Measuring Richness

Andreas Peichl
Center for Public Economics
University of Cologne
Cologne, Germany
a.peichl@uni-koeln.de
www.cpe-cologne.de

Thilo Schaefer
Center for Public Economics
University of Cologne
Cologne, Germany
schaefer@fifo-koeln.de
www.cpe-cologne.de

Abstract In this paper, we describe `richness`, a Stata program for the calculation of richness indices. Peichl, Schaefer and Scheicher (2006) propose a new class of richness measures to contribute to the debate how to deal with the financing problems that European welfare states face as a result of global economic competition. In contrast to the often used headcount, these new measures are sensitive to changes in rich person's income. This allows for a more sophisticated analysis of richness, namely the question whether the gap between rich and poor is widening. We propose to use our new measures in addition to the headcount index for a more comprehensive analysis of richness.

Keywords: richness, affluence, poverty

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1 Introduction

The financing problems of the European welfare states and the increasing pressure of global economic competition have given rise to a debate wether the gap between rich and poor is widening. It is widely believed that the rich are getting richer and the poor are getting poorer. Many proposals for reforming the tax and transfer system are critisised for redistributing from the poor to the rich. Given this debate, appropriate measures of poverty and richness are of key importance for an empirical analysis. Several income poverty indices have been developed in the long tradition of the literature on measuring poverty (see for example Zheng (1997) or Chakravarty and Muliere (2004) for recent surveys). Furthermore, there exist a lot of Stata programs for measuring poverty like, for example, `poverty` or `povdeco`. Measuring income richness is a less considered field. As far as we know, empirical studies mainly use the headcount ratio to measure income richness. Studies on income richness in Germany are for example Krause and Wagner (1997) or Merz (2004). There is a series of recent papers using the income share of the top percentile as an indicator of richness (see Atkinson (2005), Dell (2005), Piketty (2005) and Saez (2005)).

The headcount index which is often used to measure richness is not sensitive to changes in rich person's incomes. Therefore, we recently defined a new class of richness indices analogously to well-known measures of poverty (see Peichl, Schaefer and Scheicher (2006)). This approach is more sophisticated because it also takes the dimension of changes and not only the number of people beyond a given richness line into account. Applying our new measures to empirical data[1] reveals that our new measures change the results of a pure headcount analysis distinctively. Therefore, we propose to use the new measures in addition to the headcount index for a more comprehensive analysis of richness. In this paper, we describe `richness`, a Stata program for the calculation of these richness indices.

The setup of the paper is organised as follows: In section 2 we describe our new class of richness measures. Section 3 describes the new Stata program, while in section 4 several examples demonstrate its usage. Section 5 concludes.

2 New measures of richness

Little research has been done on the measurement of richness. The first challenge is to define an affluence or richness line. For an overview of the sparse literature see Medeiros (2006). Peichl, Schaefer and Scheicher (2006) define the richness line ρ analogously to the poverty line as a multiplier of a parameter of the income distribution (e.g. 200% of median or mean income).

In most studies on income richness the proportion of rich persons is used as a measure of richness:

$$R_{HC}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}_{x_i>\rho} = \frac{r}{n},$$

where $r = \#\{i|x_i > \rho,\, i = 1, 2, \ldots, n\}$ is the number of rich persons (with an income above the richness line ρ). The definition of R_{HC} resembles that of the headcount ratio. But, if we want to compare, for example, different tax and transfer reform scenarios, this is not a satisfying definition of richness: If nobody changes his or her status (rich or non-rich), neither a change in a rich person's income nor a transfer between rich persons will change this index.

Medeiros (2006) proposes to define measures of richness in analogy to the FGT indices of poverty (see Foster, Greer and Thorbecke (1984)). However, Medeiros' proposed FGT indices of richness are not standardised, which would be appropriate for the headcount but not for the FGT indices. Therefore, we propose a standardised approach of richness measures bounded to

[1] See Peichl, Schaefer and Scheicher (2006) for an analysis of longitudinal German data, a cross country comparison of the EU-15 and the effects of introducing possible flat tax reforms using the new richness measures.

the unit interval (see Peichl, Schaefer and Scheicher (2006)).

There is an obvious difference between the income classes of the poor and of the rich: The incomes of the poor are bounded by 0 and π, but the incomes of the rich only have a lower bound ρ. Therefore, we transform the incomes of the rich, relative to the richness line, $\frac{x_i}{\rho}$, to the unit interval by a strictly increasing transformation function f. We use strictly increasing transformations, because the indices of richness should be sensitive to higher incomes, and assume $\lim_{y\to\infty} f(y) = 1$.

In poverty measurement, the focus axiom is generally accepted, i.e. a poverty index is not modified if a non-poor person's income is changed and this person does not change his or her status. This can be applied analogously to the measurement of richness: A person with an income not higher than ρ should not influence the measure of richness, $f(\frac{x_i}{\rho}) = 0$, for $\frac{x_i}{\rho} \leq 1$. Examples for $f(y)$ are the functions $f(y) = 1 - \frac{1}{y}$ or $f(y) = 1 - e^{1-y}$, for $y > 1$, and $f(y) = 0$ elsewhere.

A second important difference between the measurement of poverty and richness concerns the transfer axiom (c.f. Chakravarty and Muliere (2004)). In poverty measurement decreasing the income of a very poor person shall have a larger effect than increasing the income of a relatively richer poor (minimal transfer axiom). Because of diminishing marginal utility, we define our richness index to be less sensitive to changes of very high incomes. The relative incomes $\frac{x_i}{\rho}$ have then to be transformed by a function which restriction to high incomes is concave.

Taking all this into account, we define measures of richness R by

$$R(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} v\left(f\left(\frac{x_i}{\rho}\right)\right),$$

where $f : R_+ \to [0,1]$ is strictly increasing on $(1,\infty)$, $v : [0,1] \to R_+$ (in particular $[0,1]$) is increasing and $v(f(\cdot))$ is at last concave, that is, has a concave restriction on $[a,\infty[$ for some $a \in R_+$.

If we use $f(y) := 1 - \frac{1}{y}$ for $y > 1$ and $v(y) := y^\alpha$, with $\alpha > 0$, we obtain a richness index R_α,

$$R_\alpha(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n}\left(1 - \frac{1}{\left(\frac{x_i}{\rho}\right)\mathbf{1}_{x_i>\rho}}\right)^\alpha = \frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{x_i-\rho}{x_i}\right)_+\right)^\alpha. \tag{1}$$

This richness index resembles the FGT index of poverty. In this case the richness index decreases by a regressive transfer between a rich and a very rich person. For $0 < \alpha < 1$, $(\frac{x-\rho}{x})^\alpha$ is concave on (ρ,∞) and for $\alpha > 1$, $(\frac{x-\rho}{x})^\alpha$ is at last, i.e. on $((\alpha+1)\rho/2,\infty)$, concave and by this, the second postulate that distinguishes richness from poverty measurement is fulfilled.

3 The program richness

3.1 Syntax

`richness` *varlist* [*if*] [*in*] [*weight*] [, <u>rl</u>ine(*rl*) | <u>rv</u>al(*rv*) <u>rn</u>umber(*rn*) <u>rlf</u>ix]

`aweights` and `fweights` are allowed; see `help weights`.

3.2 Description

`richness` computes the following richness measures based on the (income) distribution for each *varname* of *varlist*:

- *headcount ratio*: fraction of people above the richness line,

- *PSS*: a series of Peichl, Schaefer and Scheicher (2006) indices (see equation 1) with parameters 1, 2 and 3.

The richness line is either directly specified by the user or computed relative to the median or mean of *varname*, see under "options" below.

For the calculation of income richness, the income may not be negative. Therefore, cases with *varname* less than zero are omitted in the calculation, whereas values of zero are used for the calculation.

3.3 Options

There are two ways of defining the richness line:

`rline(rl)` manually defines a number *rl* as the (absolute) richness line (can be any positive number, macro or scalar). If `rline` is not used, the richness line is computed relatively (see below).

The relative calculation of the richness line is based on a multiplier of a parameter of the distribution of *varname*.

`rnumber`(*rn*) defines the multiplier *rn*, which can be any positive number and has to be specified in percent but without the "%" symbol (e.g. 200, which is the default value, and not "200%" if you want to specify a richness line of 200%). `rval`(*rv*) defines the distributional parameter *rv*, which can be either median (default) or mean.

`rlfix` specifies that the richness line of the first variable of *varlist* is fixed and used for all other variables of *varlist*.

If none of the options is specified, a default richness line of 200% of the median is assumed.

If both ways (absolute and relative) are specified, the (absolute) richness line defined by `rline` is used.

3.4 The output and saved results

`richness` displays a matrix of the computed results and stores the following results in r():

RR is the matrix with all stored results for *varlist*,

Rline_varname is the value of the (computed or specified) richness line for *varname*,

R0_varname is the headcount index (as a decimal) for *varname*,

R1_varname is the PSS index with alpha = 1 for *varname*,

R2_varname is the PSS index with alpha = 2 for *varname*,

R3_varname is the PSS index with alpha = 3 for *varname*.

4 Examples

We now illustrate the usage of richness by small examples. The first and second example highlight two considerable advantages of our new measures, whereas the third example illustrates a comparative analysis of different scenarios. All three examples are executed in `example.do` which also generates the artificial data.

4.1 Example 1: Change of a rich person's income

Consider two populations with income distribution

$$\mathbf{a} = (5, 5, 5, 11, 11) \text{ and } \mathbf{b} = (5, 5, 5, 100, 100) .$$

Let the richness line $\rho_{\mathbf{a}}$, $\rho_{\mathbf{b}}$ be 200% of the median income.

```
.  richness a
 Richness indices:
------------------------------------------------------
    RL   HC  R(1)        R(2)        R(3)
 a  10  .4  .03636364  .00330578  .00030053
------------------------------------------------------
 RL: richness line
 HC: headcount index
 R(a):  PSS richness indices
```

The output gives us a table reporting the results, which can be accessed using Stata's `r()` function. In the first column, the richness line is reported. By default, the richness line is computed as 200% of the median income. The following columns report the values of the richness indices.

In the next case, the same richness line is directly specified using the `rline()` option. In the case of **b**, the default richness line of 200% of the median income is, of course, 10 as well.

```
. richness b, rline(10)
 Richness indices:
```

	RL	HC	R(1)	R(2)	R(3)
b	10	.4	.35999999	.324	.29159999

```
RL: richness line
HC: headcount index
R(a):  PSS richness indices
```

The richness lines are in both cases $\rho_{\mathbf{a}} = \rho_{\mathbf{b}} = 10$ and we obtain the following values for the indices:

$$R_{HC}(\mathbf{a}) = R_{HC}(\mathbf{b}) = 40\%\,,$$

and

$$R_1(\mathbf{a}) = 3,64\% \text{ and } R_1(\mathbf{b}) = 36,00\%\,.$$

The latter appears to be the more plausible result since $R_1(\mathbf{a}) < R_1(\mathbf{b})$, because a change in a rich person's income also changes the measure of richness.

4.2 Example 2: Sensitivity to changes of very high incomes

Let:

$$\mathbf{c} = (5, 5, 5, 11, 9989) \text{ and } \mathbf{d} = (5, 5, 5, 1000, 9000)\,,$$

where $\mathbf{d}$ is obtained from $\mathbf{c}$ by a progressive transfer of 989 monetary units between the two rich persons.

```
. richness c
 Richness indices:
```

	RL	HC	R(1)	R(2)	R(3)
c	10	.4	.21798159	.20125265	.1995502

```
RL: richness line
HC: headcount index
R(a):  PSS richness indices
```

We could obtain the same (default) richness line of 200% of the median income by specifying the options `rval(median) rnumber(200)`.

```
. richness d, rval(median) rnumber(200)
```

```
Richness indices:
```

	RL	HC	R(1)	R(2)	R(3)
d	10	.4	.39777778	.3955758	.39339387

```
RL: richness line
HC: headcount index
R(a):  PSS richness indices
```

We obtain

$$R_{HC}(\mathbf{c}) = R_{HC}(\mathbf{d}) = 40\%\,,$$

but more plausible results for R_1, as our richness index is less sensitive to changes of very high incomes:

$$R_1(\mathbf{c}) = 21,80\% \text{ and } R_1(\mathbf{d}) = 39,78\%\,.$$

4.3 Example 3: Using various options

This example shows the use of a *varlist*: the indices are calculated for each variable of *varlist* with the default option of a richness line of 200% of the median.

```
. richness a b c d
Richness indices:
```

	RL	HC	R(1)	R(2)	R(3)
a	10	.4	.03636364	.00330578	.00030053
b	10	.4	.35999999	.324	.29159999
c	10	.4	.21798159	.20125265	.1995502
d	10	.4	.39777778	.3955758	.39339387

```
RL: richness line
HC: headcount index
R(a):  PSS richness indices
```

We obtain the same values of the richness indices for $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ as in the two previous subsections.

Now, all previous examples are calculated with a richness line of 200% of their respective means instead of the median. This results in varying richness lines and thus in different values of the richness indices.

```
. richness a b c d, rval(mean)
```

```
Richness indices:
```

	RL	HC	R(1)	R(2)	R(3)
a	14.8	0	0	0	0
b	86	.4	.056	.00784	.0010976
c	4006	.2	.11979177	.07175034	.0429755
d	4006	.2	.11097778	.06158034	.03417024

```
RL: richness line
HC: headcount index
R(a):  PSS richness indices
```

If we want to compare different reform scenarios with the status quo, it might be useful to use the same richness line for all scenarios. This could be either done by specifying an absolute fixed poverty line using `rline()` or by fixing the relative poverty line of the first variable of *varlist*. To fix the richness line we use the option `rlfix`. This holds the richness line constant at the value of the first variable of *varlist*.

```
.  richness a b c d, rval(mean) rlfix
Richness indices:
```

	RL	HC	R(1)	R(2)	R(3)
a	14.8	0	0	0	0
b	14.8	.4	.3408	.2903616	.24738808
c	14.8	.2	.19970367	.19940779	.19911234
d	14.8	.4	.39671111	.39346657	.39026573

```
RL: richness line
HC: headcount index
R(a):  PSS richness indices
```

The richness line is now fixed at the value of the first scenario (status quo). In comparison to the second case, this yields changes in the richness lines and indices for the other scenarios. Which procedure is more appropriate, depends on the context. See Peichl, Schaefer and Scheicher (2006) for an application to real data, where the different results of both methodologies are discussed.

5 Conclusion

In this paper, we present `richness`, a Stata program for the calculation of a new class of richness measures. It accounts for changes in the dimension of high incomes and therefore allows for a distinct analysis of structural changes at the top of the income distribution. We propose to use the new measures in addition to the headcount index for a more sophisticated analysis of richness.

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