1. Types of data

2. Survey data characteristics

3. Variance estimation

4. Estimation for subpopulations

5. Summary
Collecting data can be expensive and time consuming.

Consider how you would collect the following data:

- Smoking habits of teenagers
- Birth weights for expectant mothers with high blood pressure

Using stages of clustered sampling can help cut down on the expense and time.
Types of data

Simple random sample (SRS) data

Observations are "independently" sampled from a data generating process.

- Typical assumption: independent and identically distributed (iid)
- Make inferences about the data generating process
- Sample variability is explained by the statistical model attributed to the data generating process

Standard data

We’ll use this term to distinguish this data from survey data.
Types of data

Correlated data

Individuals are assumed not independent.

Cause:
- Observations are taken over time
- Random effects assumptions
- Cluster sampling

Treatment:
- Time-series models
- Longitudinal/panel data models
- `cluster()` option
Types of data

Survey data

Individuals are sampled from a fixed population according to a survey design.

Distinguishing characteristics:

- Complex nature under which individuals are sampled
- Make inferences about the fixed population
- Sample variability is attributed to the survey design
Types of data

Standard data

- Estimation commands for standard data:
  - proportion
  - regress

- We’ll refer to these as *standard estimation commands*.

Survey data

- Survey estimation commands are governed by the `svy` prefix.
  - `svy: proportion`
  - `svy: regress`

- `svy` requires that the data is `svyset`. 
Survey data characteristics

Single-stage syntax

```
svyset [psu][weight][, strata(varname) fpc(varname)]
```

- Primary sampling units (PSU)
- Sampling weights – `pweight`
- Strata
- Finite population correction (FPC)
Survey data characteristics

Sampling unit

An individual or collection of individuals from the population that can be selected for observation.

- Sampling groups of individuals is synonymous with cluster sampling.
- Cluster sampling usually results in inflated variance estimates compared to SRS.
Survey data characteristics

Sampling weight

The reciprocal of the probability for an individual to be sampled.

- Probabilities are derived from the survey design.
  - Sampling units
  - Strata

- Typically considered to be the number of individuals in the population that a sampled individual represents.

- Reduces bias induced by the sampling design.
Survey data characteristics

**Strata**

In stratified designs, the population is partitioned into well-defined groups, called strata.

- Sampling units are independently sampled from within each stratum.
- Stratification usually results in smaller variance estimates compared to SRS.
Finite population correction (FPC)

An adjustment applied to the variance due to sampling without replacement.

- Sampling without replacement from a finite population reduces sampling variability.
Example: `svyset` for single-stage designs
Survey data characteristics

Population 1000
Survey data characteristics

Cluster sample 20 (208 obs)
Survey data characteristics

Stratified sample 198
Survey data characteristics

Stratified-cluster sample 20 (215 obs)
Survey data characteristics

Multistage syntax

```
svyset psu [weight][, strata(varname) fpc(varname)]
   [|| ssu [, strata(varname) fpc(varname)]]
   [|| ssu [, strata(varname) fpc(varname)]] ... 
```

- Stages are delimited by “||”
- SSU – secondary/subsequent sampling units
- FPC is required at stage $s$ for stage $s + 1$ to play a role in the linearized variance estimator
Survey data characteristics

Poststratification

A method for adjusting sampling weights, usually to account for underrepresented groups in the population.

- Adjusts weights to sum to the poststratum sizes in the population
- Reduces bias due to nonresponse and underrepresented groups
- Can result in smaller variance estimates

Syntax

```
svyset ... poststrata(varname) postweight(varname)
```
Example: `svyset` for poststratification
Strata with a single sampling unit

Big problem for variance estimation

- Consider a sample with only 1 observation
- `svy` reports missing standard error estimates by default

Finding these lonely sampling units

Use `svydes`:

- Describes the strata and sampling units
- Helps find strata with a single sampling unit
Strata with a single sampling unit

Example: `svydes`
Strata with a single sampling unit

Handling lonely sampling units

1. Drop them from the estimation sample.
2. `svyset` one of the ad-hoc adjustments in the `singleunit()` option.
3. Somehow combine them with other strata.
Certainty units

- Sampling units that are guaranteed to be chosen by the design.
- Certainty units are handled by treating each one as its own stratum with an FPC of 1.
Stata has three variance estimation methods for survey data:

- Linearization
- Balanced repeated replication
- The jackknife
Variance estimation

Linearization

A method for deriving a variance estimator using a first order Taylor approximation of the point estimator of interest.

- Foundation: Variance of the total estimator

Syntax

svyset ... [vce(linearized)]

- Delta method
- Huber/White/robust/sandwich estimator
Variance estimation

Total estimator – Stratified two-stage design

- \( y_{hijk} \) – observed value from a sampled individual
- Strata: \( h = 1, \ldots, L \)
- PSU: \( i = 1, \ldots, n_h \)
- SSU: \( j = 1, \ldots, m_{hi} \)
- Individual: \( k = 1, \ldots, m_{hij} \)

\[
\hat{Y} = \sum w_{hijk} y_{hijk}
\]

\[
\hat{V}(\hat{Y}) = \sum_h (1 - f_h) \frac{n_h}{n_h - 1} \sum_i (y_{hi} - \bar{y}_h)^2 + \sum_h f_h \sum_i (1 - f_{hi}) \frac{m_{hi}}{m_{hi} - 1} \sum_j (y_{hij} - \bar{y}_{hi})^2
\]
Variance estimation

Total estimator – Stratified two-stage design

- \( y_{hijk} \) – observed value from a sampled individual
- **Strata:** \( h = 1, \ldots, L \)
- **PSU:** \( i = 1, \ldots, n_h \)
- **SSU:** \( j = 1, \ldots, m_{hi} \)
- **Individual:** \( k = 1, \ldots, m_{hij} \)

\[
\hat{Y} = \sum w_{hijk} y_{hijk}
\]
\[
\hat{V}(\hat{Y}) = \sum_h (1 - f_h) \frac{n_h}{n_h - 1} \sum_i (y_{hi} - \bar{y}_h)^2 + \sum_h f_h \sum_i (1 - f_{hi}) \frac{m_{hi}}{m_{hi} - 1} \sum_j (y_{hij} - \bar{y}_{hi})^2
\]
Variance estimation

Total estimator – Stratified two-stage design

- $y_{hijk}$ – observed value from a sampled individual
- Strata: $h = 1, \ldots, L$
- PSU: $i = 1, \ldots, n_h$
- SSU: $j = 1, \ldots, m_{hi}$
- Individual: $k = 1, \ldots, m_{hij}$

\[
\hat{Y} = \sum w_{hijk} y_{hijk}
\]

\[
\hat{V}(\hat{Y}) = \sum_h (1 - f_h) \frac{n_h}{n_h - 1} \sum_i (y_{hi} - \bar{y}_h)^2 + \sum_h f_h \sum_i (1 - f_{hi}) \frac{m_{hi}}{m_{hi} - 1} \sum_j (y_{hij} - \bar{y}_{hi})^2
\]
Example: `svy: total`
Variance estimation

Linearized variance for regression models

- Model is fit using estimating equations.
- $\hat{G}()$ is a total estimator, use Taylor expansion to get $\hat{V}(\hat{\beta})$.

$$\hat{G}(\beta) = \sum_j w_j s_j x_j = 0$$

$$\hat{V}(\hat{\beta}) = D\hat{V}\{\hat{G}(\beta)\}\big|_{\beta=\hat{\beta}} D'$$
Linearized variance for regression models

- Model is fit using estimating equations.
- \( \hat{G}(\beta) \) is a total estimator, use Taylor expansion to get \( \hat{V}(\hat{\beta}) \).

\[
\hat{G}(\beta) = \sum_j w_j s_j x_j = 0
\]

\[
\hat{V}(\hat{\beta}) = D\hat{V}\{\hat{G}(\beta)\}|_{\beta=\hat{\beta}} D'
\]
Variance estimation

Example: `svy: logit`
Balanced repeated replication

For designs with two PSUs in each of $L$ strata.

- Compute replicates by dropping a PSU from each stratum.
- Find a balanced subset of the $2^L$ replicates. $L \leq r < L + 4$
- The replicates are used to estimate the variance.

Syntax

```
svyset ... vce(brr) [mse]
```
Variance estimation

BRR variance formulas

- $\hat{\theta}$ – point estimates
- $\hat{\theta}_{(i)}$ – $i$th replicate of the point estimates
- $\overline{\theta}_{(.)}$ – average of the replicates

Default variance formula:

$$\hat{V}(\hat{\theta}) = \frac{1}{r} \sum_{i=1}^{r} \{\hat{\theta}_{(i)} - \overline{\theta}_{(.)}\} \{\hat{\theta}_{(i)} - \overline{\theta}_{(.)}\}'$$

Mean squared error (MSE) formula:

$$\hat{V}(\hat{\theta}) = \frac{1}{r} \sum_{i=1}^{r} \{\hat{\theta}_{(i)} - \hat{\theta}\} \{\hat{\theta}_{(i)} - \hat{\theta}\}'$$
Example: `svy brr: logit`
Variance estimation

The jackknife

A replication method for variance estimation. Not restricted to a specific survey design.

- Delete-1 jackknife: drop 1 PSU
- Delete-$k$ jackknife: drop $k$ PSUs within a stratum

Syntax

```
svyset ... vce(jackknife) [mse]
```
Variance estimation

Jackknife variance formulas

- $\hat{\theta}_{(h,i)}$ – replicate of the point estimates from stratum $h$, PSU $i$
- $\bar{\theta}_h$ – average of the replicates from stratum $h$
- $m_h = (n_h - 1)/n_h$ – delete-1 multiplier for stratum $h$

Default variance formula:

$$\hat{V}(\hat{\theta}) = \sum_{h=1}^{L} (1 - f_h) m_h \sum_{i=1}^{n_h} \{(\hat{\theta}_{(h,i)} - \bar{\theta}_h)(\hat{\theta}_{(h,i)} - \bar{\theta}_h)'\}$$

Mean squared error (MSE) formula:

$$\hat{V}(\hat{\theta}) = \sum_{h=1}^{L} (1 - f_h) m_h \sum_{i=1}^{n_h} \{(\hat{\theta}_{(h,i)} - \hat{\theta})(\hat{\theta}_{(h,i)} - \hat{\theta})'\}$$
Example: `svy jackknife: logit`
Variance estimation

Replicate weight variable
A variable in the dataset that contains sampling weight values that were adjusted for resampling the data using BRR or the jackknife.

- Typically used to protect the privacy of the survey participants.
- Eliminate the need to `svyset` the strata and PSU variables.

Syntax
```plaintext
svyset ... brrweight(varlist)
svyset ... jkrweight(varlist [, ... multiplier(#)])
```
Focus on a subset of the population

- **Subpopulation variance estimation:**
  - Assumes the same survey design for subsequent data collection.
  - The `subpop()` option.

- **Restricted-sample variance estimation:**
  - Assumes the identified subset for subsequent data collection.
  - Ignores the fact that the sample size is a random quantity.
  - The `if` and `in` restrictions.
Estimation for subpopulations

Total from SRS data

- Data is \( y_1, \ldots, y_n \) and \( S \) is the subset of observations.

\[
\delta_j(S) = \begin{cases} 
1, & \text{if } j \in S \\
0, & \text{otherwise}
\end{cases}
\]

- Subpopulation (or restricted-sample) total:

\[
\hat{Y}_S = \sum_{j=1}^{n} \delta_j(S) w_j y_j
\]

- Sampling weight and subpopulation size:

\[
w_j = \frac{N}{n}, \quad N_S = \sum_{j=1}^{n} \delta_j(S) w_j = \frac{N}{n} n_S
\]
Estimation for subpopulations

Variance of a subpopulation total

Sample \( n \) without replacement from a population comprised of the \( N_S \) subpopulation values with \( N - N_S \) additional zeroes.

\[
\hat{V}(\hat{Y}_S) = \left(1 - \frac{n}{N}\right) \frac{n}{n-1} \sum_{j=1}^{n} \left\{ \delta_j(S)y_j - \frac{1}{n} \hat{Y}_S \right\}^2
\]

Variance of a restricted-sample total

Sample \( n_S \) without replacement from the subpopulation of \( N_S \) values.

\[
\tilde{V}(\tilde{Y}_S) = \left(1 - \frac{n_S}{N_S}\right) \frac{n_S}{n_S-1} \sum_{j=1}^{n_S} \delta_j(S) \left\{ y_j - \frac{1}{n_S} \tilde{Y}_S \right\}^2
\]
Estimation for subpopulations

Example: `svy, subpop()`
Use **svyset** to specify the survey design for your data.

2 Use **svydes** to find strata with a single PSU.

3 Choose your variance estimation method; you can **svyset** it.

4 Use the **svy** prefix with estimation commands.

5 Use **subpop()** instead of **if** and **in**.