Measuring mobility

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Topics

- transition matrices
- matrix-based mobility measures
- other mobility measures

Focus

We will examine various means of measuring mobility, with a focus on economic mobility of individuals over time, primarily due to changes in income.

But most of these measures can be applied in many areas, and are used, for example, to measure changes in measured teacher quality in education research, or any changing states such as marital status or occupation, disability or morbidity, or changing prices or market shares.

Many (but not all) of commonly used measures rely on an estimated transition matrix, so let's start there.

Transition matrices

The usual setup for a transition matrix is to measure status s at time $t-1$ and again at time t, then to estimate the matrix M_1 (the one denotes a one-period delta):

$$s_t = M_1 s_{t-1}$$

in which case each column of M_1 sums to one, or sometimes its transpose:

$$s_t^T = s_{t-1}^T M_1^T$$

in which case each row of M_1^T sums to one.

s may measure, for example, which fifth (or half, or hundredth) of the income distribution a panel survey respondent falls into in one year, and then which fifth they fall in the next year.

Alternative transition matrices

One could also measure which fifth of the income distribution a respondent falls into, then which fifth their child appears in 30 years later (intergenerational mobility).

Or we could measure which fifth of a different economic status distribution a respondent falls into, e.g. a measure of educational attainment, then which fifth of the income distribution their child appears in 30 years later (in which case we do not have s on both sides of that equation).

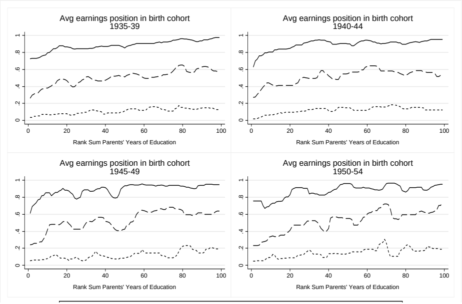
Or states defined by an absolute measure, such as the US poverty line (unlike the traditional European model for measuring poverty, the US cutoff is defined by a theoretical lower bound budget adjusted only for measured inflation). This can define multiple states as well: we can look at who is poor/nonpoor in each period, but we can also look at below the poverty line, [1,2) times the poverty line, [2,3) times the poverty line, etc.

One (related) alternative to transition matrices

We could measure at which point in the overall distribution at time $t+1$ (or generation $t+1$) each conditional quantile reaches, calculating quantiles conditional on starting points; this is similar in spirit to a transition matrix.

Can do this nonparametrically with a series of kernel-weighted quantile regressions as in Nichols and Favreault (2009).

Nichols and Favreault (2009)



One alternative to transition matrices

That paper also critiques the common "intergenerational elasticity" measure, which regresses log income of the child on log income of the parent (note that we could do this for income of a person at two points in time as well, to measure "intragenerational elasticity").

Making sure the nonparametric quantile regressions satisfy some basic adding-up constraints is no easy matter and is not even attempted in Nichols and Favreault (2009).

Quintile transition matrices

The canonical example defines s_t so it measures fifths of an income distribution, so M_1 in

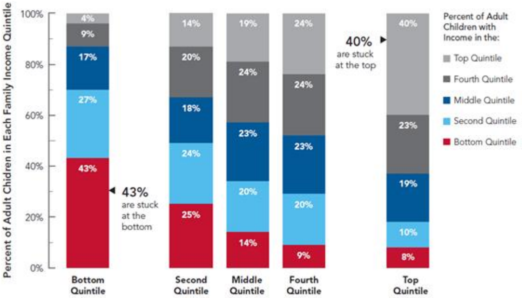
$$s_t = M_1 s_{t-1}$$

is a **quintile transition matrix** (all the same theory applies to any quantile transition matrix, but 5 categories seems to be the optimal number for our limited attention). In that case $s = (0.2, 0.2, 0.2, 0.2, 0.2)^T$ in every period, and M_1 must be bistochastic, i.e. rows and columns must sum to one.

Now there is no information in s, i.e. the "middle class" cannot grow or shrink if it is always 60 percent of the population, and we can devote our attention exclusively to the properties of M_1.

Displaying transition matrices

We don't need to show the numbers in a table, of course; transition bar charts common in reports.



Displaying comparisons of transition matrices

FIGURE 6 Chances of Getting Ahead or Falling Behind in Income Ranking, by Parental Income and Race

Markov matrices

Much of the appeal of transition matrices arises from the idea that we can use them to describe longer-run dynamics. In particular, if the probability a person winds up in a particular row of s_t depends only on which row of s_{t-1} they are in, then M_1 is a Markov matrix, and we can describe the probability they wind up in a particular row at time $t+1$:

$$s_{t+1} = M_1 s_t = M_1 M_1 s_{t-1} = M_1^2 s_{t-1}$$

or time $t+k$:

$$s_{t+k} = M_1^k s_t$$

so we don't need to compute two-period transition matrix M_2 or the three-period transition matrix M_3 or measure any longer transitions.

Markov failures

Unfortunately, it rarely is the case that the data satisfies the Markov assumption that history does not matter for transition rates, even at the first comparison point of the two-period transition matrix M_2 (recall the subscript describes the delta in time periods, whereas the superscript denotes the power i.e. M^2 is the product of M_1 and M_1, the square of single-period transition matrices):

$$s_{t+1} = M_2 s_{t-1} \neq M_1^2 s_{t-1}$$

but sometimes it is possible to expand the space over which the states in s are measured and get a transition matrix that comes close to satisfying the Markov assumption.

Markov failures

For example, instead of measuring poor/nonpoor in one period and trying to predict poor/nonpoor in the next period and all future periods, we can measure poor/nonpoor in two periods and try to predict the next two periods, and all future periods (instead of two proportions in s, now there are 4).

The meaning of the matrix changes in that case, of course, but it allows long-run projections if Markov assumptions are satisfied.

Transition estimation

Stata has a command xttrans for measuring transition rates, but it does not respect the panel structure properly. tabulate works fine, though, and svy: tabulate allows weights, cluster-robust standard errors, and tests of hypotheses.

```
webuse nlswork, clear
keep if inlist(year,70,71,72,73,77,78)
egen m=median(ln_wage), by(year)
gen above=ln_wage>m if ln_wage<.
gen lastyr=l.above
bysort idcode (year): gen wrong=above[_n-1]
gen nextyr=f.above
tab above nextyr, nofreq row
tab lastyr above, nofreq row
tab wrong  above, nofreq row
xttrans above
```

xttrans

```
. tab lastyr above, nofreq row
           |         above
    lastyr |         0          1 |     Total
-----------+----------------------+----------
         0 |     80.04      19.96 |    100.00
         1 |     12.85      87.15 |    100.00
-----------+----------------------+----------
     Total |     44.79      55.21 |    100.00

. tab wrong above, nofreq row
           |         above
     wrong |         0          1 |     Total
-----------+----------------------+----------
         0 |     78.33      21.67 |    100.00
         1 |     17.14      82.86 |    100.00
-----------+----------------------+----------
     Total |     47.41      52.59 |    100.00

. xttrans above
           |         above
     above |         0          1 |     Total
-----------+----------------------+----------
         0 |     78.33      21.67 |    100.00
         1 |     17.14      82.86 |    100.00
-----------+----------------------+----------
     Total |     47.41      52.59 |    100.00
```

Quantile transition matrix estimation

Of course, tabulate does not know that the resulting matrix is supposed to be bistochastic in the previous example, and so it predicts 45 percent will be below the median next period and 55 percent above, which cannot happen.

This error can be due to a mass of people right at quantile breaks (if a lot of people are right at the median wage) and tie-breaking rules, or an unbalanced panel.

Quantile transition matrix estimation

The latter problem is pervasive, and requires some thought: how do you want to select a balanced panel to do your estimation (a larger issue in general)?

One simple and defensible method is to reweight the data using the proportion of people in each category in the first time period who also appear in the second (dropping those who appear only in one period). This is a nonparametric propensity score approach to nonresponse/attrition adjustment (see also Nichols 2007), and can work to create a representative balanced panel of any length (representing the population of the base year). Properly done, it also requires re-estimating all the relevant quantiles for each set of balanced data using weights.

A quick and dirty approach is to simply estimate the quantiles for the subsample in period $t + 1$ that has data available in period t, which turns out to fix most problems.

A quick and dirty adjustment

```
webuse nlswork, clear
keep if inlist(year,70,71,72,73,77,78)
egen m=median(ln_wage), by(year)
egen m2=median(ln_wage) if l.ln_wage<., by(year)
gen above=ln_wage>m if ln_wage<.
gen above2=ln_wage>m2 if ln_wage<.
gen nextyr=f.above
gen nexty2=f.above2
tab above nextyr if year==70, nofreq row
tab above nexty2 if year==70, nofreq row
```

A quick and dirty adjustment, cont.

```
. tab above nextyr if year==70, nofreq row
           |        nextyr
     above |         0          1 |     Total
-----------+----------------------+----------
         0 |     73.94      26.06 |    100.00
         1 |     13.41      86.59 |    100.00
-----------+----------------------+----------
     Total |     41.67      58.33 |    100.00

. tab above nexty2 if year==70, nofreq row
           |        nexty2
     above |         0          1 |     Total
-----------+----------------------+----------
         0 |     81.92      18.08 |    100.00
         1 |     22.40      77.60 |    100.00
-----------+----------------------+----------
     Total |     50.19      49.81 |    100.00
```

Quantile transition matrix estimation

One can also use optimize to estimate the closest matrix (where close is defined using the spectral norm measure of distance for matrices) to the empirical estimate that is bistochastic, but these small deviations are unlikely to matter in practice (at least, I have never seen any real difference by moving to a properly constrained estimate of the quantile transition matrix in my own work).

Matrix properties

We've estimated a matrix; now what? Is the society highly mobile or not?

Perfect immobility would be an identity matrix as the transition matrix; perfect mobility might be any matrix with zeros on the diagonal (no one ends where they started) or everyone has equal probability of winding up in the various possible slots next period, regardless of starting positions.

Any other matrix has a large-dimensional set of possible deviations from these ideals. Hard to look at a pair of transition matrices and say, "this matrix corresponds to an unambiguously more mobile society than that one."

Mobility statistics for transition matrices

Shorrocks (1978 Econometrica) proposed measures of mobility based on quantile transition matrices, which generated a literature on matrix-based mobility measures, notably including work on ordering due to Dardanoni (1993), with a social welfare foundation.

Sommers and Conlisk (1979) and Bartholomew (1982) also defined mobility measures based on a quantile transition matrix.

Mobility measures

Let's denote each of the commonly used measures with a single letter:

- T Trace measure: $[m - Tr(M)]/(m-1)$, from Shorrocks (1978 Ecm)
- D Determinant measure: $\det(M)/(m-1)$, from Shorrocks (1978 Ecm)
- E Eigenvalue measure: one minus the modulus of the second largest eigenvalue of M, due to Sommers and Conlisk (1979)
- M Mean crossing measure: the sum over i and j (from 1 to m) of M_{ij} times $|i-j|$ divided by $m(m-1)$, due to Bartholomew (1982).

All are easy to calculate in Mata or Stata, given the estimate of the transition matrix.

Other mobility measures

Shorrocks (1978 Journal of Economic Theory) defined mobility in terms of reductions in an inequality measure due to changes in accounting period. This definition of mobility or a related one from Maasoumi and Zandvakili (1986) is used in many articles, e.g. Burkhauser and Poupore (1997), Maasoumi and Trede (2001), and Kopczuk, Saez, and Song (2010). Call this the "ratio" measure R.

Nichols (2008, 2010) defined mobility risk in terms of variability of growth paths; denote this M.

Nichols (2008, 2010) measure

The goal of my own approach was to find a measure for income mobility that would integrate measures of mobility rates, volatility, and long-run inequality.

The central insight was that an inequality measure that is additively decomposable by population subgroup, such as the generalized entropy index with parameter 2 (GE_2) or half the squared coefficient of variation, can be applied to panel data.

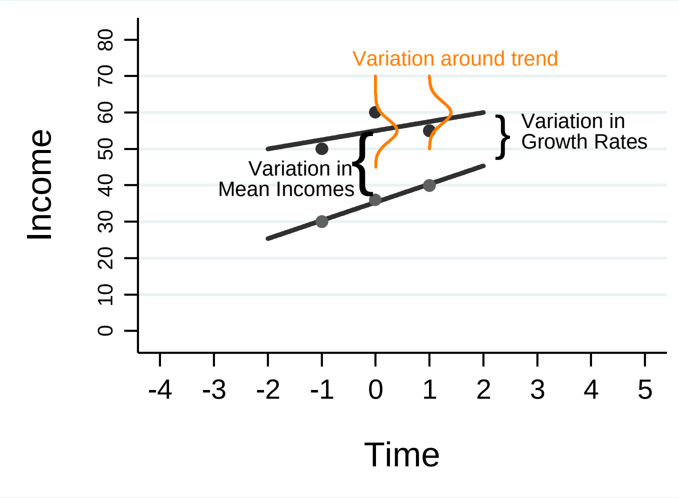
Nichols measure, cont.

Let individuals be the subgroups, and then the “between-group” inequality component measures long-run inequality across people, while the “within-group” inequality component measures inequality in individual income over time, a combination of mobility and volatility.

If we measure mobility risk as the variance of growth rates divided by squared mean income, the GE_2 decomposition maps perfectly onto a regression framework:

$$y_{it} = u_i + r_i t + e_{it}$$

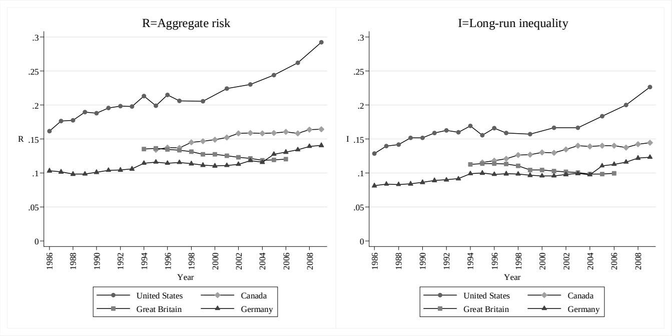
Nichols (2008) graphic



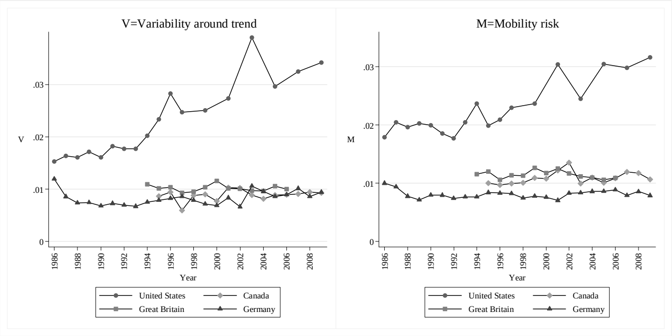
Comparing across time and space

This measure needs panel data, of course, but with a long panel, we can measure the components of income risk using short windows, say 3 or 5 years, and compute at each overlapping window of time, for estimates of the evolution of inequality, volatility, and mobility risk over time.

Comparing across time and space



Comparing across time and space



Mobility measures

Adding to our list:

- T Trace measure: $[m - Tr(M)]/(m-1)$, from Shorrocks (1978 Ecm)
- D Determinant measure: $\det(M)/(m-1)$, from Shorrocks (1978 Ecm)
- E Eigenvalue measure: one minus the modulus of the second largest eigenvalue of M, due to Sommers and Conlisk (1979)
- M Mean crossing measure: the sum over i and j (from 1 to m) of M_{ij} times $|i-j|$ divided by $m(m-1)$, due to Bartholomew (1982).
- R Ratio of multi-period to weighted average single-period inequality (Shorrocks 1978 JET)
- M Mobility risk (Nichols 2008, 2010; Nichols and Rehm 2014)

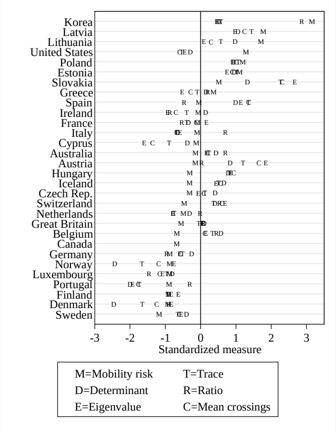
Comparing measures

All of these measure different concepts of mobility and will rate mobility in the same data differently.

For example, if we ask which country has the highest level of economic mobility in recent data, we can get quite different rankings using different measures.

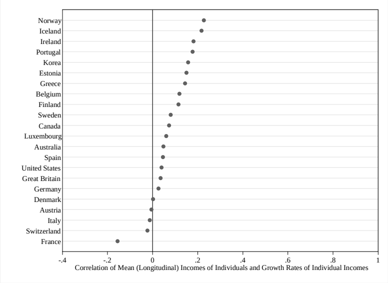
That said, they are all highly correlated in actual empirical examples (Nichols 2008; Nichols and Rehm 2014).

Comparing measures for 30 countries



Pro-poor growth

There are also many measures of whether income growth is pro-poor or pro-rich; one drops out naturally of the Nichols (2008, 2010) framework: the correlation of mean income u_i and the individual-specific growth rate r_i. Estimated in Nichols and Rehm (2014):



See also

The book by Corak (2006) contains work by many authors, touchpoints for a lot of the subsequent work on economic mobility across generations. On measuring mobility, Fields and Ok (1999) and Fields (2007) have a lot of ideas about how it should be done. On measuring poverty, see Jenkins (2006), and on measuring inequality, start with Cowell (2011) and Jenkins (2009). Some of my own work on poverty, income inequality, and mobility is accessible at:

- http://pped.org
- http://www.urban.org/economy/Economic-Insecurity.cfm
- http://www.urban.org/inequality
- http://www.urban.org/economicmobility

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