Structural Equation Modeling Using `gllamm`, `confa` and `gmm`

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Structural equation models
Formulation
Path diagrams
Identification
Estimation
Stata tools for SEM
gllamm
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Goals of the talk

1. Introduce structural equation models
2. Describe Stata packages to fit them:
   - `confa`: a 5/8" hex wrench
   - `gllamm`: a Swiss-army tomahawk
   - `gmm`: do-it-yourself kit
3. Give example(s)
   - Health: daily functioning in NHANES
   - Sociology: industrialization and political democracy
   - Psychology: Holzinger-Swineford data

First, some theory

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Structural equation modeling (SEM)

- Standard multivariate technique in social sciences
- Incorporates constructs that cannot be directly observed:
  - psychology: level of stress
  - sociology: quality of democratic institutions
  - biology: genotype and environment
  - health: difficulty in personal functioning
- Special cases:
  - linear regression
  - confirmatory factor analysis
  - simultaneous equations
  - errors-in-variables and instrumental variables regression

Origins of SEM

Path analysis of Sewall Wright (1918)

$\otimes$

Causal modeling of Hubert Blalock (1961)

$\otimes$

Factor analysis estimation of Karl Jöreskog (1969)

$\otimes$

Econometric simultaneous equations of Arthur Goldberger (1972)

Structural equations model

Latent variables:

$$\boldsymbol{\eta} = \boldsymbol{\alpha}_\eta + \boldsymbol{B}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta} \tag{1}$$

Measurement model for observed variables:

$$\boldsymbol{y} = \boldsymbol{\alpha}_y + \boldsymbol{\Lambda}_y\boldsymbol{\eta} + \boldsymbol{\varepsilon} \tag{2}$$

$$\boldsymbol{x} = \boldsymbol{\alpha}_x + \boldsymbol{\Lambda}_x\boldsymbol{\xi} + \boldsymbol{\delta} \tag{3}$$

$\boldsymbol{\xi}, \boldsymbol{\zeta}, \boldsymbol{\varepsilon}, \boldsymbol{\delta}$ are uncorrelated with one another

Jöreskog (1973), Bollen (1989), Yuan & Bentler (2007)

Implied moments

Denoting

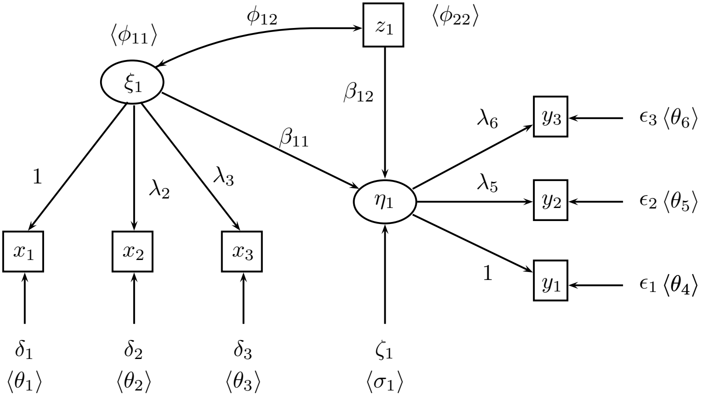
$$\mathbb{V}[\boldsymbol{\xi}] = \Phi, \quad \mathbb{V}[\boldsymbol{\zeta}] = \Psi, \quad \mathbb{V}[\boldsymbol{\varepsilon}] = \Theta_\varepsilon, \quad \mathbb{V}[\boldsymbol{\delta}] = \Theta_\delta,$$
$$R = \boldsymbol{\Lambda}_y (I - B)^{-1}, \quad z = (x', y')'$$

obtain

$$\mu(\boldsymbol{\theta}) \equiv \mathbb{E}\left[z\right] = \begin{pmatrix} \boldsymbol{\alpha}_y + \boldsymbol{\Lambda}_y (I - \boldsymbol{B})^{-1} \boldsymbol{\Gamma} \boldsymbol{\mu}_\xi \\ \boldsymbol{\alpha}_x + \boldsymbol{\Lambda}_x \boldsymbol{\mu}_\xi \end{pmatrix} \tag{4}$$

$$\Sigma(\boldsymbol{\theta}) \equiv \mathbb{V}\left[z\right] = \begin{pmatrix} \boldsymbol{\Lambda}_x \Phi \boldsymbol{\Lambda}_x' + \Theta_\delta & \boldsymbol{\Lambda}_x \Phi \Gamma' R' \\ R \Gamma \Phi \boldsymbol{\Lambda}_x' & R(\Gamma \Phi \Gamma' + \Psi) R' + \Theta_\varepsilon \end{pmatrix} \tag{5}$$

Path diagrams



Identification

Before proceeding to estimation, the researcher needs to verify that the SEM is *identified*:

$$\mathbf{Pr}\{X : f(X,\theta) = f(X,\theta') \Rightarrow \theta = \theta'\} = 1$$

Different parameter values should give rise to different likelihoods/objective functions, either globally, or locally in a neighborhood of a point in a parameter space.

Likelihood

- Normal data $\Rightarrow$ likelihood is the function of sufficient statistic $(\bar{z}, S)$:

$$-2\log L(\theta, Y, X) \sim n \ln \det\bigl(\Sigma(\theta)\bigr) + n\,\mathrm{tr}[\Sigma^{-1}(\theta)S] \\ +n(\bar{z} - \mu(\theta))'\Sigma^{-1}(\theta)(\bar{z} - \mu(\theta)) \to \min_\theta \qquad (6)$$

- Generalized latent variable approach for mixed response (normal, binomial, Poisson, ordinal, within the same model):

$$-2\log L(\theta, Y, X) \sim \sum_{i=1}^{n} \ln \int f(y_i, x_i|\xi, \zeta; \theta)\mathrm{d}F(\xi, \zeta|\theta) \qquad (7)$$

Bartholomew & Knott (1999), Skrondal & Rabe-Hesketh (2004)

Estimation methods

- (quasi-)MLE
- Weighted least squares:

$$s = \operatorname{vech} S, \quad \sigma(\theta) = \operatorname{vech} \Sigma(\theta)$$

$$F = (s - \sigma(\theta))' V_n (s - \sigma(\theta)) \to \min_{\theta} \tag{8}$$

  where V_n is weighting matrix:
  - Optimal $\hat{V}_n^{(1)} = \hat{\mathbb{V}}[s - \sigma(\theta)]$ (Browne 1984)
  - Simplistic: least squares $V_n^{(2)} = I$
  - Diagonally weighted least squares: $\hat{V}_n^{(3)} = \operatorname{diag} \hat{\mathbb{V}}[s - \sigma]$
- Model-implied instrumental variables limited information estimator (Bollen 1996)
- Bounded influence/outlier-robust methods (Yuan, Bentler & Chan 2004, Moustaki & Victoria-Feser 2006)
- Empirical likelihood

Goodness of fit

- The estimated model $\Sigma(\hat{\theta})$ is often related to the "saturated" model $\Sigma \equiv S$ and/or independence model $\Sigma_0 = \operatorname{diag} S$
- Likelihood formulation $\Rightarrow$ LRT test, asymptotically χ^2_k
- Non-normal data: LRT statistic $\sim \sum_j w_j \chi^2_1$, can be Satterthwaite-adjusted towards the mean and variance of the appropriate χ^2_k (Satorra & Bentler 1994, Yuan & Bentler 1997)
- Analogies with regression R^2 attempted, about three dozen fit indices available (Marsh, Balla & Hau 1996)
- Reliability of indicators: R^2 in regression of an indicator on its latent variable
- Signs and magnitudes of coefficient estimates

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Now, some tools

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gllamm

Generalized Linear Latent And Mixed Models (Skrondal & Rabe-Hesketh 2004, Rabe-Hesketh, Skrondal & Pickles 2005, Rabe-Hesketh & Skrondal 2008)

- Exploits commonalities between latent and mixed models
- Adds GLM-like links and family functions to them
- Allows heterogeneous response (different exponential family members)
- Allows multiple levels
- Maximum likelihood via numeric integration of random effects and latent variables (Gauss-Newton quadrature, adaptive quadrature); hence one of the most computationally demanding packages ever

`gllamm`

- One observation per dependent variable $\times$ observation
- Requires `reshape long` transformation of indicators for latent variable models
- Measurement model: `eq()` option
- Structural model: `geq() bmatrix()` options
- Families and links: `family() fv() link() lv()`
- Tricks that Stas commonly uses:
  - make sure the model is correctly specified: `trace noest` options
  - good starting values speed up convergence: `from()` option
  - number of integration points gives tradeoff between speed and accuracy: `nip()` option
  - get an idea about the speed: `dot` option

`confa` package

- CONfirmatory Factor Analysis models, a specific class of SEM
- Maximum likelihood estimation
- Arbitrary # of factors and indicators; correlated measurement errors
- Variety of standard errors (OIM, sandwich, distributionally robust)
- Variety of fit tests (LRT, various scaled tests)
- Post-estimation:
  - fit indices;
  - factor scores (predictions)

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gmm

New (as of Stata 11) estimation command `gmm`:

- Estimation by minimization of

$$g(X,\theta)'\,V_n\,g(X,\theta) \to \min_\theta$$

- Evaluator vs. "regression+instruments"
- Variety of weight matrices V_n
- Homoskedastic/`unadjusted` or heteroskedastic/`robust` standard errors
- Overidentification (goodness of fit) J-test via `estat overid`

gmm+sem4gmm

Least squares estimators can be implemented using `gmm` (Kolenikov & Bollen 2010).

1. Compute the implied moment matrix $\Sigma(\theta)$ (user-specified Mata function `ParsToSigma()`)
2. Form observation-by-observation contributions to the moment conditions $\mathrm{vech}\big[(x_i - \bar{x})(x_i - \bar{x})' - \Sigma(\theta)\big]$ (Mata function `VechData()` provided by Stas)
3. Feed into `gmm` using moment evaluator function `sem4gmm` (provided by Stas)
4. Enjoy!

LS family of estimators

- Common part:
  `gmm sem4gmm, parameters(`pars') ...`
- ULS: `... winit(id) onestep vce(unadj)`
- DWLS: `... winit(unadj, indep) wmat(unadj, indep) twostep`
- ADF: `... twostep | igmm`

Comparison of functionality

	gllamm	confa	gmm+sem4gmm
General SEM	...	–	$\surd$
Estimation	$\surd$	$\surd$	$\surd$
Overall test	–	$\surd$	$\surd$
Fit indices	–	...	–
Prediction	$\surd$	...	–
Ease of use	–	$\surd$	–
Speed	–	...	–

Finally, examples

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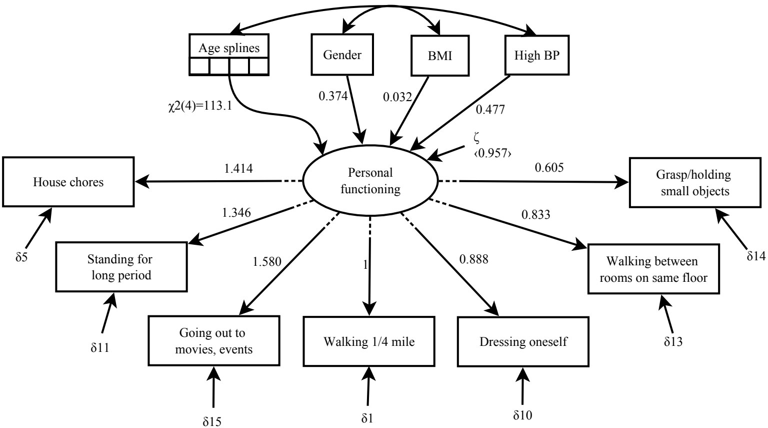
NHANES data

- NHANES 2007–08 data
- Personal functioning section: *“difficulty you may have doing certain activities because of a health problem”*
- 17 questions: Walking for a quarter mile; Walking up ten steps; Stooping, crouching, kneeling; Lifting or carrying; House chore; Preparing meals; Walking between rooms on same floor; Standing up from armless chair; Getting in and out of bed; Dressing yourself; Standing for long periods; Sitting for long periods; Reaching up over head; Grasp/holding small objects; Going out to movies, events; Attending social event; Leisure activity at home
- Response categories: “No difficulty”, “Some difficulty”, “Much difficulty”, “Unable to do”
- Research questions: How to summarize these items? What’s the relation between individual demographics and health?

Path diagram



A multiple indicators and multiple causes (MIMIC) model

NHANES example using `confa`

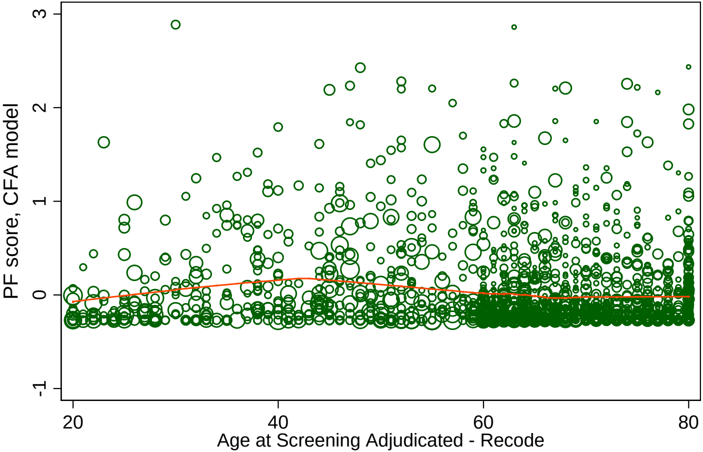
Only the measurement model can be estimated with `confa`, as a preliminary step in gauging the performance of this part of the model.

```
. confa (difficulty: pfq*), from(iv)

. confa (difficulty: pfq*), from(iv)
> missing
```

Show results: `estimates use confa_pwise`, `estimates use confa_fiml`

Factor scores



NHANES example via `gllamm`

Data management steps for `gllamm`:

1. Rename `pfq061b`$\mapsto$`pfq1`, `pfq061c`$\mapsto$`pfq2`, ...`pfq061s`$\mapsto$`pfq17`
2. `reshape long pfq, i(seqn) j(item)`
3. Generate binary indicators `q1-q17` of the items
4. Produce binary outcome measures: `bpfq`k'` = !("No difficulty") of `pfq`k'`

Model setup steps:

1. Define loading equations: `eq items: q1 q2 ...q17`
2. Come up with good starting values

NHANES example via `gllamm`

Syntax of `gllamm` command:

`gllamm ///`	
`bpfq ///`	single dependent variable
`q1 - q17, nocons ///`	item-specific intercepts
`i(seqn) ///`	“common factor”
`f(bin) l(probit) ///`	link and family
`eq(items) ///`	loadings equation
`from(...) copy`	starting values

The “common factor” is a latent variable that is constant across the `i()` panel, but can be modified with loadings

Show results in Stata: `est use cfa_via_gllamm; gllamm`

MIMIC model

Additional estimation steps:

1. Store the CFA results: `mat hs_cfa = e(b)`
2. Define the explanatory variables for functioning: `eq r1: female` *age splines*
3. Extend the earlier command: `gllamm ..., geq(r1) from( hs_cfa, skip )`

Parameter "complexity":

1. fixed effects
2. loadings
3. latent regression slopes
4. latent (co)variances

Show results in Stata: `est use mimic_bmi; gllamm`; show the diagram again.

NHANES example via gmm

Full model:

- 1 latent variable $\Rightarrow$ 1 variance
- 17 indicators $\Rightarrow$ 17 loadings, 17 variances
- 7 explanatory variables $\Rightarrow 7 \cdot 8/2$ covariances, 7 regression coefficients
- Total: 70 parameters, 300 moment conditions

Trimmed model:

- 1 latent variable $\Rightarrow$ 1 variance
- 5 indicators $\Rightarrow$ 5 loadings, 5 variances
- 4 explanatory variables $\Rightarrow 4 \cdot 5/2$ covariances, 4 regression coefficients
- Total: 25 parameters, 45 moment conditions

NHANES example: syntax and results

Show syntax: `nhanes-def-sem-reduced.do`, `nhanes-gmm-est-reduced.do`

Show results:

```
foreach eres in r_uls_homosked
r_uls_heterosked r_dwls_2step_heterosked
r_effls_2step_heterosked
r_effls_igmm_heterosked {
    est use `eres'
    est store `eres'
}
estimates table, se stats(J)
```

Main journals

Journal title	Impact factor	h-index
Structural Equation Modeling	2.4	15
Psychometrika	1.1	27
British Journal of Mathematical and Statistical Psychology	1.3	20
Multivariate Behavioral Research	1.8	30
Psychological Methods	4.3	52
Sociological Methodology	2.5	21
Sociological Methods and Research	1.2	24
JASA	2.3	74
Biometrika	1.3	48
J of Multivariate Analysis	0.7	24
Stata Journal	1.3	9

Source: http://www.scimagojr.com/, 2008 figures.

What I covered was. . .

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