Causal Inference with Formula Instruments

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Based on Borusyak, Hull, & Jaravel '22, Borusyak & Hull '23, and Borusyak, Hull, & Jaravel '24

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How can we just leverage the exogenous shocks for identification?

The Problem: non-random exposure to exogenous shocks generally makes such formulas invalid instruments

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Controlling for μ_i or using the *recentered IV* $\tilde{z}_i = z_i - \mu_i$ avoids bias

• Or controlling for w_i that are known to linearly span μ_i

(Some) Empirical Settings Where This May be Relevant

- Network spillovers: Miguel and Kremer 2004, Gerber and Green 2012, Acemoglu et al. 2015, Jaravel et al. 2018, Carvalho et al. 2020
- Effects of transportation: Baum-Snow 2007, Donaldson and Hornbeck 2016, Lin 2017, Donaldson 2018, Ahlfeldt and Feddersen 2018, Bartelme 2018
- Simulated instruments: Currie and Gruber 1996a,b, Cullen and Gruber 2000, East and Kuka 2015, Cohodes et al. 2016, Frean et al. 2017
- Shift-share/Bartik IV: Autor et al. 2013, Adão et al. 2021, Kovak 2013
- Nonlinear shift-share IV: Boustan et al. 2013, Berman et al. 2015, Basso and Peri 2015, Chodorow-Reich and Wieland 2020, Derenoncourt 2021
- IVs from assignment mechanisms: Abdulkadiroglu et al. 2017, 2019
- Weather IVs: Gomez et al. 2007, Madestam et al. 2013
- IVs for mass media access: Olken 2009, Yanagizawa-Drott 2014

Economic theory suggests transportation upgrades affect local outcomes (e.g. land value) of regions i by increasing their market access (MA):

$$\Delta \log V_i = eta \Delta \log MA_i + arepsilon_i,$$

where $MA_{it} = \sum_j au(g_t, loc_i, loc_j)^{-1} pop_j,$

for road network g_t in periods t = 1, 2, region locations loc_j (co-determining travel cost τ), and regional population pop_j

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Imagine an experiment that randomly connects adjacent regions by road

- MA only grows because of the random transportation shocks
- So can we view variation in MA growth as random and just run OLS?

No, because of the non-random components of the formula

Start from no roads, assume equal population everywhere



Randomly connect adjacent regions by road



Randomly connect adjacent regions by road and compute MA growth



Counterfactual roads and MA growth



Counterfactual roads and MA growth



Expected Market Access Growth μ_i

Some regions get systematically more MA



149 lines were built or planned (as of April 2019)



The 83 lines actually built by 2016. Suppose timing is random



A counterfactual draw of 83 lines by 2016



Expected MA growth, μ_i



Recentered MA growth

Recentered MA growth, $\Delta \log MA_i - \mu_i$



General Setting

We have a model of $y_i = \beta x_i + \varepsilon_i$ for a fixed population $i = 1 \dots N$

• Extensions: heterogeneous effects, other controls, multiple treatments, panel data...

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- Applies to any z_i which can be constructed from observed data
- Nests reduced-form regressions: $x_i = z_i$
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Assumptions:

- **(**) Shocks are exogenous: $g \perp \varepsilon \mid w$, for w = (s,q)
- 2 Conditional distribution G(g | w) is known (e.g. via randomization protocol or uniform across permutations of g)

The expected instrument, $\mu_i = \mathbb{E}[f_i(g, w) | w] \equiv \int f_i(g, w) dG(g | w)$, is the sole confounder generating bias:

$$\mathbb{E}\left[\frac{1}{N}\sum_{i} z_{i}\varepsilon_{i}\right] = \mathbb{E}\left[\frac{1}{N}\sum_{i} \mu_{i}\varepsilon_{i}\right] \neq 0, \text{ in general}$$

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Regressions which control for μ_i also identify β (implicit recentering)

- **Consistency**: with many shocks and \tilde{z}_i weakly dependent across *i*
- **Robustness** to heterogeneous treatment effects: \tilde{z}_i identifies a convex avg. of $\partial y_i / \partial x_i$ under appropriate first-stage monotonicity
- Randomization inference provides exact confidence intervals for β (under constant effects) and falsification tests

Special Case: Linear Formulas

When $z_i = \sum_k s_{ik}g_k$ is linear in the shocks (i.e. shift-share IV), we need only specify the conditional shock *mean*:

$$\mu_i = E[z_i \mid w] = \sum_k s_{ik} E[g_k \mid w]$$

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If we assume $E[g_k | w] = q'_k \gamma$ for some shock-level controls q_k , this tells us it's enough to control for $c_i = \sum_k s_{ik}q_k$

 Special case: E[g_k | w] = μ_g (i.e. unconditionally exogenous shocks), so c_i is the "sum of shares" Σ_k s_{ik} (often = 1 in practice)

Rather than focusing on the design of exogenous shocks, we could model the unobserved error ε_i 's dependence on w:

 $E[\varepsilon_i \mid g, w] = q_i' \gamma, \text{ for } q_i \in w$

E.g., if y_i and x_i are in first differences, $E[\varepsilon_i | g, w] = \gamma$ is "parallel trends"

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In practice, researchers may have stronger priors on how observed shocks are assigned than the right model for ε_i

- Does parallel trends hold in logs vs. levels? (Roth and Sant'Anna '23)
- What are the right features of *w* to include in *q_i*?

Application 1: The China Shock (Autor et al. 2013)

ADH study the effects of rising Chinese import competition on US commuting zones over two periods: 1991-2000 and 2000-2007

- Treatment x_{it}: local growth of Chinese imports in \$1,000/worker
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To address endogeneity, they use a shift-share IV $z_{it} = \sum_{n} s_{int} g_{nt}$

- *n*: 397 SIC4 manufacturing industries × two periods
- g_{nt}: growth of Chinese imports in non-US economies per US worker
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Design-based justification: random industry productivity shocks in China, jointly affecting imports in the U.S. and elsewhere, proxied by g_{nt}

• If g_{nt} is as-good-as-randomly assigned within (but not across) periods, the expected instrument is $\mu_{it} = \sum_{n} s_{int} \times Post_t$

ADH Balance Tests

Balance variable	Coef.		SE
Panel A: Industry-level balance			
Production workers' share of employment, 1991	-0.011		(0.012)
Ratio of capital to value-added, 1991	-0.007		(0.019)
Log real wage (2007 USD), 1991	-0.005		(0.022)
Computer investment as share of total, 1990	0.750		(0.465)
High-tech equipment as share of total investment, 1990	0.532		(0.296)
No. of industry-periods		794	
Panel B: Regional balance			
Start-of-period % of college-educated population	0.915		(1.196)
Start-of-period % of foreign-born population	2.920		(0.952)
Start-of-period % of employment among women	-0.159		(0.521)
Start-of-period % of employment in routine occupations	-0.302		(0.272)
Start-of-period average offshorability index of occupations	0.087		(0.075)
Manufacturing employment growth, 1970s	0.543		(0.227)
Manufacturing employment growth, 1980s	0.055		(0.187)
No. of region-periods		1,444	

- Panel A regresses industry characteristics on the g_{nt} shocks, controlling for period FE
- Panel B regresses location characteristics on the z_{it} instrument, controlling for manufacturing employment share \times period FE

ADH Estimates

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Coefficient	-0.596 (0.114)	-0.489 (0.100)	-0.267 (0.099)	-0.314 (0.107)	-0.310 (0.134)	-0.290 (0.129)	-0.432 (0.205)
Regional controls							
Autor et al. (2013) controls	\checkmark	\checkmark	\checkmark		\checkmark	\checkmark	\checkmark
Start-of-period mfg. share	\checkmark						
Lagged mfg. share		\checkmark	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Period-specific lagged mfg. share			\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Lagged 10-sector shares					\checkmark		\checkmark
Local Acemoglu et al. (2016) controls						\checkmark	
Lagged industry shares							\checkmark
SSIV first stage <i>F</i> -stat. No. of region-periods No. of industry-periods	185.6 1,444 796	166.7 1,444 794	123.6 1,444 794	272.4 1,444 794	64.6 1,444 794	63.3 1,444 794	27.6 1,444 794

Note: columns 3-7 control for mfg. employment share \times period FE

Application 2: Chinese HSR (Borusyak and Hull, 2023)

Let's return to the motivating market access application

Setting: Chinese HSR; 83 lines built 2008-2016, 66 yet unbuilt

- Market access: $MA_{it} = \sum_{k} \exp(-0.02\tau_{ikt}) p_{k,2000}$, where τ_{ikt} is HSR-affected travel time between prefecture capitals (Zheng and Kahn, 2013) and $p_{i,2000}$ is prefecture *i*'s population in 2000
- Relate to employment growth in 274 prefectures, 2007-2016

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- Relate to employment growth in 274 prefectures, 2007-2016

Design: which planned lines opened by some date is as-good-as-random, conditional on line observables (e.g. line length/complexity)

• Expected market access growth given by permuting line openings among observably similar lines

HSR Lines and Market Access



Naive OLS compares dark ("treatment") vs light ("control") regions

Naive OLS Suggests a Big Market Access Effect...



... but we probably shouldn't believe it

HSR Lines and Counterfactuals



Counterfactuals permute which lines opened by 2016, conditional on length

An Example Counterfactual HSR Network



Seems ok ...

Expected Market Access Across Counterfactuals



Darker regions see more MA growth regardless of which lines are built first

Recentered Market Access



Recentered IV compares region that saw more MA growth than expected (in red) to those that saw less MA growth than expected (in blue)

Balance Tests

	Unadjusted	Recentered		l
	(1)	(2)	(3)	(4)
Distance to Beijing	-0.292	0.069		0.089
	(0.063)	(0.040)		(0.045)
Latitude/100	-3.323	-0.325		-0.156
	(0.648)	(0.277)		(0.320)
Longitude/100	1.329	0.473		0.425
	(0.460)	(0.239)		(0.242)
Expected Market Access Growth			0.027	0.056
			(0.056)	(0.066)
Constant	0.536	0.014	0.014	0.014
	(0.030)	(0.018)	(0.020)	(0.018)
Joint RI p-value		0.489	0.807	0.536
R^2	0.823	0.079	0.007	0.082
Prefectures	274	274	274	274

Recentered MA growth can't be reliably predicted from geography

No Market Access Effect with Recentering/Controlling

	Unadjusted	Recentered	Controlled
	OLS	IV	OLS
	(1)	(2)	(3)
Panel A. No Controls			
Market Access Growth	0.232	0.081	0.069
	(0.075)	(0.098)	(0.094)
		[-0.315, 0.328]	[-0.209, 0.331]
Expected Market Access Growth			0.318
-			(0.095)
Panel B. With Geography Controls			
Market Access Growth	0.132	0.055	0.045
	(0.064)	(0.089)	(0.092)
		[-0.144, 0.278]	[-0.154, 0.281]
Expected Market Access Growth			0.213
			(0.073)
Recentered	No	Yes	Yes
Prefectures	274	274	274

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A "design-based" approach to formula IVs sheds new light on longstanding identification strategies in economics (e.g. shift-share IV)...

- ... while also suggesting novel strategies leveraging more complex instrument constructions (e.g. market access models)
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