming an estimate of winning probabilities, are discussed below in the context of competition among professional bettors.

[10]The comparative static is

$$\begin{aligned}\frac{\partial b_k}{\partial \pi_k} &= \frac{X_k}{2}\left[1 + \frac{\pi_k}{1-\pi_k\beta}\left(\frac{W\beta}{X_k} - \frac{1}{\pi_k}\right)\right]^{-1/2} \\ &\times \left[\frac{1}{(1-\pi_k\beta)^2}\left(\frac{W\beta}{X_k} - \frac{1}{\pi_k}\right) + \frac{1}{\pi_k(1-\pi_k\beta)}\right] > 0\end{aligned}$$

[11]The comparative static is

$$\begin{aligned}\frac{\partial b_k}{\partial T} &= -\frac{\partial b_k}{\partial \beta} = -\frac{X_k}{2}\left[1 + \frac{\pi_k}{1-\pi_k\beta}\left(\frac{W\beta}{X_k} - \frac{1}{\pi_k}\right)\right]^{-1/2} \\ &\times \left[\left(\frac{\pi_k}{1-\pi_k\beta}\right)\frac{W}{X_k} + \left(\frac{\pi_k}{1-\pi_k\beta}\right)^2\left(\frac{W\beta}{X_k} - \frac{1}{\pi_k}\right)\right] < 0\end{aligned}$$

[12]We owe this point and its development to an anonymous referee.

equation (3) by bringing the leading X_k term to the left-hand side, it can be seen clearly that the optimal bet on a particular horse k can be interpreted in terms of the proportion of informed bets on that horse. Rearranging the comparative static results (derived in footnotes 10 and 11) shows that they can also be interpreted in terms of the proportion of informed bets. To simplify the exposition in the remainder of the paper we will discuss the comparative statics as absolute numbers since normalizing them to proportions does not substantively alter the results.

We have assumed that the bettor knows the track odds and the pool size prior to choosing and placing a bet. As bets take real time to place and as there may be many actual profitable bets to place in a given race, the bettor may have to place bets before the final track odds have been set and thus may err ex post. This problem may not be severe if the track odds remain stable as amateurs place their bets (which would be likely if amateurs all had similar beliefs and preferences) and if the final pool size is predictable with low variance. Before turning to the question of competition among professional bettors we examine the slightly more complicated case in which there are arbitrage opportunities for more than one horse and also allow for the possibility that horses which initially are unprofitable become so as a result of the bettor's actions.

3 A Single Professional Bettor with Multiple Underbet Horses

Consider the partition of the initial odds in which the professional bettor has ranked the horses in descending order according to the difference between the initial odds net of the track take and the bettor's estimate of the true odds that the horse wins. In is case the highest expected return per dollar bet will be achieved on horse number one and the lowest on the lowest ranked horse as follows:

$$p_1^* - \frac{1}{\pi_1} \geq p_2^* - \frac{1}{\pi_2} \geq ... \geq p_H^* - \frac{1}{\pi_H} \tag{4}$$

where as before $p_i = \frac{W\beta}{X_i}$. The win pool increases as the professional bets on the profitable horses. As a result, some horses which are initially unprofitable may become attractive as the odds get better with the rising win pool. The professional's problem is to choose the bets on each horse to maximize expected profits or to maximize

$$\sum_{i=1}^{H} b_i \left[\pi_i \beta \left(\frac{W + \sum_{j \neq i} b_j + b_i}{X_i + b_i} \right) - 1 \right] \tag{5}$$

subject to the constraints $b_i \geq 0$ for $i = 1, \ldots, H$.

The optimal bet vector is characterized by the following conditions for $k = 1, \ldots, H$:

$$\begin{aligned} (\pi_k \beta - 1 + C)\, b_k^2 \quad &+ \quad 2X_k \left(\pi_k \beta - 1 + C\right) b_k \\ &+ \quad \left[\pi_k \beta \left(W + \sum_{j \neq k} b_j \right) X_k - (1 - C) X_k^2 \right] \leq 0 \end{aligned} \tag{6}$$

$b_k \geq 0$ with complementary slackness, and where $C = \beta \sum_{j \neq k} \frac{\pi_j b_j}{X_j + b_j}$. These conditions can be simplified as follows:

$$\begin{aligned} b_i \quad = \quad X_i &\left\{ \left[1 + \frac{\pi_i}{1 - \pi_i \beta - \beta \sum_{k \neq i} \frac{\pi_k b_k}{X_k + b_k}} \right. \right. \\ &\times \left. \left. \left(\frac{\beta \left(W + \sum_{k \neq i} b_k \right)}{X_i} - \frac{1 - \beta \sum_{k \neq i} \frac{\pi_k b_k}{X_k + b_k}}{\pi_i} \right) \right]^{1/2} - 1 \right\} \end{aligned} \tag{7}$$

for all i such that $b_i > 0$.[13] Otherwise, $b_k = 0$.

There are two significant differences between this solution and the case in which only a single horse is bet. First, each dollar bet on a horse increases the expected payoff on all other horses bet. This effect is captured in the term $\beta \sum_{k \neq i} \pi_k b_k/(X_k + b_k)$. When bets on more than one horse are allowed, marginal horses may receive positive bets due to the favorable effects on the odds of other horses being bet. The second difference is that the optimal bet on horse i is no longer proportional to the amount of money initially bet on horse i holding the initial odds constant.

We cannot analytically solve the equations listed above for the optimal bet vector; however, we can solve numerically for the optimal bet vector given the true win probabilities and the initial amount bet on each horse.[14] In the simulation analysis discussed below, we have solved for and reported the results of numerous alternative parameterizations of the model to show how the optimal bet vector varies in response to changes in the initial bet fractions, changes in the aggregate volume of betting, changes in the track take, and changes in the win probabilities. To simplify our simulations and to make the results more transparent, we have only reported simulations of 3-horse races. Although the simulations reported here contain only 3 horses, the analysis is fully general because any qualitative result that can be generated with a larger number of horses—say 7 or 11—can also be generated in a 3 horse race by appropriately scaling the win probabilities

[13] This will be the case for any horse for which the initial odds permit profitable bets and for any marginal horses for which the odds become favorable as bets are placed on inframarginal horses.

[14] The source code for the computer program that solves for the optimal bet vector is available from the the author's web page.

and the initial bet fractions on each horse.

We first analyze the case in which there is a single horse that is underbet to demonstrate that the optimization program provides results similar to what was derived analytically in the previous section. For simplicity, we assume that there are three horses with true win probabilities of 0.6, 0.3, and 0.1, the initial amount bet on each horse is 1000, and the track take is 18%.[15] We calculate the optimal bet on each horse, then add 10 to the initial amount bet on each horse and recalculate the optimal bet on horse 1. By doing this, we show how the optimal bet on horse 1 varied in response to the aggregate bet volume holding the initial odds constant. Figure 1 plots the locus of optimal bets on horse 1 as the initial bet volume varies from 3000 to 6000 in the manner described above. It is clear from the simulation that the optimal bet is linear in the aggregate bet volume holding initial odds constant and this shows that our simulation model is consistent with the analytical model depicted in Section 2 above.[16] In Figure 1 we also plot the expected profit to be earned from placing the corresponding optimal bet.[17] Larger win pools correspond to larger optimal bets, and this translates into larger expected profits for the professional bettor.[18]

In Figure 2 we examine a case in which two horses are initially under-

[15]The track take T at nearly all tracks is 18% of the win pool; the notable exceptions are Hong Kong and Japan where the track takes are 17.5% and 26%, respectively. Tracks also extract a fee from winners—known as breakage—by rounding winnings down to the nearest nickel or dime. We do not explicitly model the effect of breakage, although it would appear to affect favorites more than longshots.

[16]When only the ith horse is bet on, $b_k = 0$ for all $k \neq i$, so the first order condition in equation 7 is identical to the first order condition in equation 3.

[17]Since there is only one profitable horse in this simulation, the expected profit can be calculated by evaluating equation 2.

[18]Larger expected profits in turn would be expected to attract entry, and we examine the implications of this point in Section 4 below.

bet. The true win probabilities and track take are as given in the previous example, but the initial amounts bet on horses 1, 2, and 3, are 1000, 1000, and 10, respectively. We solve for the optimal bet vector for this case, then let the initial amount bet on horse 2—the only horse that is not a profitable bet—increase in increments of 1, and then recalculate the optimal bet vector. The figure reveals that the optimal bets on horses 1 and 3 increase linearly in the amount bet on the unprofitable horse. This illustrates how the optimal bets increase as the initial odds vector is increasingly out of whack with the inverse probability vector.

Finally, we examine how changes in the track take and changes in the win probabilities affect the optimal bet vector to verify that these comparative statics are the same as derived for the one-bettor-one-horse case in Section 2 above. To analyze how the track take affects the optimal bet vector, we begin with the same win probabilities listed above and 1000 bet on each horse. We then let the track take vary and calculate the optimal bet vector for each alternative value of the track take. Figure 3 plots the optimal bet vector as a function of the track take. For very high levels of the track take—greater than or equal to 0.45—no bets are profitable. However, as the track take decreases, horse 1 becomes a profitable bet. As the track take continues to fall, horse 2 also becomes a profitable bet and the slope of the optimal bet on horse 1 increases because bets on horse 2 make bets on horse 1 more profitable.[19] To analyze how the win probability affects the

[19] There is a convergent feedback mechanism between bets on profitable horses which can be intuitively understood to operate as follows: A one dollar bet on horse 1 improves the odds for horse 3, inducing a bet on horse 3 (of less than a dollar) which improves the odds for horse 1, inducing a further bet on horse 1, and so on. As a result, when there is only one bettable horse the effect of changes in the track take on bets is linear, but when

optimal bet vector, we began with 1000 bet on each horse and we solved for the optimal bet vector as the win probability on horse 1 varied from 0.01 to 0.99 where the win probabilities on horses 2 and 3 were set to 3/4 and 1/4 of the remaining probability; i.e. the odds ratio for horses 2 and 3 was fixed at 3. Figure 4 shows the optimal bet vector for alternative win probabilities on horse 1. The figure confirms our earlier comparative static (derived in fn. 8) that the optimal bet is increasing in the win probability.

The effects on the final track odds of the professional bettor's participation are not sensitive to the aggregate volume of betting holding the initial odds constant. Increasing the win pool by proportionately increasing the initial bets on all horses causes the optimal bets to be scaled up accordingly. Thus, with a single professional bettor the final track odds cannot be driven very close to the inverse win probabilities. However, as the volume of betting rises so do expected profits and this will attract entry of professional bettors. In the following section we analyze a model of Nash competition in odds arbitrage among professional bettors.

4 Odds-Arbitrage Competition among Professional Bettors

Without barriers to entry, risk-neutral professional bettors will be expected to enter as long as expected revenue covers the cost of participation. If all professionals have access to the same information then their estimates of the true odds for any given race should be identical.[20] Given that placing

more than one horse can be bet the effects are non-linear.

[20]This is true under the assumption that all bettors use the best available predicting technology. The authors of this paper are aware of two successful professional bettors in

bets takes place in real time, it is also reasonable to assume that there is an optimal time to begin placing bets and an optimal order of betting when multiple bets are placed by each bettor. In the absence of any evidence to suggest otherwise, we therefore assume that the betting strategies of every professional bettor are identical and we will look for an equilibrium.[21] The natural extension to the single bettor model is to look at Cournot strategies for betting in which each of several bettors chooses the number of dollars to bet on each horse where bets are placed simultaneously.

Consider first the case in which n risk-neutral professional bettors bet on only one horse. Let b_{ij} be bettor j's bet on horse i and assume that horse i is the only profitable horse to bet on. Then bettor j's problem is to choose b_{ij} to

$$\max \; b_{ij}\left[\pi_i\beta\left(\frac{W+b_{ij}+\sum_{k\neq j}b_{ik}}{X_i+b_{ij}+\sum_{k\neq j}b_{ik}}\right)-1\right] \tag{8}$$

subject to the constraints that $b_{ij}\geq 0$. Solving the implied best response function for strictly positive b_{ij} yields

$$b_{ij}=\left(X_i+\sum_{k\neq j}b_{ik}\right)\left(\sqrt{\frac{\pi_i\beta}{1-\pi_i\beta}}\times\sqrt{\frac{W+\sum_{k\neq j}b_{ik}}{X_i+\sum_{k\neq j}b_{ik}}-1}-1\right) \tag{9}$$

In the Nash equilibrium, all bettors choose the same strategy so $b_{ij}=b_{ik}=b_i$ and the equilibrium bet must satisfy the following equation:

$$b_i-\left(X_i+(n-1)\,b_i\right)\left(\sqrt{\frac{\pi_i\beta}{1-\pi_i\beta}}\times\sqrt{\frac{W+(n-1)b_i}{X_i+(n-1)b_i}-1}-1\right)=0 \tag{10}$$

Hong Kong who use similar models to predict the true odds.

[21]There is potentially a strategic advantage to being the first professional to place a bet. By placing the first bet, the bettor changes the odds which opponents face when they make their own betting decisions. As a result, the first bettor would be able to place a higher bet and earn higher expected profits. Hence, there is a clear incentive to be a first mover. This is tempered to some extent by the fact that early betting is more risky from the perspective of estimating the final track odds and the pool size.

There is no analytical solution for b_i which easily yields comparative statics. However, as was the case with a single bettor, the equilibrium bid is increasing in W, π and β, holding W/X_i constant.[22] The equilibrium bid is also decreasing in n, the number of bettors, while aggregate professional betting nb_i is increasing in n. This last result is of primary interest because it means that the final track odds will get closer to the true odds as the number of professionals who can be supported at a track increases.[23] The number of professional bettors participating will depend on the cost of participating, the size of the pool and the track take, and on the distribution of initial track odds. The cost of participating depends on the number of horses and riders for which the bettors have to collect and maintain data and the speed at which bets can be placed.

Let C_j be the cost of participating for bettor j in any given race.[24] We have not characterized the Nash betting equilibrium for the case of n bettors and H horses.[25] However, assuming such an equilibrium exists we can characterize the long-run equilibrium in which the number of bettors is determined by a zero-profit constraint. Let b_i^n be the equilibrium bet,

[22]For brevity we do not report here the simulation results showing that these comparative statics hold. Instead we focus on the more interesting comparative statics relating to the number of professional bettors.

[23]This is analogous to the standard result that the Cournot equilibrium approaches the competitive equilibrium as the number of competitors increases. See, for example, Novshek (1980).

[24]Most of the participation costs will be incurred as fixed costs in data collection and in estimating the odds. These costs can be thought of as amortized over the betting lifetime of the player. There are some marginal costs involved with actually placing bets which may be empirically important. We have assumed that these costs are nonexistent for the purposes of developing the model because including them does not change the qualitative results while it does complicate the algebra.

[25]The equilibrium for this case is characterized by H expressions involving third degree polynomials.

for each of n professional bettors, on horse i when there are H horses. Entry will occur until an additional bettor would cause profits to be negative in expectation in a representative race. This can be summarized in the following inequality:

$$\sum_{i=1}^{H} \pi_i \beta b_i^n \left(\frac{W + \sum_{k=1}^{H} n b_k^n}{X_i + n b_i^n} \right) - b_i^n \geq C \geq \sum_{i=1}^{H} \pi_i \beta b_i^{n+1} \left(\frac{W + \sum_{k=1}^{H} (n+1) b_k^{n+1}}{X_i + (n+1) b_i^{n+1}} \right) - b_i^{n+1} \tag{11}$$

The far left hand side of the inequality is a representative professional bettor's expected return in equilibrium when there are a total of n professionals playing and given the initial size of the betting pool, W, the initial amounts bet on each of H horses, the X_i, and the probabilities of each horse winning, the π_i. The far right hand side of the inequality is identical except for the addition of the $n+1$ professional player. The middle term in the inequality, C, is the amortized costs of playing for the representative professional. It is clear that anything that increases the return of a representative player above the cost of entry will attract more players until.

We solve numerically for the optimal Nash equilibrium bets when n professional bettors compete in betting on one horse.[26] Because the comparative statics regarding the track take, win probabilities, volume of betting, etc., are the same as were shown earlier, we focus here on how the final track odds are affected by changes in variables making professional betting more profitable. We have already shown that professionals' bets are increasing in the probability that a horse wins, and that their bets are also increasing and

[26]The source code for the program that computes the Nash equilibrium bets for multiple bettors is available on the author's web page.

proportional to the size of the win pool, holding the initial odds constant. The potential profits from a large win pool would lead to entry of professional bettors and we would expect this to cause the odds to move closer to the inverse probabilities. We examine this hypothesis in detail below.

First, we examine the effect of the number of bettors on the professionals' individual and aggregate betting behavior. We set the win probability to 0.3, the track take set to 0.18, the initial amount bet on the horse 1000, and the initial aggregate volume of betting 10000. We then varied the number of bettors from 2 to 30 and calculated the individual bets (b) and the aggregate volume of professional betting ($b * n$). These quantities versus the number of bettors are plotted in Figure 5. As the number of bettors increases, the individual bets decrease, due to the nature of Cournot competition. We expect that as the aggregate volume of professional betting increases the final track odds will be driven closer to the inverse win probability.

We next examined the effect of the initial aggregate volume of betting, holding initial odds constant, on the final track odds for varying numbers of professional bettors competing in odds-arbitrage on a horse with a win probability 0.3.[27] Figure 6 shows the final track odds after the professionals have placed their optimal bets for $n = 2, 3, 4$, and 5 bettors. It is apparent from the figure that, with a fixed number of professional bettors, the final odds vector is invariant to an increase in the volume of betting holding initial odds constant.

We next examined the expected profits associated with different sized

[27] The initial amount bet on the horse is 100; the initial volume of betting is 1000; and the track take is 0.18. The initial volumes bet on the horse and the initial total volume of betting are scaled up by the same factor, so that the initial odds remain at $\frac{1000 \times 0.82}{100}$.

win pools and numbers of professional bettors, holding the initial odds constant.[28] Figure 7 shows the expected profit per professional bettor for different initial win pools. The expected profit per bettor is increasing in the size of the win pool for a given number of competitors. The increase in expected profits comes entirely from the increase in the size of bets required to drive the odds to the profit-maximizing level.[29] In as much as profits increase in the size of the win pool, more professional bettors can be expected to enter as the size of the initial win pool grows.

We have already demonstrated that, holding the initial odds and the number of professionals constant, the final odds vector is invariant to the volume of betting. We also showed that an increase in the initial odds causes expected profits per professional bettor to rise and therefore would be expected to attract more professional bettors. Finally, we showed that the final odds for the underbet horse fall as the number of professional bettors increases. Figure 8 combines all of this information into a single graph that shows how final track odds for the underbet (overbet) horse fall (rise) toward the zero-profit odds. The final track odds converge rapidly in the number of professional bettors towards the inverse win probability of 3.33.[30] Analytically, it is straightforward to show that a single professional bettor would bet so that the final track odds would be about 5.13.[31] With

[28]Using the same parameter values as discussed in footnote 25, the number of professional bettors is increased from two to five to examine how expected profits per bettor falls as entry occurs.

[29]Recall that the final track odds are invariant to the size of the win pool, holding the number of professional bettors constant.

[30]The final track odds approach but can never reach the inverse win probabilities due to the track take T.

[31]From equation (3) above, a single bettor's optimal bet would be $714, and this would cause the final track odds to become 5.13.

2 Cournot competitors the simulated final odds are about 4.25, and they fall to 3.8 with the entry of another 2 competitors. Even a small number of competitors can have a dramatic impact in driving the final track odds towards the level implied by the true win probabilities, with the divergence depending on the level of the track take parameter β.

5 Conclusion

In this paper we have presented a competition-based model of betting that is consistent with the empirical regularity of market inefficiency at horse tracks with low volumes of betting and the lack of such a bias at tracks with high volumes of betting. We modeled the effects of profit-maximizing professional bettors and found that the participation of professionals caused the final track odds to converge toward the levels implied by the true win probabilities when expected profits were sufficiently large to attract entry. Because the potential profits of bettors are increasing in the volume of betting, professional bettors' participation would be expected to drive the odds toward the levels implied by the horses' win probabilities at tracks with large win pools because this increases the proportion of informed bets. And a higher proportion of informed bets limits the divergence of the actual odds from the correct/true odds, though the degree of divergence would nonetheless be dependent on the track take.

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Figure 1: Optimal Bet and Expected Profit with 1 Profitable Horse

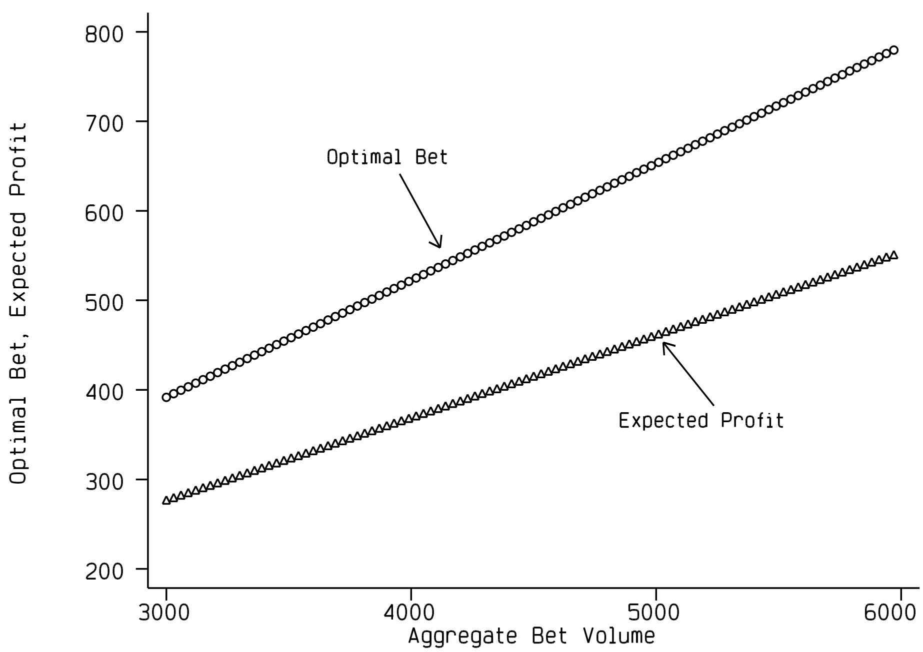

Figure 2: Optimal Bets with 2 Profitable Horses

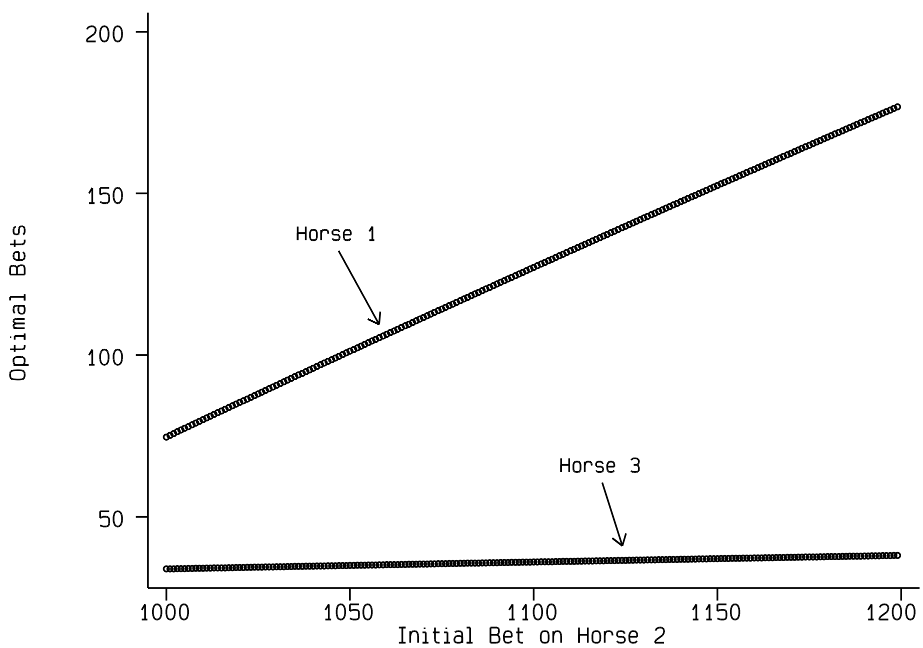

Figure 3: Optimal Bets versus Track Take

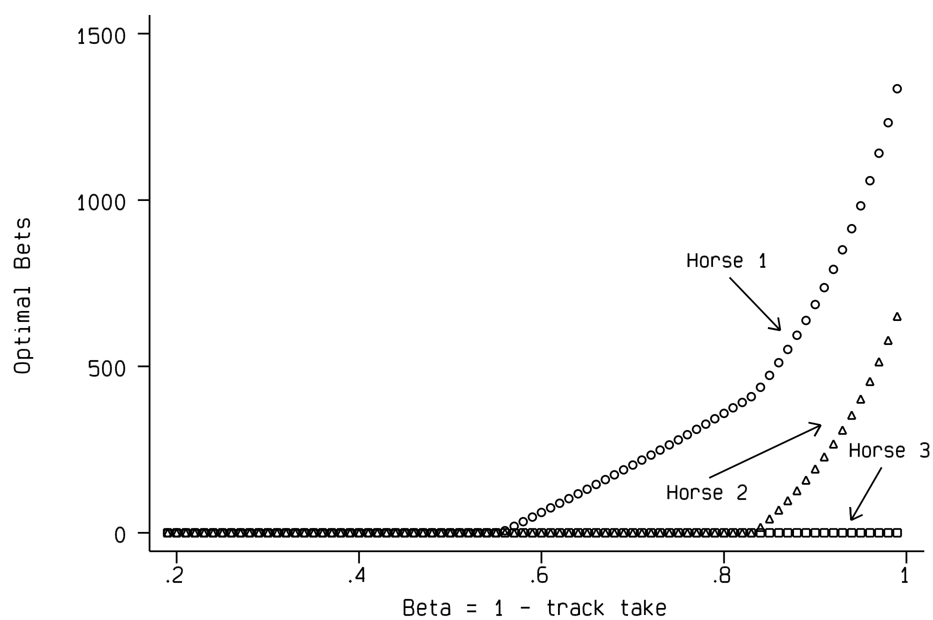

Figure 4: Optimal Bets versus Win Probability

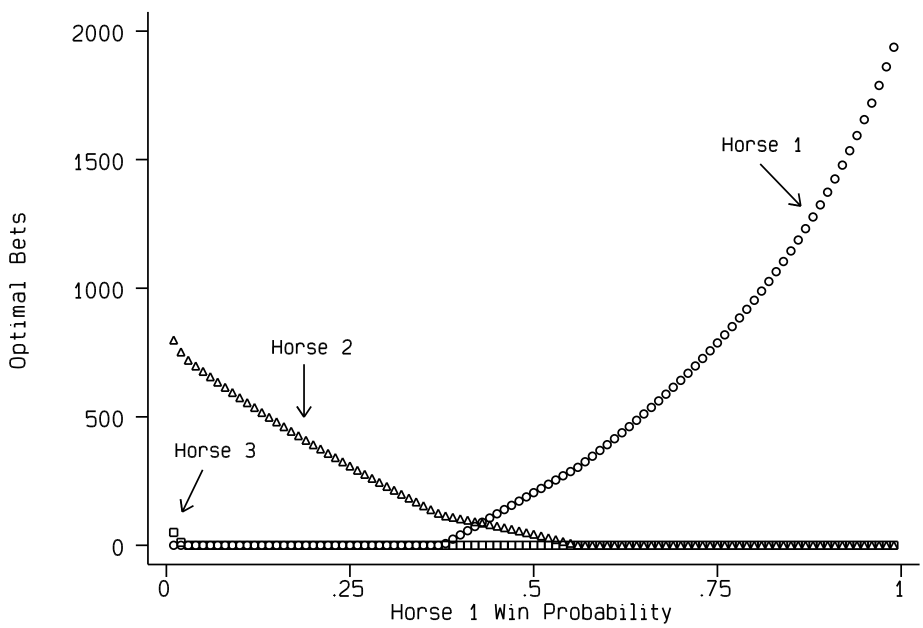

Figure 5: Number of Bettors versus Individual and Aggregate Bet Volume

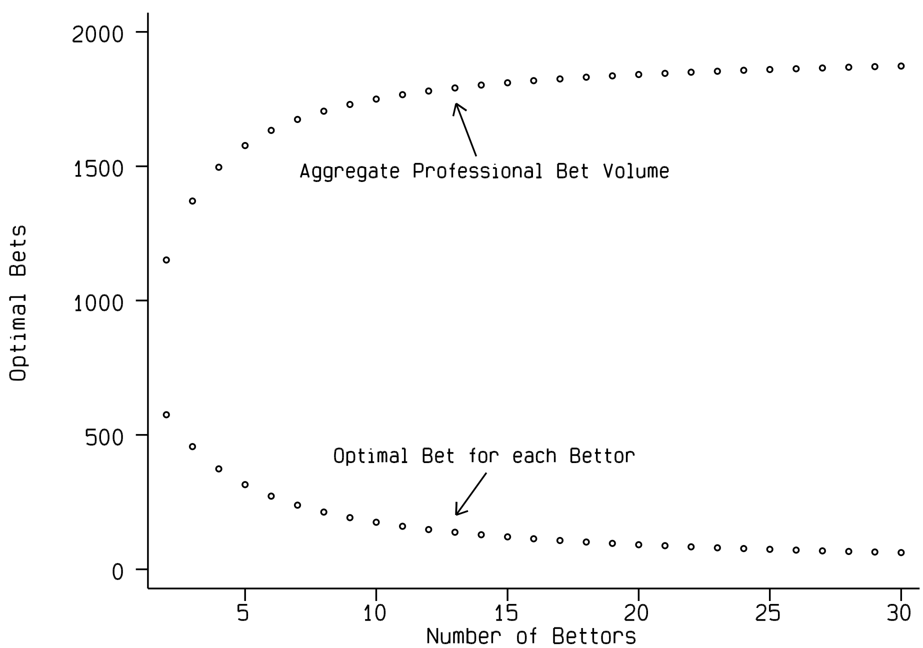

Figure 6: Effects of Number of Professionals and Bet Volume on Final Track Odds

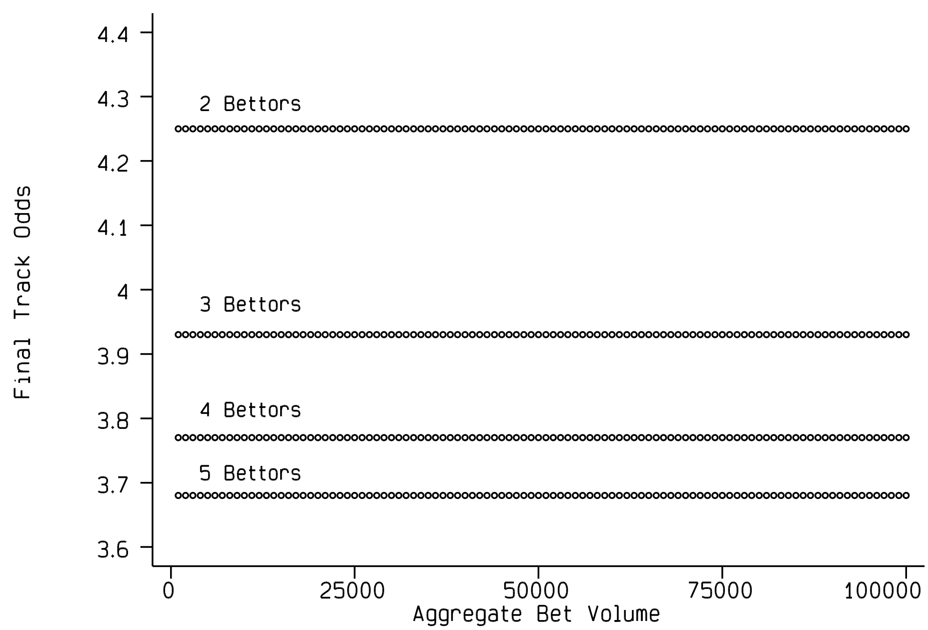

Figure 7: Effects of Number of Professionals and Bet Volume on Expected Profit

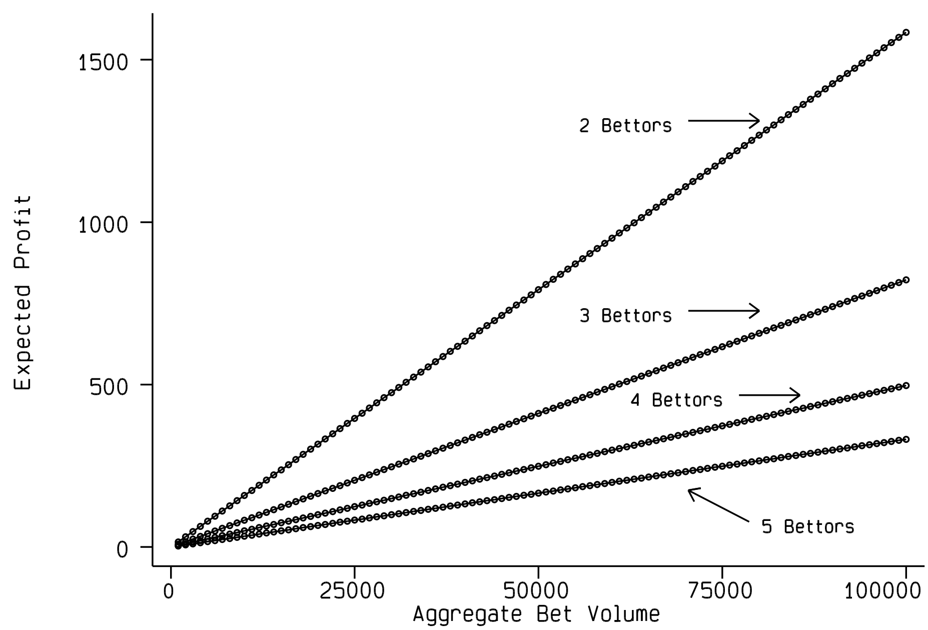

Figure 8: Effects of Number of Professional Bettors on Final Track Odds