Measuring Richness

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Introduction

- Debate on welfare state reform: increasing gap between rich and poor
- Several (income) poverty indices have been developed in the long tradition of the literature
- Measuring richness is a less considered field
- Peichl, A., Schaefer, T. and Scheicher, C. (2006) Measuring Richness and Poverty - A micro data application to Germany and the EU-15, CPE discussion paper 06 - 11.
- Peichl, A. and Schaefer, T. (2006) RICHNESS: Stata module to compute measures of income richness, Statistical Software Components (SSC), Boston College Department of Economics

Outline

1. Poverty and Richness: Poverty measurement

- Net income distribution $x = (x_1, x_2, \ldots, x_n) \in R^n_+$,
  π : poverty line (eg. 60% of median income),
  $p = \#\{i | x_i < \pi,\ i = 1, 2, \ldots, n\}$ number of poor people
- Head count index (fraction poor people):

$$\varphi_{HC}(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{x_i < \pi} = \frac{p}{n},$$

- Foster-Greer-Thorbecke (1984, FGT):

$$\varphi_{FGT}(x) = \frac{1}{n} \sum_{i=1}^{n} \left(\left(\frac{\pi - x_i}{\pi} \right)_+ \right)^{\alpha},$$

($\alpha > 0$ und $y_+ := \max\{y, 0\}$.)

1. Poverty and Richness: Richness measurement

- ρ richness line, $r = \#\{i | x_i > \rho,\ i = 1, 2, \ldots, n\}$ number rich people.
- Head count index: $R_{HC}(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{x_i > \rho} = \frac{r}{n}$.
- Peichl, Schaefer & Scheicher (2006): Richness measure R_α in addition to the head count index:
  - Incomes of the rich only have a lower bound $\rho \Longrightarrow$ transformation of the incomes to the unit interval by a strictly increasing transformation function f,
  - with $f(y) := 1 - \frac{1}{y}$, $y > 1$ and $v(y) := y^\alpha$, $\alpha > 0 \Longrightarrow$ richness index R_α resembling the FGT poverty index:

$$R_\alpha(\mathbf{x}) = \frac{1}{n} \sum_{i=1}^{n} \left(\left(\frac{x_i - \rho}{x_i} \right)_+ \right)^\alpha .$$

2. The Stata program richness: Syntax

- `richness` *varlist* [*if*] [*in*] [*weight*]
  [, `rline(`*rl*`)` | `rval(`*rv*`)` `rnumber(`*rn*`)` `rlfix`]

- There are two ways of defining the richness line:
  1. `rline(rl)` manually defines a number *rl* as the (absolute) richness line (can be any positive number, macro or scalar).
  2. The relative calculation of the richness line is based on a multiplier of a parameter of the distribution of *varname*.

  - `rnumber(`*rn*`)` defines the multiplier *rn*, which can be any positive number (e.g. 200, if you want to specify a richness line of 200%)
  - `rval(`*rv*`)` defines the distributional parameter *rv*, which can be either median (default) or mean.

- `rlfix` specifies that the richness line of the first variable of *varlist* is fixed and used for all other variables of *varlist*.

2. The Stata program richness: The output and saved results

`richness` stores the following results in r():

- *RR* is the matrix with all stored results for *varlist*,
- *Rline_varname* is the value of the (computed or specified) richness line for *varname*,
- *R0_varname* is the headcount index for *varname*,
- *R1_varname* is the PSS index with alpha $= 1$ for *varname*,
- *R2_varname* is the PSS index with alpha $= 2$ for *varname*,
- *R3_varname* is the PSS index with alpha $= 3$ for *varname*.

3. Examples: Change of a rich person's income

Let $\mathbf{a} = (5, 5, 5, 11, 11)$ and $\mathbf{b} = (5, 5, 5, 100, 100)$

```
. richness a

 Richness indices:
------------------------------------------------------------------------------
    RL   HC   R(1)         R(2)         R(3)
a   10   .4   .03636364    .00330578    .00030053
```

By default, the richness line is computed as 200% of median income. The same richness line can be directly specified using the `rline()` option:

```
. richness b, rline(10)

 Richness indices:
------------------------------------------------------------------------------
    RL   HC   R(1)         R(2)   R(3)
b   10   .4   .35999999    .324   .29159999
```

3. Examples: Sensitivity to changes of very high incomes

Let: $\mathbf{c} = (5, 5, 5, 11, 9989)$ and $\mathbf{d} = (5, 5, 5, 1000, 9000)$ where $\mathbf{d}$ is obtained from $\mathbf{c}$ by a progressive transfer of 989 monetary units between the two rich persons.

```
.  richness c
```

	RL	HC	R(1)	R(2)	R(3)
c	10	.4	.21798159	.20125265	.1995502

We could obtain the same (default) richness line of 200% of the median income by specifying the options `rval(median) rnumber(200)`.

```
.  richness d, rval(median) rnumber(200)
```

	RL	HC	R(1)	R(2)	R(3)
d	10	.4	.39777778	.3955758	.39339387

3. Examples: Using options

- Richness line 200% of mean instead of median:

```
. richness a b c d, rval(mean)
```

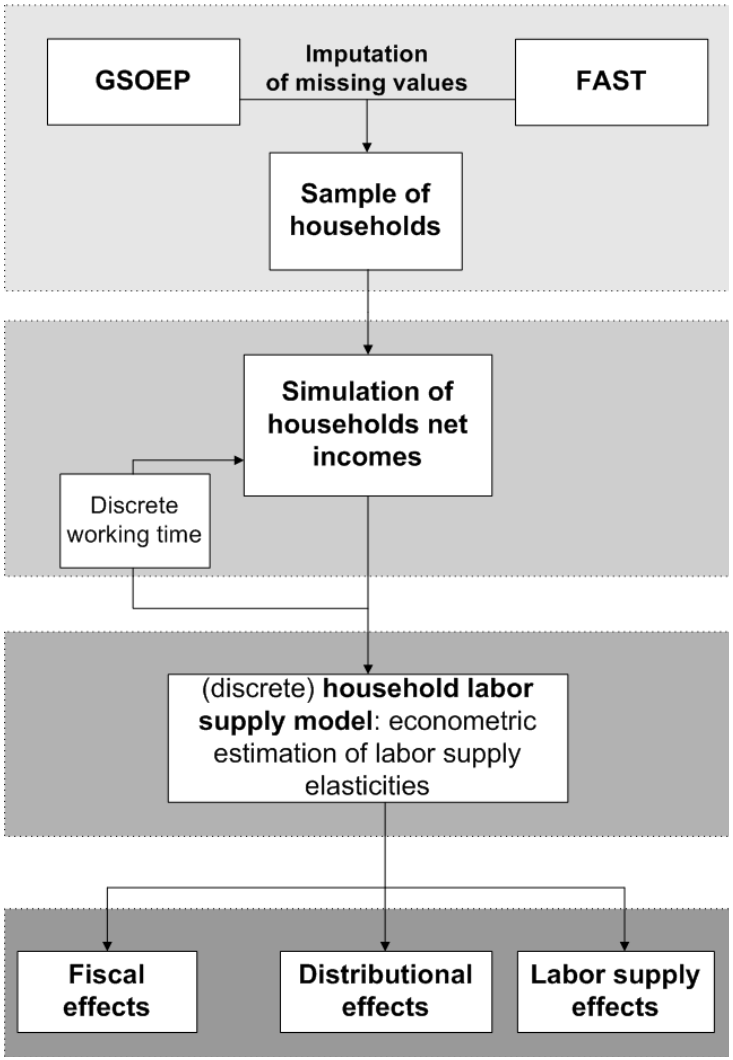
	RL	HC	R(1)	R(2)	R(3)
a	14.8	0	0	0	0
b	86	.4	.056	.00784	.0010976
c	4006	.2	.11979177	.07175034	.0429755
d	4006	.2	.11097778	.06158034	.03417024

- Fixed richness line for scenario comparison:

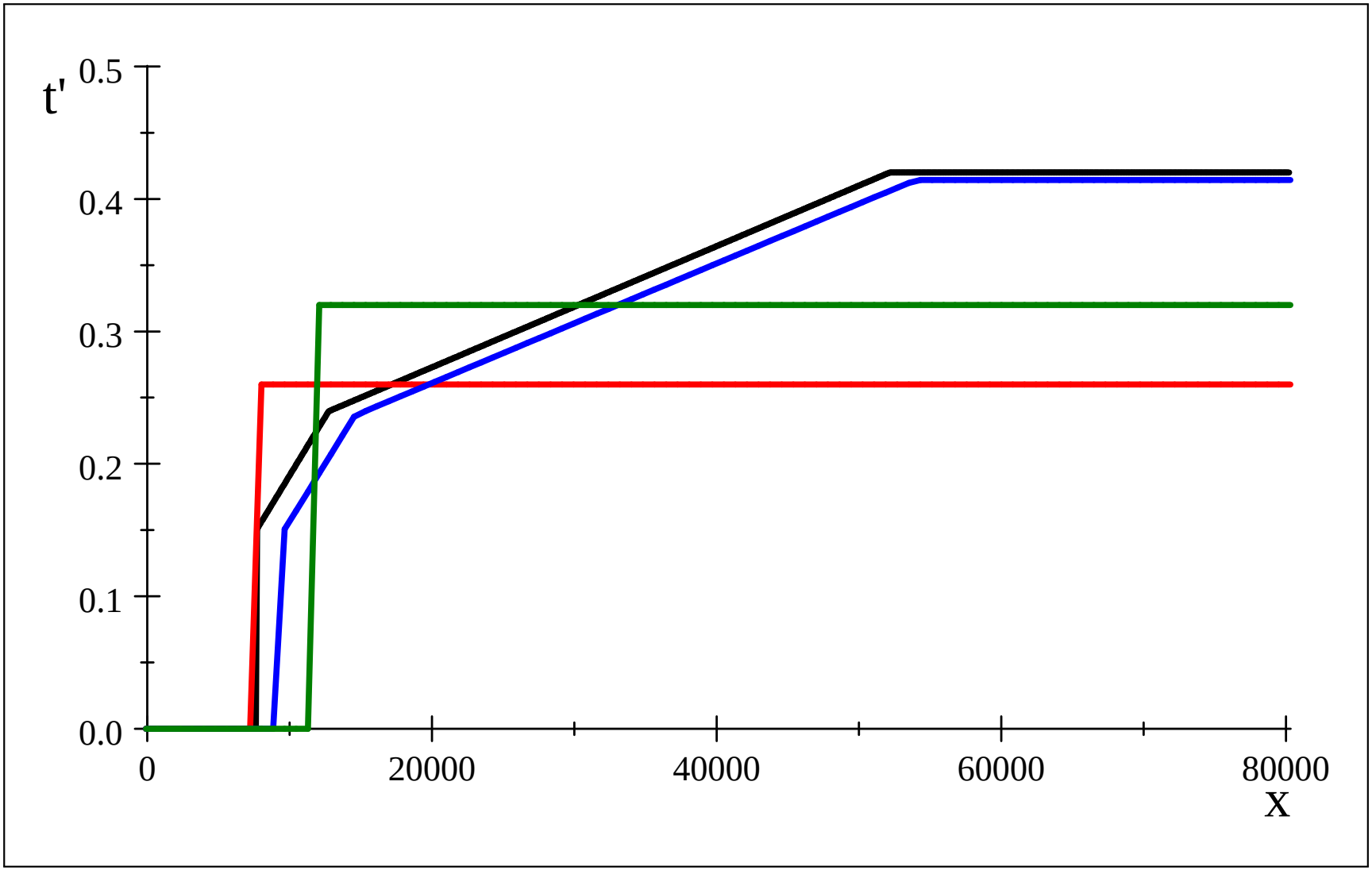
```
. richness a b c d, rval(mean) rlfix
```

	RL	HC	R(1)	R(2)	R(3)
a	14.8	0	0	0	0
b	14.8	.4	.3408	.2903616	.24738808
c	14.8	.2	.19970367	.19940779	.19911234
d	14.8	.4	.39671111	.39346657	.39026573

4. Empirical results: Model FiFoSiM



4. Empirical results: Tax simplification



4. Empirical results: Tax simplification

variabel	φ_{HC}	φ_{FGT} $\alpha=1$	φ_{FGT} $\alpha=2$	R_{HC}	R_α $\alpha=1$	R_α $\alpha=2$
status quo	15,21	4,34	2,53	6,25	1,52	0,57
progr. adjustment	15,20	4,34	2,53	6,06	1,43	0,54
FT1 (26%, 7664)	15,22	4,34	2,53	6,64	1,77	0,73
FT2 (32%, 11650)	15,20	4,33	2,52	5,63	1,39	0,55

fix	φ_{HC}	φ_{FGT} $\alpha=1$	φ_{FGT} $\alpha=2$	R_{HC}	R_α $\alpha=1$	R_α $\alpha=2$
status quo	15,21	4,34	2,53	6,25	1,52	0,57
progr. adjustment	15,20	4,34	2,52	6,19	1,47	0,55
FT1 (26%, 7664)	15,28	4,35	2,53	6,30	1,67	0,69
FT2 (32%, 11650)	15,19	4,33	2,52	5,86	1,46	0,58

Summary

- We propose a new class of richness measures which increase with rich person's income.
- We show that a flat tax with a low marginal tax rate and basic allowance increases poverty and richness, whereas a flat tax with higher tax parameters decreases both. Hence, the first one redistributes in favour of the very high incomes at the expense of the low incomes, whereas the second does the opposite.

Conclusion

- The analysis shows that new richness measures lead to different results in comparison to the headcount index for some of the periods, countries and reform scenarios.
- Our approach is more sophisticated, because it also takes the dimension of changes and not only the number of people beyond a given richness line into account.
- Therefore our approach allows for a distinct analysis of structural changes at the top of the income distribution.
- We propose to use the new measures in addition to the headcount index for a more sophisticated analysis of richness.

End

Thank you for your attention!