

# A Linear Panel Model with Heterogeneous Coefficients and Variation in Exposure<sup>1</sup>

Liyang Sun<sup>1</sup>    Jesse M. Shapiro<sup>2</sup>

<sup>1</sup>CEMFI

<sup>2</sup>Harvard and NBER

2022 Stata Economics Virtual Symposium

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<sup>1</sup>NBER Working Paper #29976, forthcoming at Journal of Economic Perspectives.

## Introduction

- ▶ Social scientists often seek to evaluate the effects of a certain event, such as the adoption of a national policy
- ▶ Running example

THE  
QUARTERLY JOURNAL  
OF ECONOMICS

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Vol. CXXII

February 2007

Issue 1

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THE AGGREGATE EFFECTS OF HEALTH INSURANCE:  
EVIDENCE FROM THE INTRODUCTION OF MEDICARE\*

AMY FINKELSTEIN

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- ▶ One heuristic model:

$$\begin{aligned} \text{Outcome} &= \text{Unit effect} + \text{Time effect} \\ &+ \text{Coefficient (Event} \times \text{Exposure)} + \text{Error} \end{aligned}$$

- ▶ Event  $\times$  Exposure is the term of greatest interest, as it captures the fact that different units are affected differently by the event because of their different exposure to it
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## More examples

- ▶ The heuristic model is usually estimated by the two-way fixed effects (TWFE) estimator
- ▶ Dube and Vargas (2013, equation 1): impact of income shocks on violence in Colombia
- ▶ Dafny et al. (2012, equation 5): impact of a merger on health insurance premiums
- ▶ Nunn and Qian (2011, equation 3): impact of potatoes on Old World population growth
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# Outline

1. Using the Medicare example to illustrate a model of treatment effects heterogeneity
2. Discuss some identification challenges due to unmodeled heterogeneity when there is no group totally unaffected by the event
  - 2.1 there exists *no* estimator that is guaranteed to estimate even a weighted average of unit-specific effects
  - 2.2 TWFE is no exception
3. Solutions: with a group that is totally unaffected by the event
  - 3.1 de Chaisemartin and D'Haultfœuille (2018): estimate an average effect by replacing the TWFE with an average of difference-in-differences estimators
  - 3.2 Stata implementations

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# The effect of Medicare on health care expenditures

- ▶ Medicare is a US government program introduced in 1965 to provide health insurance to *all* the elderly
- ▶ Stylized version of Finkelstein's (2007) study
- ▶ We observe per capita health care expenditures  $y_{st}$  on the elderly for each US state  $s$  in each of two time periods  $t$ :
  - ▶ let  $t = 0$  denote the period before the introduction of Medicare
  - ▶ let  $t = 1$  denote the period after

# Exposure measured by penetration of private insurance

TABLE I  
SHARE OF ELDERLY WITHOUT HOSPITAL INSURANCE, 1963

	Blue Cross	Any insurance
New England (CT, ME, MA, NH, RI, VT)	0.49	0.37
Middle Atlantic (NJ, NY, PA)	0.60	0.41
East North Central, Eastern Part (MI, OH)	0.55	0.32
East North Central, Western Part (IL, IN, WI)	0.75	0.42
West North Central (IA, KS, MN, MO, NE, ND, SD)	0.81	0.47
South Atlantic, Upper Part (DE, DC, MD, VA, WV)	0.75	0.45
South Atlantic, Lower Part (FL, GA, NC, SC)	0.81	0.50
East South Central (AL, KY, MS, TN)	0.88	0.57
West South Central (AR, LA, OK, TX)	0.85	0.55
Mountain (AZ, CO, ID, MT, NV, NM, UT, WY)	0.78	0.50
Pacific (OR, WA, CA, AK, HI)	0.87	0.52
National Total	0.75	0.45

Data are from individuals aged 65 and over in the 1963 National Health Survey. Sample size is 12,757. Minimum sample size for a subregion is 377.

- ▶ Medicare had a relatively small effect on rates of insurance coverage for e.g. a New England state v.s. a Pacific state

## Linear panel data model

- ▶ Formally, let  $x_{st}$  be the fraction of elderly with health insurance in a given state  $s$  at time  $t$ 
  - ▶  $x_{s0}$  measures the fraction of elderly with private insurance in state  $s$  prior to Medicare
  - ▶  $x_{s1}$  as being equal to 1 for all states  $s$  due to the universal coverage afforded by Medicare
- ▶ A linear panel data model of health care expenditures – what we will refer to as the linear model – might then take the form

$$y_{st} = \alpha_s + \delta_t + \beta x_{st} + \varepsilon_{st} \quad (\text{linear model})$$

- ▶ The parameter  $\beta$  measures the causal effect of going from no coverage ( $x_{st} = 0$ ) to full coverage ( $x_{st} = 1$ )

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## Exposure model

- ▶ We can rewrite the linear panel data model closer to the heuristic model
- ▶ Since  $x_{s1} = 1$  for all states, the linear panel data model implies that

$$y_{st} = \tilde{\alpha}_s + \delta_t + \beta(1 - x_{s0})t + \varepsilon_{st} \quad (\text{exposure model})$$

where we have redefined the state fixed effect as

$$\tilde{\alpha}_s = \alpha_s + \beta x_{s0}$$

- ▶ Here  $(1 - x_{s0})$  is the observed exposure variable and the term  $t$  is an indicator for whether the observation is from the post-Medicare period

# TWFE

- ▶ The exposure model is

$$y_{st} = \tilde{\alpha}_s + \delta_t + \beta(1 - x_{s0})t + \varepsilon_{st}$$

- ▶ We can estimate the unknown coefficient  $\beta$  by a two-way fixed effects (TWFE) estimator  $\hat{\beta}$
- ▶ Appealing properties:
  - ▶ if the exposure model holds, and  $\varepsilon_{st}$  is unrelated to  $x_{st}$ , then  $\hat{\beta}$  is unbiased for  $\beta$
  - ▶ if further  $\varepsilon_{st}$  are homoskedastic and not clustered / serially correlated, then  $\hat{\beta}$  is also efficient
- ▶ The exposure model implies that the effect of Medicare on expenditures is  $\beta(1 - x_{s0})$ 
  - ▶ The per-unit effect of insurance on expenditures is the same across states
  - ▶ Effects differ across states only due to different effect of Medicare on insurance rate:  $(1 - x_{s0})$

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## Heterogeneous coefficients

- ▶ Imagine that each state  $s$  has its own coefficient  $\beta_s$  describing the effect of insurance on expenditures in the state:

$$y_{st} = \alpha_s + \delta_t + \beta_s x_{st} + \varepsilon_{st} \quad (\text{heterogeneous model})$$

- ▶ For example, a state with a less healthy uninsured population may see expenditures rise more in response to a given expansion in insurance
  - ▶ Only the least healthy elderly remain uninsured so that the uninsured population is less healthy in states with greater insurance penetration prior to Medicare (high  $x_{s0}$  and high  $\beta_s$ )

## Behavior of the TWFE estimator

- ▶ We are still maintaining that the error term  $\varepsilon_{st}$  is unrelated to  $x_{st}$  as before, so absent changes in the insurance levels  $x_{st}$ , all states would follow identical average trends over time
- ▶ How reasonable would the TWFE estimator  $\hat{\beta}$  be, which is based on the exposure model that assumes all states have the same  $\beta$ ?
- ▶ Recent literature has investigated the expected value of the TWFE estimator  $\hat{\beta}$  under common trends assumptions
  - ▶ Most closely related: Callaway, Goodman-Bacon, and Sant'Anna (2021)

## Expected value of the TWFE estimator

- ▶ Under the heterogeneous model, the expected value of the two-way fixed effects (TWFE) estimator of the exposure model, given the data  $x = \{x_{10}, \dots, x_{S0}\}$  for states  $s \in \{1, \dots, S\}$ , is given by

$$E(\hat{\beta}|x) = \frac{\text{Cov}(\beta_s(1 - x_{s0}), (1 - x_{s0}))}{\text{Var}(1 - x_{s0})}$$

claim

- ▶ In certain situations,  $\hat{\beta}$  is still centered on an average of the true state-level coefficients  $\beta_s$ .
  - ▶ One situation is where  $\beta_s$  is unrelated to (i.e., statistically independent of)  $(1 - x_{s0})$
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## A numerical example

- ▶ Let the coefficient  $\beta_s$  vary across states according to the equation

$$\beta_s = 1 + 0.5\lambda - \lambda x_{s0} \quad (\text{numerical example})$$

- ▶  $\lambda$  is a parameter that governs how the state-level coefficient  $\beta_s$  is related to the fraction of elderly with insurance before Medicare
- ▶ When  $\lambda = 0$ , the coefficient  $\beta_s$  is equal to 1 in all states regardless of prior insurance penetration
- ▶ When  $\lambda < 0$ , states with greater insurance penetration prior to Medicare have a larger coefficient  $\beta_s$
- ▶ Set  $x_{s0} = 0.245 + s/100$  so that no matter the value of  $\lambda$ , the average value of  $\beta_s$  across all states is always 1

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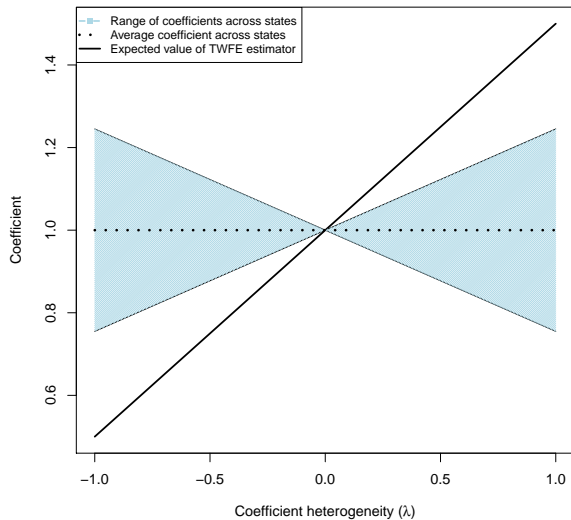
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# Illustration



identification

## Intuition

- ▶ When  $\lambda > 0$ , states with a larger increase in insurance coverage,  $(1 - x_{s0})$ , also have larger coefficients  $\beta_s$
- ▶ Following Medicare's introduction, expenditure therefore grows more in states with larger  $(1 - x_{s0})$  because
  - ▶ these states experience a larger increase in insurance coverage
  - ▶ these states experience a larger change in expenditure for a given change in insurance coverage
- ▶ TWFE estimator  $\hat{\beta}$  conflates them, thus overstating the effect of insurance on expenditure.
- ▶ In the numerical example, this conflation is so severe that the expected value of the TWFE estimator falls outside the range of the true coefficients  $\beta_s$

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## Identification challenge

- ▶ The TWFE estimator  $\hat{\beta}$  cannot, in general, be guaranteed
  - ▶ to center around the average  $\beta_s$  across the states
  - ▶ to center around a value inside the range of the true coefficients  $[\min_s \beta_s, \max_s \beta_s]$
- ▶ This phenomenon is not specific to the TWFE estimator
- ▶ Without any restriction on coefficients  $\beta_s$  and if  $x_{s0} \in (0, 1)$  for all states, then no estimator is guaranteed to center around a value inside  $[\min_s \beta_s, \max_s \beta_s]$  claim

## Proof:1/2

- ▶ Consider the special case with  $S = 2$ , some  $x_{s0}$ 's with  $0 < x_{20} \leq x_{10} < 1$ ,  $\beta_1 < \beta_2$ , and  $\delta_0$  known to be zero
- ▶ The model for the data is then

$$y_{s0} = \alpha_s + \beta_s \cdot x_{s0} + \varepsilon_{s0}$$

$$y_{s1} = \alpha_s + \delta_1 + \beta_s + \varepsilon_{s1}$$

with parameters  $\theta = (\{(\alpha_s, \beta_s)\}_{s=1}^2, \delta_1, F_{\varepsilon|X})$ , for  $F_{\varepsilon|X}$  the distribution of  $(\varepsilon_{s0}, \varepsilon_{s1})$  conditional on  $x_{s0}$

- ▶ The distribution of the data we observe is then

$$F_{Y_0, Y_1|X}(y_0, y_1 | x_{s0} = x; \theta)$$

## Proof:1/2

- ▶ Given any parameter  $\theta$ , define the distinct parameter

$$\theta' = \left( \left\{ \left( \alpha'_s, \beta'_s \right) \right\}_{s=1}^2, \delta'_1, F_{\varepsilon|X} \right) \text{ given by}$$

$$\theta' = \left( \left\{ \left( \alpha_s + \frac{\Delta \cdot x_{s0}}{1 - x_{s0}}, \beta_s - \frac{\Delta}{1 - x_{s0}} \right) \right\}_{s=1}^2, \delta_1 + \Delta, F_{\varepsilon|X} \right)$$

for some  $\Delta > (\beta_2 - \beta_1) \cdot (1 - x_{20}) > 0$ .

- ▶ Parameter  $\theta$  and  $\theta'$  are observationally equivalent:

$$F_{Y_0, Y_1|X}(y_0, y_1 \mid x_{s0} = x; \theta') = F_{Y_0, Y_1|X}(y_0, y_1 \mid x_{s0} = x; \theta)$$

### calculations

- ▶ For any estimator  $\hat{\beta}'$  that depends on the data, the expected value must be the same under  $\theta$  and  $\theta'$
- ▶ However, the  $\Delta$  is chosen such that

$$\beta'_1 = \beta_1 - \frac{\Delta}{1 - x_{10}} < \beta_2 - \frac{\Delta}{1 - x_{20}} = \beta'_2 < \beta_1 < \beta_2$$

TWFE

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## A Difference-in-Differences Perspective

- ▶ This identification challenge applies to all estimators
- ▶ Consider an exposure-adjusted difference-in-differences estimator, which provides one possible way to account for changes in insurance rates and addresses the conflation problem of TWFE

$$\hat{\beta}_{s,s'}^{DID} = \frac{(y_{s1} - y_{s0}) - (y_{s'1} - y_{s'0})}{(1 - x_{s0}) - (1 - x_{s'0})}$$

## A Difference-in-Differences Perspective

- ▶ de Chaisemartin and D'Haultfœuille (2018) call  $\hat{\beta}_{s,s'}^{DID}$  a Wald-difference-in-differences estimator because it consists of the ratio of the difference-in-differences estimator for the outcome (in our case, expenditures) to the one for exposure (insurance)

$$\hat{\beta}_{s,s'}^{DID} = \frac{(y_{s1} - y_{s0}) - (y_{s'1} - y_{s'0})}{(1 - x_{s0}) - (1 - x_{s'0})}$$

- ▶ As with the TWFE estimator, this estimator can be centered around a value outside the range of coefficients, including in our numerical example if  $x_{s0}, x_{s'0} \in (0, 1)$

# Solutions

- ▶ **Impose further structure on the coefficients  $\beta_s$** 
  - ▶ For example, suppose that a researcher is willing to posit a linear relationship between  $\beta_s$  and  $x_{s0}$ , but does not know the value of the parameter  $\lambda$  that governs this relationship
  - ▶ Then substitute the expression for  $\beta_s$  to arrive at a linear panel model whose unknown parameter,  $\lambda$ , can be estimated by a two-way fixed effects estimator
- ▶ Bounds on variation in coefficients and mean of the error term (Manski and Pepper, 2018)
- ▶ More data: a “close to” totally unaffected state ( $x_{s'0} = 1$ ) and/or a control state ( $x_{s'0} = x_{s'1} = 0$ )

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  - ▶ Then this  $\hat{\beta}_{s,s'}^{DID}$  is unbiased for  $\beta_s$ , the true coefficient for the affected state  $s$

$$\hat{\beta}_{s,s'}^{DID} = \frac{(y_{s1} - y_{s0}) - (y_{s'1} - y_{s'0})}{(1 - x_{s0})}$$

- ▶ The presence of an unaffected state brings it closer to the classical difference-in-differences setting of Card and Krueger, 1994)
- ▶ Average of  $\hat{\beta}_{s,s'}^{DID}$  is centered around the average of true coefficients for all affected states  $s \neq s'$ .

# Solutions

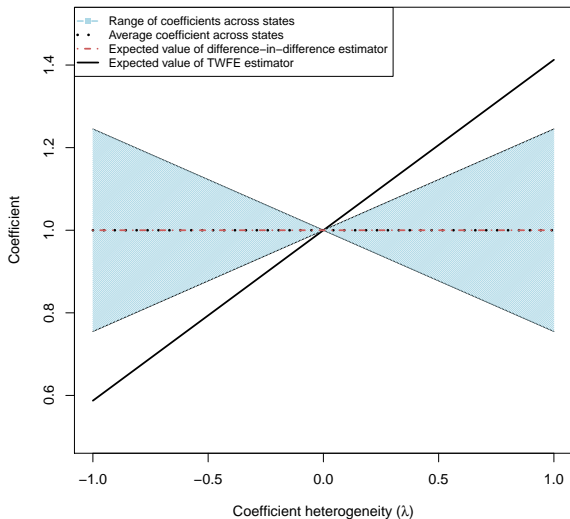
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# Solutions

- ▶ It does not repair the TWFE estimator  $\hat{\beta}$



# Solutions

- ▶ Recent papers propose alternative estimators in a range of settings
  - ▶ `csdid` implements Callaway and Sant'Anna (2021)
  - ▶ `fuzzydid` and `did_multipliedgt` implement de Chaisemartin and D'Haultfœuille (2018, 2020) respectively
  - ▶ `drdid` implements Sant'Anna and Zhao (2020)
  - ▶ `eventstudyinteract` implements Sun and Abraham (2021)
  - ▶ among others, more detailed reviews in Freyaldenhoven, Hansen, Pérez, and Shapiro (2021), de Chaisemartin and D'Haultfœuille (2022), Roth, Sant'Anna, Bilinski, and Poe (2022)

## Conclusion

- ▶ An active literature tries to interpret the two-way fixed effects (TWFE) estimator, in the presence of unmodeled coefficient heterogeneity
- ▶ We illustrate some implications for the case where the research design takes advantage of variation across units (say, US states) in exposure to some treatment
- ▶ TWFE can still fail to estimate the average of the units' coefficients
  - ▶ With unmodeled heterogeneity and without totally unaffected states, there exists *no* estimator that is guaranteed to estimate a value inside the true range
- ▶ Building on the literature, we note that when there is a totally unaffected unit, it is possible to estimate an average effect by an average of difference-in-differences estimators

## References

- ▶ Callaway, Brantly, Andrew Goodman-Bacon, and Pedro Sant'Anna, "Difference-in-Differences with a Continuous Treatment," ArXiv:2107.02637 [Econ], 2021.
- ▶ Callaway, Brantly and Pedro H.C. Sant'Anna, "Difference-in-Differences with Multiple Time Periods," *Journal of Econometrics*, 2021, 225 (2), 200–230. Themed Issue: Treatment Effect 1.
- ▶ Card, David and Alan B. Krueger, "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania," *American Economic Review*, September 1994, 84 (4), 772–793.
- ▶ de Chaisemartin, Clément and Xavier D'Haultfœuille, "Fuzzy Differences-in-Differences," *Review of Economic Studies*, April 2018, 85 (2), 999–1028.
- ▶      and     , "Two-Way Fixed Effects Estimators with Heterogeneous Treatment Effects," *American Economic Review*, September 2020, 110 (9), 2964–2996.



## References

- ▶ Finkelstein, Amy, “The Aggregate Effects of Health Insurance: Evidence from the Introduction of Medicare,” *Quarterly Journal of Economics*, February 2007, 122 (1), 1–37.
- ▶ Sant’Anna, Pedro H. C. and Jun Zhao, “Doubly Robust Difference-in-differences Estimators,” *Journal of Econometrics*, November 2020, 219 (1), 101–122.
- ▶ Sun, Liyang and Sarah Abraham, “Estimating Dynamic Treatment Effects in Event Studies with Heterogeneous Treatment Effects,” *Journal of Econometrics*, 2021, 225 (2), 175–199. Themed Issue: Treatment Effect 1.
- ▶ de Chaisemartin, Clément and d’Haultfœuille, Xavier, “Two-Way Fixed Effects and Differences-in-Differences with Heterogeneous Treatment Effects: A Survey” Forthcoming at *Econometrics Journal*

## References

- ▶ Freyaldenhoven, Simon, Christian Hansen, Jorge Pérez Pérez, and Jesse M. Shapiro. “Visualization, identification, and estimation in the linear panel event-study design.” *Advances in Economics and Econometrics: Twelfth World Congress* (Forthcoming)
- ▶ Jon Roth, Pedro Sant’Anna, Alyssa Bilinski, and John Poe. What’s Trending in Difference-in-Differences? A Synthesis of the Recent Econometrics Literature Forthcoming at *Journal of Econometrics*

## Expected value of the TWFE estimator

### Claim

Under the heterogeneous model, the expected value of the two-way fixed effects (TWFE) estimator of the exposure model, given the data  $x = \{x_{10}, \dots, x_{S0}\}$  for states  $s \in \{1, \dots, S\}$ , is given by

$$E(\hat{\beta}|x) = \frac{\text{Cov}(\beta_s(1 - x_{s0}), (1 - x_{s0}))}{\text{Var}(1 - x_{s0})}$$

where  $\text{Cov}(\cdot, \cdot)$  and  $\text{Var}(\cdot)$  denote the sample covariance and variance, respectively, and the expectation  $E(\hat{\beta}|x)$  is taken with respect to the distribution of the errors  $\varepsilon_{st}$  conditional on the data  $x = \{x_{10}, \dots, x_{S0}\}$ . [back](#)

# Underidentification

## Claim

There exists no estimator  $\hat{\beta}'$  that can be expressed as a function of the data  $\{(x_{s0}, y_{s0}, y_{s1})\}_{s=1}^S$  and whose expected value is guaranteed to be contained in  $[\min_s \beta_s, \max_s \beta_s]$  for any heterogeneous model and any  $\{x_{s0}\}_{s=1}^S$ .

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## Details

We show that the two parameter values  $\theta$  and  $\theta'$  are observationally equivalent, which means the expected value of  $\hat{\beta}'$  must be the same under  $\theta$  and  $\theta'$ . To see this, note that the distribution of  $(y_{s0}, y_{s1})$  conditional on  $x_{s0}$  is the same under  $\theta$  and  $\theta'$ :

$$\begin{aligned} & F_{Y_0, Y_1 | X}(y_0, y_1 \mid x_{s0} = x; \theta) \\ &= \Pr \{ \varepsilon_{s0} \leq y_0 - \alpha_s - \beta_s \cdot x, \varepsilon_{s1} \leq y_1 - \alpha_s - \delta_1 - \beta_s \mid x_{s0} = x; \theta \} \\ &= \Pr \{ \varepsilon_{s0} \leq y_0 - \alpha_s - \beta_s \cdot x, \varepsilon_{s1} - \varepsilon_{s0} \leq y_1 - y_0 - \delta_1 - \beta_s (1 - x) \mid x_{s0} = x; \theta \} \\ &= \Pr \left\{ \begin{array}{l} \varepsilon_{s0} \leq y_0 - \left( \alpha_s + \frac{\Delta \cdot x}{1-x} \right) - \left( \beta_s - \frac{\Delta}{1-x} \right) \cdot x, \\ \varepsilon_{s1} - \varepsilon_{s0} \leq y_1 - y_0 - (\delta_1 + \Delta) - \left( \beta_s - \frac{\Delta}{1-x} \right) (1 - x) \end{array} \mid x_{s0} = x; \theta \right\} \\ &= \Pr \left\{ \begin{array}{l} \varepsilon_{s0} \leq y_0 - \alpha'_s - \beta'_s \cdot x, \\ \varepsilon_{s1} - \varepsilon_{s0} \leq y_1 - y_0 - \delta'_1 - \beta'_s (1 - x) \end{array} \mid x_{s0} = x; \theta' \right\} \\ &= F_{Y_0, Y_1 | X}(y_0, y_1 \mid x_{s0} = x; \theta'). \end{aligned}$$

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