North-South Trade, Capital Accumulation and Personal Distribution of Wealth and Income*

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Abstract

Within a two-country, North-South framework of capital accumulation, the effects of trade on long-run capital stock and personal distribution of wealth and income are analyzed. Households face idiosyncratic preference shocks, resulting in nondegenerate distribution of wealth and income. In the North, free trade in goods lowers the aggregate stock of capital – a source of long-run social welfare loss. Further, it increases the variance of capital holding across households. Thus wealth inequality, measured by the coefficient of variation, increases. Income inequality and welfare inequality also increase. The opposite holds in the South. With free trade in goods being the initial situation, free international borrowing increases (decreases) the long-run capital stock in the North (South). But at the same time inequality further increases in the North and declines in the South.

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Key Words: Capital Accumulation, Income Inequality, International Borrowing/Lending, International Trade, Wealth Inequality

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1 Introduction

The issue of international trade policy and income distribution has a long history, going as far back as Ricardo and the debate in England on the effect of free trade on workers and capitalists. The theoretical culmination of this debate lies in the famous Stolper-Samuelson Theorem, discovered in 1940s. Since then, the "Theorem" is seen as the central proposition concerning the *long-run* effect of trade policy changes on income distribution (while the sector-specific model is appealed to evaluate the short-run effects). The implications of the Theorem (or those of the sector-specific model) relate to *functional* distribution of income. However, if equity is the concern, what is more important is *personal* distribution of wealth, income and welfare. It is because the modern economic society is quite different from the "classical" dichotomous "capitalist" society consisting of workers without significant capital ownership or capitalists without significant labor income. The transaction costs of acquiring and disposing assets are quite low today and we observe – both in developed as well as developing countries – a vast cross-section of a "mixed class" households having labor income and nonlabor income from assets. Therefore, there is no straightforward one-to-one implication of changes in functional distribution toward personal distribution. As Atkinson (1997), echoing Dalton on the state of economic analysis, has aptly written, "..the relationship between the factor [functional] distribution with the personal distribution of income is typically not spelled out... The factor [functional] distribution is certainly part of the story, but it is only part, and the other links in the chain need to receive attention".

The effect of international trade on personal distribution of wealth and income – almost a totally neglected subject matter thus far – is the issue addressed in this paper. This requires that economic agents be heterogeneous in some sense. Heterogeneity can, of course, result from various sources. Also, the basis of trade can be different. Thus the issue at hand is quite multidimensional. Das

(1999b) has analyzed it in the context of North-North trade, i.e., trade among similar countries. Households or dynasties are heterogeneous in terms of their preferences and preferences remain constant over generations. Capital is tradable and reproducible. It is shown that the degree of risk-aversion over consumption is a critical factor in how free trade may affect inequality (measured by the coefficient of variation). In a similar heterogeneous-cum-stable preference framework, the implications of North-South trade have been considered in Das (1999c). The tradable asset is not capital but 'land' whose endowment is given. Hence there is only distributional dynamics, no aggregate dynamics. Free trade affects the wage/rental ratio as per the Theorem. This, in turn, affects the demand for land, rate of return on land and thereby the distribution of land (or asset) and income. It also incorporates labor-leisure choice and hence labor income inequality. Free trade is shown to increase inequality in the North and reduce it in the South. Moreover, three classes, namely, upper class, middle class and lower class, are defined in a precise way and it is shown that free trade leads to a shrinkage of the size of the middle class in the North and an expansion of the same in the South.

This paper also examines wealth and income distribution in the context of North-South trade, but it departs from Das (1999c) in three major respects. First, instead of 'land', it considers reproducible capital (but no endogenous growth). That North (South) is a relatively capital (labor) abundant country is *derived* from more primitive differences. In the process, the aggregate model departs from the standard Heckscher-Ohlin (HO) model in many significant ways. Second, preferences are not stable: they change from one time period to the next in a random fashion. This is similar to Lucas (1992) and Atkeson and Lucas (1992). In other words there are noninsurable, idiosyncratic preference shocks. As noted by Lucas (1992), this permits predictions on wealth-income mobility. Third, not only the effects of free trade in goods but also those of free trade in assets among countries are assessed. Hence this paper presents a more general and comprehensive analysis compared to Das (1999c). As a price of all this, the only less general aspect is the lack of labor income inequality and the three-class structure.

More specifically, this paper develops and explores a two-country, three-good, three-factor, two-asset model of the world economy. Three goods are two consumer goods and an investment (or a capital) good. Labor, capital and a specific factor (called M) in the investment good sector are the three factors. The difference in the endowment of the specific factor makes one country (North) capital-rich and the other (South) capital-poor. Hence it is an endowment difference at a more primitive level which implies the standard endowment differences in the static theory. Besides capital, loans are the other asset.

While distribution in the focus, the analysis yields interesting results on the aggregate effects of trade also. Some of the paper's findings are as follows. Free trade in goods between North and South leads to a decline (an increase) in the long-run capital stock in the North (South). In the North for example, the wage/rental ratio falls due to free trade, via the Theorem. By income effect this lowers the household demand for capital as an asset. In equilibrium less capital is produced. The opposite holds in the South. This is a source of long-run welfare loss (gain) for the North (South).

Free trade, via the Theorem, raises (lowers) the rate of return on capital in the North (South) The variance of capital holding is shown to be positively related to the rate of return on capital and thus the former rises in the North and falls in the South. Coupled with the effect on the capital stock, it follows that wealth inequalilty, measured by the coefficient of variation, increases in the North and decreases in the South. The same holds for inequality in terms of income and individual welfare. This echoes the corresponding finding in Das (1999c) and underscores that it is a robust result.

Wealth-income mobility is shown to vary positively with 'fixed' (non-capital) income. This falls (rises) in the North (South), implying that such mobility decreases in the North and increases in the South.

Finally, given that social welfare is defined as an increasing function of aggregate welfare (sum of individual welfares) and a negative function of variance of welfare, an overall important result is that the South unambiguously gains from free trade, while the North may gain or lose.

In contrast to free trade in goods, free trade in assets (together with free trade in goods) tends to equalize the rate of return on capital and thereby tends to lower (raise) the rate of return on capital in the North (South). This implies that (i) capital stock in the North (South) increases (decreases) and (ii) in equilibrium the North (South) is the debtor (creditor) country. The impact on inequality and mobility is however similar to that of free trade in goods but mechanisms are quite different.

The paper proceeds as follows. Section 2 develops the basic elements of the our analysis and considers capital accumulation and distribution in a two-good, small open economy. This facilitates our analysis of North-South trade, our ultimate aim. Sections 3, 4 and 5 respectively consider autarky, free trade in goods and free movement of loans (along with free trade in goods). Concluding remarks are made in Section 6.

2 Model Elements & The Small Open Economy

The objective is to study the distributional – and in the process – the aggregate effects of free trade in goods and free movement of loans, along with capital accumulation, in a two-country world economy. We begin with model elements and characterizing a small open economy *without* international borrowing. This would pave the way toward building the two-country model.

Our first aim is to articulate a generalized HO production model in which (a) exports are capital-intensive for one country and capital-poor for the other, (b) capital is reproducible and (c) one country (North), in equilibrium, would use relatively more capital in producing the two consumption goods than would the other (South). There are many ways in which this can happen. Our model chooses one, which is as good as any other; but at the same time it embraces the endowment difference as the basis of trade at a more primitive level than in the standard paradigm. Let there be two consumption goods, x and y, which are produced by capital and labor, and let good x be relatively more labor-intensive in production. (Labor is viewed as unskilled labor.) In addition, let there be a nontraded sector producing the investment (capital) good k. For simplicity, let capital fully depreciate after one period (all results remain in tact when the depreciation rate is partial). That one country would have more capital for use in the production of consumption goods is ensured by assuming that there is a specific factor in the nontraded sector whose endowment is greater in one country than in the other. This is in keeping with the spirit of the static, standard HO view in that some endowment differences at some tier of productive activity form the basis of international trade.[1] Let $\bar{M}$ denote the endowment of the specific factor. Later, in the two-country scenario, North and South will be distinguished by a difference in $\bar{M}$, that is, $\bar{M}^N > \bar{M}^S$, where the superscripts N and S denote North and South. This factor may represent lab, infrastructure etc., the relative abundance (scarcity) of which makes North (South) a capital-rich (capital-poor) country. This is the primitive of the world economy; how North acquired the abundance of lab, infrastructure etc. in the first place is not questioned here.

Furthermore, we assume that, beside $\bar{M}$, the production of the investment good require capital.

[1] One country may be capital rich compared to another due to other factors like differences in the savings rate or in the rate of time preference.

It captures – in an extreme fashion – that this sector is more capital intensive in relation to (unskilled) labor, compared to the consumption good sectors. The underlying notion is that the production of capital goods – machinery in general – is a relatively sophisticated process and intensive in the use of capital itself, not in unskilled labor.[2]

There are constant returns in each sector. For tractability, the production function in each sector is assumed Cobb-Douglas, i.e.,

$$Q_j = \alpha_j^{-\alpha_j}(1-\alpha_j)^{-(1-\alpha_j)} K_j^{\alpha_j} L_j^{1-\alpha_j},\ j = x, y;\ Q_k = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} K_k^{\alpha} M^{1-\alpha}.$$

The Q's are the respective outputs; Q_k is the production of the investment good: addition to the aggregate capital stock. The multiplicative technology coefficient is normalized such that the unit cost function is of form, $r^{\alpha_x} w^{1-\alpha_x}$, for sector x and similarly for the other two sectors. The α's, strictly between zero and one, are the share of capital in the respective sector.

Each individual in the economy possesses the same endowment of labor that is supplied inelastically to the market. The number of households and the total endowment of labor are normalized to one. All markets are perfectly competitive.

It will be convenient to take the investment good as the numeraire. Denote the product price ratio p_x/p_y by p, which is given for a small open economy. Although Cobb-Douglas technologies are assumed, in order to obtain an overall understanding of the production side, we write in general

[2]Otherwise, if, for example, the investment good sector were very labor intensive, there is a possibility that a highly labor abundant economy may produce more capital to the extent that the overall capital labor ratio for the two consumption good sectors together is higher – compared to a less labor abundant country – which, in turn, would lead the former country to export the relatively capital intensive good. This is ruled out here.

in terms of the familiar zero-profit and full-employment conditions:

$$c_x(w_t, r_t) = pp_{yt};\ c_y(w_t, r_t) = p_{yt};\ c_k(r_t, m_t) = 1 \tag{1}$$

$$a_{Lx}(\omega_t)Q_{xt} + a_{Ly}(\omega_t)Q_{yt} = 1 \tag{2}$$

$$a_{Kx}(\omega_t)Q_{xt} + a_{Ky}(\omega_t)Q_{yt} + a_{Kk}(r_t/m_t)Q_{kt} = K_t \tag{3}$$

$$a_{Mk}(r_t/m_t)Q_{kt} = \bar{M}. \tag{4}$$

In these equations, $c_i(.)$, $i = x, y, k$, is the unit cost function of good i; a's are the respective input coefficients; p_{yt} is the relative price good y in terms of the investment good; w_t, r_t and m_t are respectively the reward of labor, capital and the factor M, $\omega_t = w_t/r_t$, and finally, K_t is the total capital stock.

For a small open economy, p_t is given for all t. Also, whether small or large, $\bar{M}$ is given for a country. Thus these are indicated without the time subscript. The history at t is characterized by a known value of K_t. Given p and K_t, eqs. (1) – (4), together with a demand side equation for assets (to be specified), determine w_t, r_t, m_t, p_{yt}, Q_{xt}, Q_{yt} and Q_{kt}.

On the preference side it is assumed that every individual or dynasty lives for one period and derives utility from own consumption as well as bequests left to posterity. Hence it is the so-called 'warm-glow' model with one period life time (Aghion and Bolton (1997) and Piketty (1997)). There is borrowing and lending. Along with capital, loans (positive or negative) can be passed as bequests. It is assumed either that the net bequest value cannot be negative (with perfect foresight), or, if negative, is bounded. Results do not change between the two alternative assumptions. We however assume the latter since it buys considerable simplicity. This will be made more specific later.

Dynasties, indexed by h, are heterogeneous in terms of preferences (and the bequest they inherit). They face an idiosyncratic preference shock, i.i.d. over time (Atkeson and Lucas (1992) and Lucas (1992)).[3] The utility function is given by

$$U_{ht} = \left(\Gamma D_{xh}^{\gamma} D_{yh}^{1-\gamma}\right)^{1-\beta} \left(K_{ht+1} - \frac{D_{ht+1}}{r_{t+1}} - z_{ht}\right)^{\beta}, \; \Gamma > 0;\; 0 < \beta,\; \gamma < 1. \tag{5}$$

The first part represents static utility; γ is share of expenditure allocated to good x. The second part represents utility from bequest. K_{ht+1} is the capital stock bequeathed. D_{ht+1} is the fresh, one-period loan (gross of interest payments to be made) incurred by household h in period t. D_{ht+1}/r_{t+1} is the value of the loan in terms of physical capital and $B_{ht+1} \equiv K_{ht+1} - D_{ht+1}/r_{t+1}$ is the net value of the bequest in terms of the capital good.[4] Bequest being expressed in the unit of capital implies three things. (a) capital and loans are perfect substitutes. (b) Overall, utility is defined in terms of units of all three goods produced in the economy. (c) Good k is *not* a *pure* capital good or a pure asset; it has direct consumption value as well.[5]

The term z_{ht} indicates noninsurable preference shock, a continuous random variable, having a finite support $(\underline{z}, \bar{z})$ and being i.i.d. over time. It is also idiosyncratic, meaning that the proportion of the households experiencing $z \leq z_0$ is equal to $\Pr(z \leq z_0)$. The magnitude of z_{ht} reflects propensity to leave bequest. Higher the value of z_{ht}, greater is the marginal utility from bequest, and, hence, intuitively, higher is the propensity to leave bequest. All our results hold when $E(z) \geq 0$. Thus we can assume $\underline{z} \geq 0$ and rule out negative bequest. But, for algebraic simplicity, we will suppose that $E(z) = 0$. This means that negative realizations of z and hence negative bequests ($B_{ht+1} < 0$) are possible, but with $|\underline{z}|$ as the upper bound in terms of magnitude. Moreover, for notational simplicity, we normalize the variance to one, i.e. $\sigma_z^2 = 1$.[6]

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[3]These authors assume infinite lifetime however.

[4]With less than full depreciation rate, equal to δ, we would have $r_{t+1} + 1 - \delta$ instead of r_{t+1}. If some other good were chosen as the numeraire, it would have been $r_{t+1} + (1-\delta)p_{kt+1}$.

[5]Later, this implies that factor price equalization does *not* lead to asset price equalization.

[6]Preference heterogeneity can also be introduced in the subutility from current consumption, e.g., $\Gamma D_x^{\gamma} D_y^{1-\gamma} - z_{ht}$ in place of just $\Gamma D_x^{\gamma} D_y^{1-\gamma}$. This would imply that the index of relative risk aversion is not equal to one unless z_{ht}

The static utility function being Cobb-Douglas, the demand functions for x and y are $\gamma E_{ht}/p_{xt}$ and $(1-\gamma)E_{ht}/p_{yt}$ respectively. The indirect static utility has the form, $\tilde{U}_{ht} = p_{xt}^{-\gamma} p_{yt}^{-(1-\gamma)} E_{ht}$ when Γ is normalized such that $\Gamma\gamma^{\gamma}(1-\gamma)^{1-\gamma} = 1$. Accordingly, rewrite the utility function as:

$$U_{ht} = \left[p_{xt}^{-\gamma} p_{yt}^{-(1-\gamma)} E_{ht}\right]^{1-\beta} (B_{ht+1} - z_{ht})^{\beta}. \tag{6}$$

where, recall, $B_{ht+1} \equiv K_{ht+1} - D_{ht+1}/r_{t+1}$.

Specifying the budget constraint requires what the ownership pattern of the factor M is. It is assumed that the government owns M and redistributes the income equally among the households. Alternatively and equivalently, we can assume that factor M is equally and privately owned by all dynasties and it is nontraded. Either assumption is designed to abstract the analysis from inequality arising from unequal ownership of this specific and nonreproducible factor and focus instead on capital as a reproducible factor. Given this, the intertemporal budget constraint is:

$$D_{ht} + E_{ht} + K_{ht+1} - \frac{D_{ht+1}}{1+i_t} \leq \bar{M}m_t + w_t + r_t K_{ht}, \tag{7}$$

where i_t is interest on consumption loans and D_{ht} is the loan inherited. A dynasty maximizes U_{ht} subject to this budget constraint. It is easily verified that the partials of the Lagrangian with respect to K_{ht+1} and D_{ht+1} are both zero only if $1 + i_t = r_{t+1}$. This conforms that our bequest specification is consistent with capital and loans being perfect substitutes. Given this arbitrage

is zero. Our specification implies that it is one from everyone. In Das (1999b) the subutility function is assumed to be of the form, $\Gamma D_x^{\gamma} D_y^{1-\gamma} + a$, where a is same across hous eholds (and, moreover, heterogeneity is present in the subutility from bequests). It is shown there that the effect of North-North trade on distribution depends critically on whether $a \gtrless 0$.

condition, the budget constraint can be shortened as

$$E_{ht} + B_{ht+1} \leq \bar{M}m_t + w_t + r_t B_{ht}. \tag{8}$$

There is one initial condition: B_{ht} is given. The choice variables are effectively E_{ht} and B_{ht+1}. The first-order condition and the resulting asset demand functions are:

$$\frac{E_{ht}}{B_{ht+1} - z_{ht}} = \frac{1-\beta}{\beta} \tag{9}$$

$$B_{ht+1} = \beta(\bar{M}m_t + w_t + r_t B_{ht}) + (1-\beta)z_{ht}. \tag{10}$$

For distributional implications, the last equation is central. Moreover, it is valid for a small and a large economy. Aggregating this over the dynasties and recalling that the number of dynastied is normalized to one and $E(z_{ht}) = 0$, we obtain the aggregate demand function for assets:

$$B_{t+1} = \beta(\bar{M}m_t + w_t + r_t B_t). \tag{11}$$

Without international borrowing/lending, $B_t = K_t$, the aggregate stock of capital. Thus

$$K_{t+1} = \beta(\bar{M}m_t + w_t + r_t K_t). \tag{12}$$

This is the demand-side equation. Using the identity

$$K_{t+1} = Q_{kt}, \tag{13}$$

we have altogether 8 equations, namely, (1)-(4), (12) and (13), which describe the aggregate dynamics of the small open economy.

Solution of the Model

The dimension of the model is reduced as follows. Cobb-Douglas technology implies:

$$\bar{M}m_t = (1-\alpha)Q_{kt} = (1-\alpha)K_{t+1}. \tag{14}$$

Thus we can write (12) as

$$K_{t+1} = \frac{\beta r_t}{1-\beta+\alpha\beta}\left(\omega + K_t\right). \tag{15}$$

Note that, for a small open economy, ω is uniquely given by the product price ratio p.

Also, we have the unit cost function $c_k(.) = r^{\alpha}m^{1-\alpha}$. Hence the zero-profit condition $c_k(.) = 1$ yields $r^{\alpha}m^{1-\alpha} = 1$. Substituting this into (14), we have

$$K_{t+1} = \frac{\bar{M}}{1-\alpha} r_t^{-\alpha/(1-\alpha)}. \tag{16}$$

Finally we eliminate r_t from the last two equations and obtain

$$K_{t+1} = \eta(\omega + K_t)^{\alpha}, \quad \text{where } \eta = \left(\frac{\beta}{1-\beta+\alpha\beta}\right)^{\alpha}\left(\frac{\bar{M}}{1-\alpha}\right)^{1-\alpha}. \tag{17}$$

This is the basic difference equation. Given K_0, it charts a unique path of K_t.

Steady State

The right-hand-side of (17), as a function of K_t, satisfies the Inada condition as $K_t \to \infty$ and has a positive intercept. Hence a steady state exists, it is unique and the adjustment path is monotonic. From now on we will concentrate on steady state only. Along the steady state

$$K = \eta(\omega + K)^{\alpha}. \tag{18}$$

A variable without the time subscript represents its steady state value. Given K, eq. (16) determines r:

$$r = \left[\frac{\bar{M}}{(1-\alpha)K}\right]^{(1-\alpha)/\alpha}. \tag{19}$$

This is essentially a supply-side, negative, relationship between the rate of return of capital and the long-run capital stock, which will be used repeatedly as it holds for a small open economy and a large economy with or without international trade. Given the yield on capital, it is equivalent to a positive (supply-side) relationship between the price of capital and the long-run capital stock. (Recall that r is the rental rate in term of the capital good.)

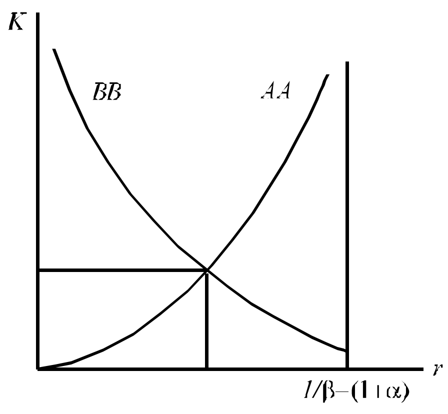

Figure 1: Steady State: A Small Open Economy

The solution of K and r is shown in Figure 1. Eq. (15) yields a positive locus between K and r, given by

$$K = \frac{\beta \omega r}{1 - \beta + \alpha\beta - \beta r}. \tag{20}$$

This is denoted by the AA curve. Along this curve $K \to \infty$ as $r \to 1/\beta - (1+\alpha)$. Eq. (19) depicts a downward schedule such as BB. An intersection in the positive quadrant, determining K and r, is ensured. We see that $r < 1/\beta - (1+\alpha)$.

A desirable feature we would like to ensure is that the interest rate is positive, i.e., the gross rate of return on capital, r, exceeds one. This holds if β is small enough so that K is not large enough; we assume so.

In what follows, four comparative static results are noted, which will be used in the two-country analysis later.

Result 1: An increase in the wage/rental ratio leads to a greater capital stock in the long run. The elasticity of response is less than one.

Result 2: An increase in $\bar{M}$ implies an increase in K, K_c and r, where K_c is the capital employed in the two consumption good sectors combined.

Result 3: (a) An increase in p, the relative price of the labor-intensive good, leads to an increase in K and K_c. Moreover, (b) $\hat{K}_c/\hat{p} < \hat{K}/\hat{p}$.

Result 4: The own price elasticity of relative output may be positive or negative but it exceeds -1, i.e.,

$$\frac{\widehat{Q_x/Q_y}}{\hat{p}} > -1.$$

Proofs are given in Appendix 1. Intuitively, an increase in ω has a positive income effect on asset demand (eq. 10). In equilibrium, more capital is produced. This explains Result 1. Result 2 says that an increase in the factor specific in the production of the capital good leads to more production of the capital good and more use of capital in the two consumption sectors together. The specific factor and capital being complements of each other in the technology of the investment good sector implies that an increase in $\bar{M}$ increases r, the marginal product of capital in that sector. That $dK/dp > 0$ in the first part of Result 3 follows from the Theorem and Result 1. An increase in K results in more capital being used in producing the two consumption goods together. Result 4 deals with the slope of long-run supply curve, that is, how a change in p affects Q_x/Q_y directly as well as through a change in K_c. The direct effect is simply a movement along a given production posssibility frontier, which is positive. By Result 3, $dK_c/dp < 0$. Through the Rybczinski effect however, a decrease in K_c lowers Q_x/Q_y, the relative output of the labor intensive good. Hence the net effect of an increase in p on the output ratio is positive or negative.

3 Autarky

From the perspective of an autarkic economy, the features of a small open economy describe the supply side. The (static) demand side equation is given by: $D_x/D_y = \gamma/[(1-\gamma)p]$, with elasticity equal to -1. Since the relative demand curve is asymptotic to both price and quantity axes, an intersection with the relative supply curve is assured. Moreover, the supply elasticity being greater than -1 (Result 4), it follows that, at the point of intersection, the relative supply curve, if negatively sloped, is less elastic than the relative demand curve. Hence the steady state autarky equilibrium is unique and for qualitative purposes we can depict the supply curve as upward sloping (see Figure 3 later).

Whether or not the economy is along the steady state, at any t, from the demand side, the quantity ratio is related one-to-one with p_t and hence with ω_t. From the supply side, it is uniquely related to K_{ct}. Thus ω_t and K_{ct} are uniquely related. Given Cobb-Douglas technology and static preferences, we specifically have

$$\omega_t = \chi K_{ct}, \quad \text{where } \chi \equiv \frac{1}{\gamma\alpha_x + (1-\gamma)\alpha_y} - 1. \tag{21}$$

Next, using this expression along with (A1) – the first equation in Appendix 1 – and (17) yields

$$K_{ct} = \frac{(1-\beta)K_t}{1-\beta+\alpha\beta(1+\chi)} \tag{22}$$

$$K_{t+1} = \eta\left[\frac{(1+\chi)(1-\beta+\alpha\beta)}{1-\beta+\alpha\beta(1+\chi)}\right]^{\alpha} K_t^{\alpha}. \tag{23}$$

The last one is the basic aggregate, dynamic equation in the closed economy. There is monotonic adjustment towards this steady state and (23) yields a closed-form solution of the steady-state capital stock. In view of (19), there is also a closed-form solution of the rate of return to capital.

$$K_a = \eta^{1/(1-\alpha)}\left[\frac{(1+\chi)(1-\beta+\alpha\beta)}{1-\beta+\alpha\beta(1+\chi)}\right]^{\alpha/(1-\alpha)} \tag{24}$$

$$r_a = \frac{1-\beta+\alpha\beta(1+\chi)}{\beta(1+\chi)}, \tag{25}$$

where the subscript a represents autarky.

Turning to distribution, recall that eq. (10) is the central equation governing its dynamics. We will be concerned with invariant distribution in the long-run. As m_t, w_t and r_t approach their steady state values, the question is whether the stochastic process,

$$B_{ht+1} = \beta(\bar{M}m + w + rB_{ht}) + (1-\beta)z_{ht} \tag{26}$$

converges to an invariant distribution, that is, B_{ht+1} and B_{ht} have the same distribution.[7] Given $\beta < 1$ and that z_{ht} has finite support, the Stokey-Lucas (1989) and Hopenhayn-Prescott (1992) results on the existence and uniqueness of invariant distribution apply. Hence the process (26) does converge to a unique invariant distribution (see Appendix 2 for details). The set of possible states of B_{ht} in the invariant distribution is a compact set. In what follows, this set is identified and the monotone-mixing condition is checked. Along with other conditions given and verified in Appendix 2, it completes the proof of the existence and uniqueness of the invariant distribution of B_{ht}.

Let $\underline{B}$ and $\bar{B}$ be the steady state values of B corresponding to $\underline{z}$ and $\bar{z}$, i.e., let

$$\underline{B} \equiv \frac{\beta(\bar{M}m + w) + (1-\beta)\underline{z}}{1-\beta r}; \ \bar{B} \equiv \frac{\beta(\bar{M}m + w) + (1-\beta)\bar{z}}{1-\beta r}.$$

The interval $(\underline{B}, \bar{B})$ is then the compact set to which B_h belongs in the invariant distribution. Consider Figure 2 which depicts B_{ht+1} as a function of B_{ht} for the two extreme values of z, denoted $f(.,\underline{z})$ and $f(.,\bar{z})$. Since $\beta < 1$, the *non*stochastic processes $f(.,\underline{z})$ and $f(.,\bar{z})$ are stable. This implies two things. First, irrespective of the initial distribution of B_h, in the limit, B_h is confined to the interval $(\underline{B}, \bar{B})$. Second, if $B_{ht} = \underline{B}$ (or $\bar{B}$), there is a positive probability that the same dynasty will end up with $\bar{B}$ (or $\underline{B}$). The monotone-mixing condition is thus met.

From (10), the mean and variance of the invariant distribution of B_{ht} can be solved. We have

[7] For a similar approach to study invariant distribution of wealth in a different context, see Picketty (1997).

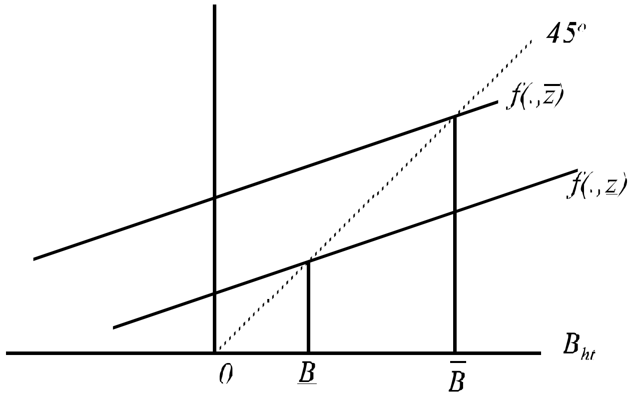

Figure 2: Monotone Mixing Condition

$\mu_B = \beta(\bar{M}m + w + r\mu_B)$, implying

$$\mu_B = \frac{\beta(\bar{M}m + w)}{1 - \beta r} = \frac{\beta(\bar{M}m(r) + \omega r)}{1 - \beta r} = \frac{\beta r(\bar{M}r^{-1/(1-\alpha)} + \omega)}{1 - \beta r}. \quad (27)$$

Since there is no international borrowing or lending, $\mu_B = K$. Next, $\sigma_B^2 = \beta^2 r^2 \sigma_B^2 + (1-\beta)^2 \sigma_z^2$. Recall that σ_z^2 is normalized to one. Thus[8]

$$\sigma_B = \frac{1 - \beta}{\sqrt{1 - \beta^2 r^2}}. \quad (28)$$

[8]Note that if preferences were assumed to be stable, (26) would have implied that, the steady state,

$$B_h = \frac{\beta(\bar{M}m + w) + (1-\beta)z_h}{1 - \beta r}.$$

and thus

$$\sigma_B^c = \frac{1 - \beta}{1 - \beta r}.$$

It is easy to check that σ_B below is less than σ_B^c (as long as $\beta r < 1$). This is because variability of z_h over time makes it possible for a rich household to become relatively poor in the next generation and vice versa and hence acts as a factor toward less inequality.

We shall measure inequality by the coefficient of variation. Thus wealth inequality is given by

$$\psi_{Ba} = \frac{\sigma_B}{\mu_B} = \frac{1-\beta}{K\sqrt{1-\beta^2 r^2}}. \tag{29}$$

Define personal income as $I_h = \bar{M}m + w + rB_h$, so that $\mu_I = \bar{M}m + w + r\mu_B = r(\bar{M}m/r + \omega + \mu_B)$. From the zero profit condition in the investment good sector, $m/r = r^{-1/(1-\alpha)}$. Thus, $\mu_I = r[\bar{M}r^{-1/(1-\alpha)} + \omega + \mu_B]$ and $\sigma_I = r\sigma_B = r(1-\beta)/\sqrt{1-\beta^2 r^2}$. Hence,

$$\psi_{Ia} = \frac{1-\beta}{[\bar{M}r^{-1/(1-\alpha)} + \omega + K]\sqrt{1-\beta^2 r^2}}. \tag{30}$$

Finally we develop the expression for inequality in terms of individual welfare levels. After the substitution of the first-order condition of intertemporal optimization into the utility function, we can write

$$U_{ht} = \left[\frac{1-\beta}{\beta} p_{xt}^{\gamma} p_{yt}^{1-\gamma}\right]^{1-\beta} (B_{ht+1} - z_{ht}).$$

Thus, at the autarky steady state,

$$\mu_U = \left[\frac{1-\beta}{\beta} p_x^{\gamma} p_y^{1-\gamma}\right]^{1-\beta} K; \quad \sigma_U = \left[\frac{1-\beta}{\beta} p_x^{\gamma} p_y^{1-\gamma}\right]^{1-\beta} \sqrt{\sigma_B^2 + 1},$$

$$\Rightarrow \psi_{Ua} = \frac{1}{K}\sqrt{\frac{(1-\beta)^2}{1-\beta^2 r^2} + 1}. \tag{31}$$

An important feature which is common for the variance of wealth or income – and which is crucial for understanding the effect of trade policy on inequality indices – is that *it is unaffected by a change in m or w, but it increases with r, the rate of return on capital.* The reason is the following. A change in m or w has an income effect on the demand for assets (as indicated in (10)). However, at the margin, these effects are same for all dynasties. Hence, the variance of wealth holding and therefore the variance of income or utility as well are not affected. A change in r, on the other hand, also has an income effect on the demand for assets, but it is proportional to initial wealth holding. Along the invariant distribution, it then has a *multiplying effect* on the variance.

North and South in Autarky

Having laid out the features of a closed economy, we now compare North and South in autarky, which would set the stage for analyzing the effect of international trade. Technologies are assumed the same between North and South. Moreover, preferences including the distribution of z are also assumed the same. In terms of the primitives, the only difference is that $\bar{M}^N > \bar{M}^S$. Accordingly, we can call North and South as the M-rich country and the M-poor country.

First, we appeal to Result 2. It implies that, at any given p, $K_c^N > K_c^S$. By virtue of the Rybczinski theorem, $(Q_x/Q_y)^N < (Q_x/Q_y)^S$. Thus, as shown in Figure 3, the relative supply curve in the North lies to the left of that in the South. As a result, $p_a^N > p_a^S$, i.e., North (South) has comparative advantage in the capital- (labor-) intensive good. This is an all too familiar tool of trade theorists. The only difference however is that one of the factor endowments is variable.

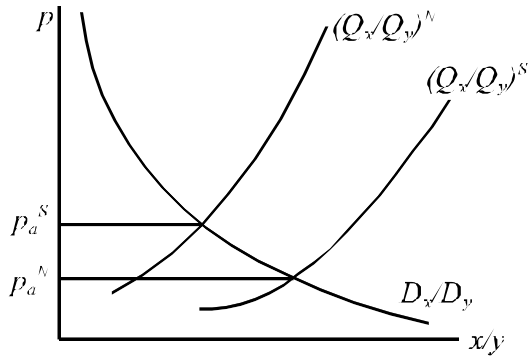

Figure 3: Autarky Equilibria: Endogenous Capital Stock

Moreover, from (24), it is seen that $K_a^N > K_a^S$, as expected (since $\eta^N > \eta^S$). This implies that $K_{ca}^N > K_{ca}^S$. These aggregate implications are summarized in our first proposition.

Proposition 1: In autarky equilibria, $p_a^N > p_a^S$, $K_a^N > K_a^S$ *and* $K_{ca}^N > K_{ca}^S$.

There are further implications which are specific to the assumption of Cobb-Douglas technology. First, in view of (25), that $r_a^N = r_a^S$, i.e., the rate of return to capital is equalized in the absence of trade.[9]

Second, since $K_a^N > K_a^S$ and $r_a^N = r_a^S$, (29) implies that $\psi_{Ba}^N < \psi_{Ba}^S$. Likewise, it is seen that $\psi_{Ia}^N < \psi_{Ia}^S$ and $\psi_{Ua}^N < \psi_{Ua}^S$. Thus, *in autarky equilibria, wealth, income and welfare inequalities are less in the North than in the South.* Consider wealth inequality for example. In autarky, the M-rich North possesses more capital than South does; thus, the mean wealth is greater in the North. As ready discussed earlier, the variance of wealth is positively related to r. However, r is same and hence variance of wealth holding is the same between the North and the South. It follows then that wealth inequality, measured by the coefficient of variation, is less in the North. Similar reasoning leads to the conclusion that income and welfare inequalities also less in the North.

4 Free Trade in Goods

Starting from autarky, let the two countries open up free trade in goods. To isolate the effect of this, assume for now that loan market is not opened.

[9]Even if the technologies were not Cobb-Douglas, the difference between r^N and r^S is likely to be small. The M-rich North would produce more capital in equilibrium. Hence the price of capital (in terms of good x or y) would be lower. Also, more capital is available to factor market, which implies a lower factor price (or yield). Hence the ratio of the yield to price – equal to the rate of return on capital – is not likely to differ much between North and South.

Aggregate Effects

From Figure 3, it is obvious that the free-trade price ratio lies inbetween (p_a^N, p_a^S). As a result, at the free trade equilibrium, $(Q_x/Q_y)_g^N < (Q_x/Q_y)_g^S$ and $K_{cg}^N > K_{cg}^S$, where the subscript g denotes free trade in goods, and the North (South) exports the capital- (labor-) intensive good. Result 2 implies that $r_g^N > r_g^S$.

Compared to autarky, as North (South) experiences a decrease (an increase) in p, Result 3 implies that, starting from autarky, K and K_c fall (increase) in the North (South). The negative relationship between K and r (in (16)) means that r increases (decreases) in the North (South). Intuitively, as free trade opens up, the M-rich North produces relatively more of the capital-intensive good and exports it. In the process, the wage/rental ratio falls. By the income effect there is less demand for capital as an asset and hence less capital is produced in the long run. Less demand tends to lower the price of capital relative to its yield. Thus there is an increase in the rate of return on capital. Just the opposite happens in the South. It is interesting to note that free trade in goods leads to a divergence of rate of return on capital between North and South.

Turn to the aggregate wealth dynamics equation (12). Along the steady state, $K(1-\beta r) = \beta(\bar{M}m + w)$. Since K falls (rises) and r rises (falls) in the North (South), it follows that the 'fixed income', $\bar{M}m + w$, which is invariant across households, falls (rises) in the North (South). As will be seen a bit later, this will be important in understanding how wealth and income mobility is affected by trade policy. In summary we have

Proposition 2: Starting from autarky, free trade in goods between North and South leads to

(a) a decrease (an increase) in the long-run capital stock in the North (South) and hence a narrowing of the difference between the capital stocks and capital used in the consumption sectors between the two countries;

(b) an increase (a decrease) in the rate of return to capital in the North (South); at the free trade equilibrium, the rate of return to capital in the North (r_g^N) exceeds that in the South (r_g^S).

(c) The fixed income falls (rises) in the North (South).

Starting from the well-known Prebisch-Singer hypothesis of 'secularly' declining terms of trade facing the South, it is believed that South may lose out from freer North-South trade. Proposition 2 is noteworthy in delivering an opposite message: free trade may lower growth rate of the North and improve that of the South.

Distributional Effects

In the absence of an international loan market, similar to autarky, the aggregate (mean) capital stock in the respective country is equal to the respective aggregate (mean) wealth. We have seen that a movement from autarky to free trade in goods lowers (increases) the mean capital stock in the North (South). Thus, in the North (South), mean wealth decreases (increases). As discussed earlier, the variance of wealth holding is an increasing function of the rate of return on capital – which in turn increases in the North and decreases in the South. Hence, the variance of wealth holding increases (decreases) in the North (South). From the changes in the mean and variance, it follows that wealth inequality increases in the North and decreases in the South. It can be checked that the same holds for income and welfare inequalities. Hence

Proposition 3: Free international trade between North and South leads to more inequality in the North and less in the South.

This substantially generalizes the corresponding result obtained in Das (1999c) – in allowing for reproducible capital and random preferences. It is worth noting that inequalities in the North (South) increase (decrease) not only because of the adjustment in variances but also due to changes in the respective means. In recent decades, wealth and income inequalities have increased a great deal in many developed countries (Das (1993)). Proposition 3 suggests that freer trade between North and South may be one of the factors. Casual observations also indicate that developing countries like India and Mexico have experienced in recent years a so-called middle-class boom suggesting a decrease in inequality and this also accords with Proposition 3. However, this ought to be interpreted with great caution since formal empirical evidence of the movement of inequalities in developing countries are either nonexistent or fragmented at best.[10]

In closing, note that the results of this section are valid whether or not there is factor price equalization (FPE). It is however interesting to note in passing that since free trade narrows difference between K_c^N and K_c^S, this is an additional favorable effect of free trade toward FPE. Later, we analyze the implications of free movement of loans (along with free trade in goods) and for tractability with regard to characterizing the equilibrium and comparing it with free trade in goods only, we do assume that $\bar{M}^N - \bar{M}^S$ not so large, such that FPE holds in both equilibria.

Social Mobility

Preferences following a stochastic process, the model permits mobility along the wealth-income ladder or what may be called 'social mobility' (Piketty (1995)). In the long run (with invariant wealth and income distribution), there is a positive probability that a dynasty at the ith percentile of wealth or income at any given time will move to the jth percentile, for any i, j, $i \neq j$, at some future date. Unfortunately however, a well accepted measure of such mobility does not seem to exist. But in simple terms and within the purview of the model, we can ask how the downward (upward) mobility of an above-average (below-average) dynasty at a given point in time is affected by a regime change.[11]

[10] Note that this is related to but different from the increase in skilled-unskilled wage differential observed in developed countries. For a recent theoretical analysis of such differential and international trade, see Das (1999a).

[11] Note that wealth or income mobility may be viewed in reference to the median wealth/income. Unfortunately, not much is known about the density function of invariant distributions in general. Hence characterizing a change in

Consider the long-run distributional dynamics dictated by (26). Suppose, at a given date t, the wealth of a dynasty is $100\alpha\%$ greater than the mean wealth holding, i.e.,

$$B_{ht} = (1+\alpha)B_t = (1+\alpha)\frac{\beta(\bar{M}m + w)}{1-\beta r}. \tag{32}$$

We then ask: what happens to the probability that the wealth holding of this dynasty at $t+1$ is less than B_{ht}, i.e., what happens to $\Pr(B_{ht+1} < B_{ht})$? Simiarly for upward mobility. Thus downward and upward mobilities are respectively indicated by

$$\Pr\,[B_{ht+1} < (1+\alpha)B_t] = \Pr[\beta(\bar{M}m + w + rB_t) + (1-\beta)z_{ht}] < (1+\alpha)B_t \tag{33}$$

$$\Pr\,[B_{ht+1} > (1-\alpha)B_t] = \Pr[\beta(\bar{M}m + w + rB_t) + (1-\beta)z_{ht}] > (1-\alpha)B_t. \tag{34}$$

We note from these expressions that mobility is affected by changes in the rate of return on capital and the fixed income. In general there are three ways in which changes in r and $\bar{M}m + w$ would impact $|B_{ht+1} - (1+\alpha)B_t|$. (a) They affect $B_t = K_t$ along the steady state. (b) They influence B_{ht+1} directly through the income effect: the effect of a change in the fixed income has the same effect for all h, while that of a change in r is proportional to the wealth holding of h. (c) The implied change in B_t (in (a)) affects B_{ht+1} also. These effects of a change in r or $\bar{M}m + w$ on B_t and B_{ht+1} are all positive. And it turns out that the net effect of r is zer o. It is because the direct effect of a change in r on B_{ht+1} equals the previous period's wealth holding itself which varies across households. On the contrary, the direct effect of a change in the fixed income on B_{ht+1} is symmetric across households while B_{ht} varies. Hence wealth mobility depends critically on how

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the median is a difficult task. But our analysis obviously remains unchanged in the special case where the distribution of z is symmetric which implies that the invariant distribution of wealth is also symmetric and the median is equal to the mean wealth. Otherwise, simulation exercise would be necessary to assess changes in the degree of mobility.

the fixed income is affected by the change in the trading regime.

Algebraically, if we substitute (32) into (33) or (34), we get

$$\Pr\left[B_{ht+1} < (1+\alpha)B_t\right] = \Pr\left[z_{ht} < \frac{\alpha\beta}{1-\beta}(\bar{M}m + w)\right] \tag{35}$$

$$\Pr\left[B_{ht+1} > (1-\alpha)B_t\right] = \Pr\left[z_{ht} > \frac{\alpha\beta}{1-\beta}(\bar{M}m + w)\right] \tag{36}$$

Notice that these expressions are independent of r, and, moreover, both downward and upward mobility probabilities vary directly with fixed income.

The effect of free trade in goods on wealth mobility is now immediate from part (c) of Proposition 2. Since the fixed income, $\bar{M}m + w$, falls in the North and rises in the South, wealth mobility is less in the North and more in the South. Intuitively, in the North for example, the household, which, in the wealth ladder, is $\alpha\%$ higher/lower above/below the mean is even further away from the mean on the absolute scale. So the chance of coming down/ coming up on the ladder is greater.

The same holds for income mobility too. The mean income is equal to

$$\mu_I = \bar{M}m + w + \frac{r\beta(\bar{M}m + w)}{1-\beta r} = \frac{\bar{M}m + w}{1-\beta r}$$

and let income at time t be $I_{ht} = (1+\alpha)\mu_I$. We have $I_{ht+1} = \bar{M}m + w + r[\beta(\bar{M}m + w + rB_{ht}) + (1-\beta)z_{ht}] = \bar{M}m + w + r\beta I_{ht} + r(1-\beta)z_{ht}$. Thus

$$\Pr[I_{ht+1} < I_{ht}] = \Pr\left[z_{ht} < \frac{\alpha\beta}{1-\beta} \cdot \frac{\bar{M}m + w}{r}\right].$$

The only difference with the corresponding expression relevant for wealth mobility is that there is 'r' in the denominator. However, in the North for example, free trade in goods leads to an increase in r (and a decrease in $\bar{M}m + w$). Therefore, income mobility is also less in the North and more in the South.

Proposition 4: Compared to autarky, in the free trade equilibrium mobility across wealth and income is less in the North and more in the South.

Normative Implications

The model is not amenable to defining aggregate or social welfare in an unambiguous fashion even while accepting Scitovsky-type compensation criterion. First of all, among other difficulties, tracking welfare changes out of steady state would have required a discount rate and this would have been arbitrary in terms of our model since individuals have finite lifetime. In what follows, we confine ourselves to welfare comparisons along different steady states only. Since welfare levels replicate along th e steady state, the problem discussed above does not arise. But ignoring off-the-steady-state changes while individual households decide current consumption and savings irrespective of whether the economy is along the steady state implies that overall welfare changes are not fully accounted for; this allows for the possibility that, even without any distortions and with all markets competitive, more trade may imply less steady state welfare. This is clearly a serious limitation. Nonetheless, how aggrega te welfare changes occur from one long run equilibrium to another may be of some interest and we proceed with this presumption.

Since individual preferences differ, there is a further difficulty in defining aggregate welfare in the conventional additive way. However, this is no more serious than the standard practice of defining aggregate welfare as the sum of identical individual welfare functions in the presence of risk-aversion (e.g. Atkinson and Stiglitz (1980) and Strawczinski (1998)); the marginal weights on individual welfare levels are different even though utility functions may be identical. Linearly homogenous but heterogenous preferences across dynasties present a similar situation. Hence in the same vein, we can define here, in the utilitarian fashion, the sum of utilities or mean utility, μ_U, as the index of aggregate welfare.

Furthermore, if equity is considered as a concern on its own right, apart from its possible effects on efficiency (although there is no such effect in our model per se), one may wish to define the (Samuelsonian) social welfare as a function of individual welfare levels such that the distribution of welfare matters. However, not much is known about the density function of the invariant wealth distribution (except for its mean and standard deviation). Hence the scope for generality in defining a tractable social welfare function, taking to consideration distributional effects, is very limited.

One particular class which seems tractable and 'reasonable' is where this function (a) has two arguments, mean-welfare and its standard deviation, with partial derivatives positive and negative respectively, (b) is strictly quasi-concave and (c) is linearly homogeneous:

$$\mathcal{U} = \mathcal{U}(\mu_U,\ \sigma_U). \tag{37}$$

This can be rewritten as

$$\mathcal{U} = \mu_U \mathcal{U}(1,\ \psi_U).$$

Thus, ceteris paribus, inequality is perceived as a "bad". Further, if the negative of ψ_U is taken as "equity" and μ_U as "efficiency", the implied social indifference curve creates an efficiency-equity trade off that looks like the standard indifference curve.[12]

How ψ_U, for North or South, changes from autarky to free trade is already discussed. Consider changes in μ_U now. Along the steady state, substituting the first-order condition (9) into U_h given in (6) and aggregating over dynasties, we get

$$\mu_U = \left(\frac{\beta}{1-\beta}\right)^{\beta} \left[p_x^{-\gamma} p_y^{-(1-\gamma)}\right]^{1-\beta} E,$$

where E is the aggregate or mean expenditure. Next, aggregate the first-order condition (9) to obtain

$$E = \frac{(1-\beta)B}{\beta} = \frac{(1-\beta)K}{\beta}. \tag{38}$$

Substituting this into μ_U and rearranging,

$$\mu_U = \left[p_x^{-\gamma} p_y^{-(1-\gamma)} E\right]^{1-\beta} K^{\beta} = \left[p^{-\gamma}\tilde{E}\right]^{1-\beta} K^{\beta},$$

where the tilde represents 'in terms of good y'. The effect of trade in goods on aggregate welfare is captured through a change in p. Log-differentiating μ_U with respect to p,

$$\frac{d\ln\mu_U}{dp} = (1-\beta)\left[-\frac{\gamma}{p} + \frac{1}{\tilde{E}}\frac{d\tilde{E}}{dp}\right] + \frac{\beta}{K}\frac{dK}{dp}. \tag{39}$$

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[12] A specific example is: $\mathcal{U} = \mu_U - b\sigma_U, b > 0$. We would need to assume b to be sufficiently small such that (a) $1 - b\psi_U > 0$ and (b) the Pareto-superiority criterion is met – i.e. an increase in utility of one individual, holding others' utility constant, would increase social welfare.

A relationship between $d\tilde{E}$ and dK is obtained from aggregating budget constraint (8):

$$\tilde{E} + \tilde{p}_k K = \bar{M}\tilde{m} + \tilde{w} + \tilde{r}K \equiv Y(p, \tilde{p}_k, K), \tag{40}$$

where $Y(.)$ is the national income function, having the envelope properties: $\partial Y/\partial p = Q_x$, $\partial Y/\partial \tilde{p}_k = Q_k$ and $\partial Y/\partial K = \tilde{r}$. If we now totally differentiate this budget constraint, use the envelope properties and that $Q_k = K$, we have $d\tilde{E} = Q_x dp + (\tilde{r} - \tilde{p}_k)dK$. This implies

$$\frac{d\tilde{E}}{dp} = Q_x + (\tilde{r} - \tilde{p}_k)\frac{dK}{dp}.$$

We now substitute this into (39), use the static demand function $D_x = \gamma\tilde{E}/p$ and obtain

$$\begin{aligned} \frac{d\ln\mu_U}{dp} &= \frac{1-\beta}{\tilde{E}}(Q_x - D_x) + \left[\frac{1-\beta}{\tilde{E}}(\tilde{r} - \tilde{p}_k) + \frac{\beta}{K}\right]\frac{dK}{dp} \\ &= \frac{1-\beta}{\tilde{E}}(Q_x - D_x) + \left[\frac{\beta}{\tilde{p}_k K}(\tilde{r} - \tilde{p}_k) + \frac{\beta}{K}\right]\frac{dK}{dp} \\ &= \frac{1-\beta}{\tilde{E}}(Q_x - D_x) + \frac{\beta r}{K}\frac{dK}{dp}, \end{aligned} \tag{41}$$

where we have used $r = \tilde{r}/\tilde{p}_k$ and (38). The first term captures the standard, static, gains-from-trade expression. The second term is the gains/losses due a change in the long-run capital stock, which arise because only the long-run welfare effects are considered. Since p decreases (increases), for the North (South), the latter is negative (positive) in sign.

We are now in a position to assess all components of social welfare changes. For the South, both aggregate effects – the standard terms of trade effect and the effect through a change in the long run capital stock – of free trade are positive. Inequality also falls. Thus, there is an unambiguous increase in social welfare. For the North, the terms of trade effect is positive but the long-run capital stock effect is negative as capital stock declines. Increase in inequality also has a negative effect. Thus

Proposition 5: As countries open up free trade, in terms of our social welfare function, South unambiguously gains while North may gain or lose.

Put differently, in the South, the production possibility frontier expands and wealth/income inequality moves the 'right way'. These are sources of additional gain compared to standard production and consumption gains. Opposite outcomes in the North constitute sources of welfare loss.

Proposition 5 is a culmination of results obtained thus far pointing that it is the North, not the South, who may benefit less compared to South or potentially lose from free trade.

5 Free Movement of Loans

In addition to free trade in goods, suppose the two countries open their loan markets. This will equalize the rate of return on capital. Let $r^N = r^S \equiv r$. In view of the zero-profit condition in the consumer goods and investment good sectors, it will imply $w^N = w^S$ and $m^N = m^S \equiv m$. We can then write the dynamics of aggregate wealth in the country j as:

$$B^j_{t+1} = \beta(\bar{M}^j m_t + w_t + r_t B^j_t). \tag{42}$$

Adding over j, loans cancel out and $K^o_{t+1} = \beta(\bar{M}^o m_t + 2w_t + r_t K^o_t)$, where the superscript 'o' denotes 'world level', e.g., $\bar{M}^o = \bar{M}^N + \bar{M}^S$. Hence, along the steady state, $K^o = \beta(\bar{M}^o m + 2w)/(1 - \beta r)$. Moreover, we have $(1-\alpha)K^j = \bar{M}^j m$, from a profit-maximizing condition in the investment good sector, implying $(1-\alpha)K^o = \bar{M}^o m$. Substituting this into the earlier expression of K^o,

$$K^o = \frac{2\beta\omega r}{1-\beta r-\beta(1-\alpha)}. \tag{43}$$

Also, $r^\alpha m^{1-\alpha} = 1$ – the zero-profit condition in the investment good sector – gives $m = r^{-\alpha/(1-\alpha)}$. Thus

$$(1-\alpha)K^o = \bar{M}^o r^{-\alpha/(1-\alpha)}. \tag{44}$$

The other first-order condition, $\alpha K^j = rK_k^j$ gives $\alpha K^o = rK_k^o = r(K^o - K_c^o)$, or

$$K_c^o = \left(1-\frac{\alpha}{r}\right)K^o. \tag{45}$$

Given FPE, ω and $K_c^o/2$ (the average per-country capital stock in use in the two consumption sectors) are related just as in a closed economy, that is,

$$\omega = \chi\frac{K_c^o}{2}. \tag{46}$$

The last four equations determine K^o, K_c^o, ω and r. Indeed, they yield closed-form solutions. In particular,

$$r_l = \frac{1-\beta+\alpha\beta(1+\chi)}{\beta(1+\chi)} \tag{47}$$

$$\omega_l = \frac{\bar{M}^o(1-\beta)\chi}{2(1-\alpha)}\frac{[\beta(1+\chi)]^{\alpha/(1-\alpha)}}{[1-\beta+\alpha\beta(1+\chi)]^{1/(1-\alpha)}} \tag{48}$$

$$K_l^o = \frac{\bar{M}^o}{1-\alpha}\left[\frac{\beta(1+\chi)}{1-\beta+\alpha\beta(1+\chi)}\right]^{\alpha/(1-\alpha)}, \tag{49}$$

where the subscript l denotes free movement of loans. From now on, we will call the equilibrium with free movement of goods and loans as **full integration**. Note that $r_l = r_a$, that is, the world rate of return on capital with full integration is same as the (common) autarky rate of return. This is, however, an artifact of the assumption of Cobb-Douglas technology. The general implication is that free movement of loans brings down the rate of return in the North where it was higher and raises it in the South where it was lower. From the supply-side negative relationship between the capital stock and the rate of return on capital, it follows that the capital stock in the North rises and that in the South falls. Further, as shown in Appendix 3, the world capital stock increases however. This implies that capital used in the world economy in the two consumer good sectors together increases.[13] Consequently, the relative price of the labor-intensive good increases and ω rises.

Moreover, (42) implies that in the steady state

$$(1 - \beta r_l)B_l^j = \beta(\bar{M}^j m_l + w_l). \tag{50}$$

Since $\bar{M}^N > \bar{M}^S$, we have $B_l^N > B_l^S$, i.e. North holds more wealth. The rate of return being the same but $\bar{M}^N > \bar{M}^S$ imply (from (19)) that $K_l^N > K_l^S$, i.e., North holds more capital as well.

It is interesting that although North holds more capital and more wealth, it is the debtor country. The proof is as follows. In (50), substitute $B_l^j = K_l^j - D_l^j/r_l$ and $\bar{M}^j m_l = (1-\alpha)K_l^j$. On rearrangement, and aggregation over the two countries,

[13]From Appendix 1 (Result 3), $dK_c^j = (1 - 1/r^j)dK^j$, $j = N,\ S$. Summing over N and S, $d(K_c^N + K_c^S) = (1 - 1/r^N)dK^N + (1 - 1/r^S)dK^S$. Since K^N increases, K^S falls and $K^N + K^S$ rises, the transition from free trade in goods only to free movement of goods and loans can be seen as an increase in K^N (i.e. $dK^N > 0$), together with a change in K^S such that $-1 < dK^S/dK^N < 0$. Moreover, along the transition, $r^N \geq r^S$. Thus $d(K_c^N + K_c^S) = [1 - 1/r^N + (1 - 1/r^S)dK^s/dK^N]dK^N > 0$.

$$K_l^j[1-\beta(1-\alpha)-\beta r_l] = \frac{D_l^j(1-\beta r_l)}{r_l} + \beta w_l;\ K_l^o[1-\beta(1-\alpha)-\beta r_l] = 2\beta w_l.$$

Dividing these two equations and using $D_l^N + D_l^S = 0$,

$$\frac{K_l^j}{K_l^o} = \frac{1}{2} + \frac{(1-\beta r_l)D_l^j}{2w_l r_l}.$$

Hence $K_l^N > K_l^S$ implies $D_l^N > D_l^S$, i.e., $D_l^N > 0 > D_l^S$, i.e., North (South) is the debtor (creditor) country; this is because, starting from zero debt associated with free trade in goods only, the North and the South respectively experience a decline and an increase in the interest rate. It follows then that the North (South) holds trade surplus (deficit) surplus (deficit) and service deficit (surplus). Of course, the current account and the capital account are in balance along the steady state.

In summary,

Proposition 6: As the economies allow free movement of loans (and attain full integration), (a) the rate of return on capital falls (rises) and the capital stock increases (decreases) in the North (South); (b) the world capital stock increases; (c) the wage/rental ratio and the relative price of the labor-intensive good in the world economy increase; (d) at the equilibrium, North (South) holds more (less) capital and wealth and is the debtor (creditor) country.

We now turn to distributional effects via Eq. (26). An invariant distribution of B_{ht} exists and it is unique for the same reasons as discussed earlier. However, in the presence of international loan market the mean (aggregate) wealth of a country in terms of capital is not equal to the mean capital stock; as a result, the effects on inequality indices are complex. Using μ_B as given in (27),

$$\psi_B^j = \frac{1-\beta}{\beta}\sqrt{\frac{1-\beta r}{1+\beta r}} \cdot \frac{r^{\alpha/(1-\alpha)}}{\bar{M}^j + \omega r^{1/(1-\alpha)}} \tag{51}$$

$$\psi_I^j = (1-\beta)\sqrt{\frac{1-\beta r}{1+\beta r}} \cdot \frac{r^{1/(1-\alpha)}}{\bar{M}^j + \omega r^{1/(1-\alpha)}} = \beta r \psi_B^j \tag{52}$$

$$\psi_U^j = \psi_B^j \frac{\sqrt{(1-\beta)^2 + 1 - \beta^2 r^2}}{1-\beta}. \tag{53}$$

The variances are governed by the change in r. As discussed earlier, an increase in r, through a *multiplying effect*, increases the variance. Since r falls in the North and rises in the South, the variance of each indicator decreases (increases) in the North (South). This tends to lower (increase) inequality in the North (South).

The effects on the means are complicated however. Going back to (27), we see that the mean wealth holding can change due to changes in ω, r and m. An increase in ω would increase the mean wealth holding by an income effect. An increase in r has (i) a direct income effect (at given ω), (ii) an indirect and opposite effect through implied change in m, the reward to factor M in the investment good sector, and (iii) a multiplying (positive) effect (via the denomenator).

Hence it is quite difficult to analytically predict the overall effect on wealth inequality, measured by the coefficient of variation. But the magnitude of various effects can be assessed however. The global increase in ω tends to increase mean wealth and lower inequalilty in both countries. But recall that free movement of loans increases the capital stock in the North and lowers it in the South; hence, the overall effect on the global capital stock and the wage/rental ratio, although positive, is likely to be small. Thus its effect on inequality is also likely to be marginal. Turning next to the effects of r, observe that it has symmetric multiplying effects on standard deviation and mean that exactly offset each other. Hence only the effects (i) and (ii) remain.

Concentrating on (ii), we see that if $\alpha \simeq 0$, capital as in input in the production of capital is negligible, the supply of capital is very inelastic (i.e. it is almost like 'land'), the coefficient of factor M is almost a constant and so is its competitive reward m. In the other extreme, if $\alpha \simeq 1$, $m = r^{-\alpha/(1-\alpha)} \to 0$ in both equilibria (as r, the gross interest rate, exceeds one in both equilibria). Therefore, if $\alpha \simeq 0$ or 1, the change in m is negligibl e. The effect (i) remains dominant, implying that the fall (rise) in r in the North (South) tends to lower (raise) mean wealth and thereby increase (decrease) wealth inequality. For the intermediate values of α, it is hard to predict whether (i) dominates (ii) or vice versa.

This leaves us with the choice of simulation. Our analytical model contains all functional forms. So only the parametric values needed to specified, namely those of α_x, α_y, α, $\bar{M}^N$, $\bar{M}^S$, and the preference parameters, γ and β. All parameters except $\bar{M}^N$ and $\bar{M}^S$ are bounded between 0 and 1. These were varied at .01 grid. $\bar{M}^N$ and $\bar{M}^S$ were chosen respectively greater and less than one with the difference between them in an increasing order until FPE was violated.[14] Parametric configurations were further restricted so that the rate of return on capital in both equilibria were greater than one (i.e. the interest rate were ensured to be positive). The results indicated a very consistent pattern: for values of α from 0.1 to 0.9, the impact on inequality through the change in ω w as small, and the effect (i), the direct income effect of r on mean wealth, dominates the effect (ii), the indirect and opposite effect through the change in m.

As an illustrative example, Table 1 presents the results for a specific case where $\alpha_x = .45$; $\alpha_y = .55$; $\beta = .2$; $\gamma = .5$, $\bar{M}^N = 1.05$ and $\bar{M}^S = .95$.

Table 1: Comparison between Free Trade in Goods Only and Full Integration

[14] This was ascertained by assuming FPE and checking out that the output of good y in the North and good x in the South exceeded zero.

	North			South		
α	Wealth Ineq.	Income Ineq.	Welfare Ineq.	Wealth Ineq.	Income Ineq.	Welfare Ineq.
.2	.83 (.85)	.375 (.376)	1.25 (1.28)	.90 (.88)	.389 (.387)	1.36 (1.32)
.3	.87 (.89)	.407 (.409)	1.29(1.32)	.93 (.91)	.421 (.420)	1.40 (1.36)
.4	.95 (.97)	.463 (.465)	1.40 (1.43)	1.01 (.99)	.478 (.476)	1.51 (1.47)
.5	1.12 (1.14)	.569 (.571)	1.64 (1.68)	1.19 (1.16)	.586 (.583)	1.76 (1.72)
.6	1.53 (1.55)	.807 (.810)	2.23 (2.27)	1.61(1.58)	.827 (.823)	2.36 (2.31)
.7	2.83 (2.87)	1.548 (1.552)	4.10 (4.17)	2.95 (2.91)	1.579 (1.573)	4.30 (4.22)
.8	11.70 (11.81)	6.602 (6.617)	16.81 (17.01)	12.06 (11.81)	6.700 (6.677)	17.39 (17.16)
No parenthesis: free trade in goods Parenthesis: full integration						

The following proposition is then presented (in which 'S' indicates 'simulation-based').

Proposition 7 (S): As the countries allow free movement of loans, inequality rises in the North and falls in the South.

Moreover, the inequality indices across the two countries at the full integration equilibrium can be ranked. Because the rates of return are equalized, the variances are the same. Together with FPE and that mean wealth is greater in the North, the mean wealth, income or welfare is higher in the North. Thus,

Proposition 7: At the full integration equilibrium, inequality is less in the North than in the South.

How does free movement of loans affect social mobility? Similar ambiguities as for inequality arises here. As already discussed, mobility depends on fixed income, $\bar{M}^j m^j + w^j = \bar{M}^j m^j + \omega r^j$. The global increase in ω tends to increase mobility in both countries, but the magnitude of this effect is small. Thus it is governed by effects (i) and (ii). As discussed earlier, it is hard to pin down the net effect analytically and simulations showed that the (i) outweighs (ii). Recalling that an increase in the fixed income lowers inequality and increases mobility, the mobility implications are then opposite to that of inequality. We have

Proposition 8 (S): Mobility of wealth and income at the full integration equilibrium is less in the North and more in the South compared to free trade in goods only.

It should be noted that while full integration could not be compared analytically with free trade in goods, it can be with autarky. Since $r_a^j = r_l$, the variance of wealth holding is the same. But $r_a^j = r_l$ implies $K_a^j = K_l^j$ for country j; together with the North (South) being the debtor (creditor) country, it follows that mean wealth with full integration, compared to autarky, is less (greater) in the North (South). Hence, wealth inequality relative to autarky is unambiguously greater (less) in the North (South). The same holds for income or welfare inequality. Consider next the fixed income. We have $r_a^j = r_l$ and as a result, $m_a^j = m_l^j$. But $\omega_l^N < \omega_a^N$ and $\omega_l^S > \omega_a^S$ because of commodity trade. Thus fixed income – and hence social mobility – at the full integration equilibrium is less (more) in the North (South).

Finally, insofar as the social welfare implication is concerned, we have just seen that the effect on inequality is ambiguous. As will be discussed below, the effect on mean welfare is also ambiguous. Hence the net effect on welfare of either country could be positive or negative.

The movements of individual components of the aggregate (mean-welfare) effect are however quite different from those when countries move from autarky to free trade in goods; moreover, there is a new component (or source). Consider North for example. Since the price of the labor-intensive good rises, it faces a terms of trade decline which is a source of mean-welfare loss. However, its capital stock increases, the production possibility frontier expands and this is a source of gain. These are just the opposite of what they were when the world economy moves from autarky to free trade in goods. In addition, there is a new effect arising from whether the country is debtor or a creditor country. For example, we know that North becomes a debtor country, which, ceteris paribus, reflects a source of loss of wealth and implies mean-welfare loss.

Using the same method of derivation of mean-welfare change from autarky to free trade in goods, we obtain the following expression of mean welfare change from free trade in goods to free movement of loans. For country j,

$$d\mu_U^j = \frac{1-\beta}{\tilde{E}}(Q_x^j - D_x^j)dp + \frac{\beta r^j}{B}dK^j - \frac{\beta}{B}d(D^j/r^j).$$

The three terms corresponds to the three effects discussed above.

6 Concluding Remarks

This paper has presented a baseline, North-South, factor-endowment model of trade, capital accumulation and personal distribution of wealth and income. Inequality in terms of unequal capital holding and earning from capital is considered. The source of inequality lies in idiosyncratic preference shocks (as in Atkenson and Lucas (1992) and Lucas (1992)). Capital, as an asset, is tradable. A central result is that globalization between North and South in terms of free movement of goods and loans leads to more inequality and less mobility along the wealth or income ladder in the North and less inequality and more mobility in the South.

The predictions of the model for aggregate changes also appear interesting. For example, free trade in goods leads to an increase (a decrease) in the total capital stock in the North (South). The South unambiguously gains in terms of social welfare, whereas the North may gain or lose. Moreover, in the presence of free trade in goods as well as free movement of assets, North (South) is the debtor (creditor) country.

Research on trade and personal distribution is in its infancy. There are numerous aspects left out in our analysis. For example, although capital accumulation is considered, long-run, endogenous growth is not. There is already a huge literature on trade and endogenous growth on one hand and distribution and endogenous growth on the other. It is high time to integrate these two strands of literature. Also, our analysis only looks at inequality in terms of capital holding and capital income, not wage or 'skill-wage' inequality.

Another issue is commercial policy meaning tariff, production subsidies etc., for small and large economies rather than this extreme case of a movement from autarky to free trade. Our analysis would imply that, for a small open economy for example, an increase in tariff on imports will increase or decrease wealth/income inequality according as the import-competing sector is capital or labor intensive. If, in particular, we presume that developing countries import relatively capital intensive goods, it then follows that trade protection increases inequality. There is indeed some empirical support for this hypothesis (Bourguignon and Morrisson (1990)). Taking a cross-sectional sample of developing countries, these authors find that "trade protection has a large potential for worsening the income distribution".

Another obvious but important extension would be to consider the political economy of trade and trade related policies in the presence of endogenous wealth and income distribution. The well-known analysis of Mayer (1984) assumes an exogenous distribution of wealth but would seem to serve as the point of departure.

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Appendix 1

Results 1-4 are proven here.
Result 1: The first part is immediate from (18). Next, in view of (19), $dK/d\omega > 0 \Rightarrow dr/d\omega < 0$. Now, log-differentiation of (20) gives

$$\frac{\hat{K}}{\hat{\omega}} = 1 + \frac{1-\beta(1-\alpha)}{1-\beta(1-\alpha)-\beta r}\frac{\hat{r}}{\hat{\omega}} < 1, \text{ since } \frac{dr}{d\omega} < 0.$$

Result 2: Turn to Figure 1. An increase in $\bar{M}$ shifts out the BB curve. Thus $dK/d\bar{M}$ and $dr/d\bar{M}$ are both positive. It remains to show that $dK_c/d\bar{M} > 0$. In the investment good sector, we have $\alpha K_{t+1} = r_t K_{kt}$, where K_{kt} is capital employed in this sector. Thus, in view of (15), $K_{kt} = \alpha K_{t+1}/r_t = \alpha\beta(\omega + K_t)/(1-\beta+\alpha\beta)$. Hence

$$K_{ct} = K_t - K_{kt} = \frac{(1-\beta)K_t - \alpha\beta\omega}{1-\beta+\alpha\beta}, \tag{A1}$$

is an increasing function of K_t. Since $dK/d\bar{M} > 0$, it follows that $dK_c/d\bar{M} > 0$.
Result 3: By the Stolper-Samuelson effect, $d\omega/dp > 0$; also $dK/d\omega > 0$ from Result 1. Thus $dK/dp > 0$. Next, in the investment good sector, $rK_k = \alpha Q_k = \alpha K$. Thus $\hat{r} + \hat{K}_k = \hat{K}$. In view of (19), $\hat{r} = -[(1-\alpha)/\alpha]\hat{K}$. Substituting this into the earlier relation, $\hat{K}_k = \hat{K}/\alpha$. Thus

$$dK_c = d(K - K_k) = K\hat{K} - K_k\hat{K}_k = \left(K - \frac{K_k}{\alpha}\right)\hat{K} = \left(K - \frac{K}{r}\right)\hat{K} = \frac{r-1}{r}dK.$$

Since $dK/dp > 0$ and we have imposed conditions such that $r > 1$, it follows that $dK_c/dp > 0$. Furthermore, from the above expression,

$$\frac{dK_c}{\hat{p}} = \left(K - \frac{K_k}{\alpha}\right)\frac{\hat{K}}{\hat{p}} < (K - K_k)\frac{\hat{K}}{\hat{p}} = K_c\frac{\hat{K}}{\hat{p}}$$

$$\Rightarrow \frac{\hat{K}_c}{\hat{p}} < \frac{\hat{K}}{\hat{p}}.$$

Result 4: Given Cobb-Douglas technology in sectors x and y, we can solve the 'two-sector' model and obtain

$$\frac{\widehat{Q_x/Q_y}}{\hat{p}} = \frac{1}{|\lambda|(\alpha_y - \alpha_x)} - 1 - \frac{1}{|\lambda|}\frac{\hat{K}_c}{\hat{p}}, \tag{A2}$$

where $\alpha_y > \alpha_x$ and $|\lambda| \equiv \lambda_{Lx}\lambda_{Ky} - \lambda_{Kx}\lambda_{Ly}$; moreover, sector x being more labor-intensive, $|\lambda| > 0$. The first two terms are the standard price-output effect which is positive ($|\lambda|(\alpha_y - \alpha_x)) < 1$). The last term is the product of the Rybczinski effect (negative since an increase in K_c reduces the relative output of the labor-intensive good) and the impact of p on K_c (positive *a la* Result 3). It is sufficient to show that

$$\frac{1}{\alpha_y - \alpha_x} - \frac{\hat{K}_c}{\hat{p}} > 0, \text{ or } \frac{1}{\alpha_y - \alpha_x} - \frac{\hat{K}}{\hat{p}} > 0 \quad \text{since } \frac{\hat{K}}{\hat{p}} > \frac{\hat{K}_c}{\hat{p}} \text{ via Result 3.}$$

In this Cobb-Douglas economy, $\hat{\omega}/\hat{p} = 1/(\alpha_y - \alpha_x)$ and $\hat{K}/\hat{p} = (\hat{K}/\hat{\omega}) \cdot (\hat{\omega}/\hat{p}) = (\hat{K}/\hat{\omega}) \div (\alpha_y - \alpha_x)$. Thus the above inequality is equivalent to $1 > \hat{K}/\hat{\omega}$, which is true (Result 1).

Appendix 2

The existence and uniqueness of invariant distributions and its applicability to wealth distribution in the autarkic economy is discussed here.

Consider (a) two measurable spaces, $(V, \mathcal{H})$, $(Z, \mathcal{L})$, and construct a third one $(S, \mathcal{H}\times\mathcal{L})$ where $S \equiv V \times Z$. (b) Let $Q(z, \mathcal{L})$ be the transition function defined on $(Z, \mathcal{L})$; and (c) Let $v_{t+1} = f(v_t, z_t)$ be the 'policy function' where $v \in V$ and and $z \in Z$.

This defines the transition function on $(S, \mathcal{H} \times \mathcal{L})$:

$$P[(v,z), E \times F] = \begin{cases} Q(z,F) & \text{if } f(v,z) \in E \\ 0 & \text{otherwise.} \end{cases}$$

Let P^N be the N-th iteration of it. Then we have the following result based on Theorems 9.14 and 12.10 in Stokey and Lucas (1989).

Result: Let (i) V be a compact, convex subset of R^l with $\mathcal{H}$ as a collection of Borel subsets of V; (ii) Either (a) Z a countable subset of R^k and $\mathcal{L}$ the collection of *all* subsets of Z, or, (b) Z a compact, convex subset of R^k with $\mathcal{L}$ a collection of Borel subsets of Z and $Q(.,.)$ have the Feller property; and (iii) The function $f(.,.)$ be continuous in (v, z). Then $P[.]$ has an invariant probability measure. $\|$

In simpler language, the Markov process of (v, z) generated by $Q(.,.)$ and $f(.,.)$ has a stationary or invariant distribution, i.e., v_{t+1} has the same distribution as does v_t and z_{t+1} has the same distribution as does z_t.

In our model, $Z = (\underline{z},\ \bar{z})$ and $Q(.,.)$ is based on the distribution of z. The set V is the set of B_{ht} in the limit and $f(\cdot)$ is the function given in (10). As $t \to \infty$, m_t, w_t and r_t approach steady state values m, w and r respectively, so that (10) can be regarded as B_{ht+1} being a function of B_{ht} and z_t. Turn now to Figure 2 which depicts the function f for the two extreme values of z. Given $\beta < 1$, it follows that, irrespective of the initial distribution of B_h, as $t \to \infty$, the distribution of B_h is restricted to the range $\underline{B}$ and $\bar{B}$. Hence $V = (\underline{B},\ \bar{B})$. Obviously, (i) is satisfied. (ii) – condition (b) – is also met; $Q(.,.)$ satisfies Feller property because the distribution of z is continuous and i.i.d. In view of (10), the function $f(\cdot)$ is continuous and hence (iii) is satisfied. It follows then that an invariant distribution of B_{ht} exists.

The next result on uniqueness and convergence is based on Stokey and Lucas, Theorem 12.12.

Result: In addition, (iv) Let $S = [a, b] \subset R^l$; (v) $Q(.,.)$ be monotone in the stochastic sense; (vi) The function $f(\cdot)$ be monotonic with respect to both arguments; and (vii) $\exists\ c \in S,\ \epsilon > 0$ and $N \geq 1$ such that $P^N(a, [c, b]) \geq \epsilon$ and $P^N(b, [a, c]) \geq \epsilon$.

Then P has a unique invariant probability measure and the sequence $\{T *^N \lambda_0\}$ converges weakly to it for any initial probability measure λ_0, where $T * \lambda(F)$ is defined as $\int Q(z, F)\lambda(dz)$ for all $F \in \mathcal{L}$. $\|$

Given $V = (\underline{B},\ \bar{B}) \in R$ and $Z = (\underline{z},\ \bar{z}) \in R$, (iv) is satisfied. (v) is also met since the distribution of z is i.i.d. From (10), it is seen that $f(\cdot)$ is monotonic with respect to both arguments and hence (vi) is fulfilled. The most critical condition for uniqueness and stability is (vii), the monotone-mixing condition. Following the argument in the text, this is also satisfied.

A significant generalization of Stokey-Lucas results is given in Hopenhayn and Prescott (1992). That is, the continuity of $f(.,.)$ is not needed. (The standard fixed point arguments are not helpful in proving the existence or uniqueness of the Markov process.) See, especially, their Corollary 5 and Theorem 2.

Appendix 3

We prove that $K_l^o > K_g^o$, i.e. the world capital stock with free movement of goods and loans is greater than that with free movement of goods only. The former is given in (49). Consider free movement of goods only.

Given FPE, the relationship between ω and K_c^o holds as in (46). Also, summing (A1) over the two countries, in the steady state,

$$K_c^o = \frac{(1-\beta)K_g^o - 2\alpha\beta\omega_g}{1-\beta+\alpha\beta}.$$

Thus, substituting (46) into it and eliminating K_c^o,

$$\omega_g = \xi K_g^o, \text{ where } \xi = \frac{(1-\beta)\chi}{2[1-\beta+\alpha\beta(1+\chi)]}. \tag{A3}$$

Eq. (18) also holds for respective countries, i.e.,

$$K_g^N = \eta^N(\omega_g + K_g^N)^\alpha \tag{A4}$$

$$K_g^S = \eta^S(\omega_g + K_g^S)^\alpha. \tag{A5}$$

Noting that $K_g^o = K_g^N + K_g^S$, eqs. (A3)-(A5) determine ω_g, K_g^N and K_g^S.

Define $s^j \equiv K_g^j/K_g^o$. Substituting (A3) into (A4)-(A5) and dividing the resulting expressions by K_g^o,

$$s^N = \frac{\eta^N}{K_g^{o\,1-\alpha}}\left(\xi + s^N\right)^\alpha \tag{A6}$$

$$s^S = \frac{\eta^S}{K_g^{o\,1-\alpha}}\left(\xi + s^S\right)^\alpha \tag{A7}$$

Dividing (A6) by (A7) and noting $s^N + s^S = 1$,

$$\frac{s^N}{1-s^N} = \frac{\eta^N}{\eta^S}\left(\frac{\xi+s^N}{\xi+1-s^N}\right)^\alpha, \quad \text{or}$$

$$h(s^N) \equiv \frac{s^N}{(\xi+s^N)^\alpha} - \frac{\eta^N}{\eta^S}\left(\frac{1-s^N}{(\xi+1-s^N)^\alpha}\right) = 0. \tag{A8}$$

The last equation solves s^N. The function $h(s^N)$ is increasing in s^N. Moreover, as $s^N \to 0$, $h(.) < 0$ and as $s^N \to 1$, $h(.) > 0$. Thus $h(s^N) = 0$ yields a unique solution in the open interval (0,1). Given s^N, (A6) or (A7) determines K_g^o. Once s^N and K_g^o are known, K_g^N and K_g^S are solved.

We now prove that $s^N < \bar{M}^N/\bar{M}^o \equiv b$. Using this, it would be shown that $K_g^o < K_l^o$. From the expression of $h(s^N)$, it follows directly that

$$h(b) = \frac{\bar{M}^{N\,1-\alpha}\bar{M}^{S\,\alpha}}{\bar{M}^{o\,1-\alpha}(\xi\bar{M}^o + \bar{M}^N)^\alpha}\left[\left(\frac{\bar{M}^N}{\bar{M}^S}\right)^\alpha - \left(\frac{\xi\bar{M}^o+\bar{M}^N}{\xi\bar{M}^o+\bar{M}^S}\right)^\alpha\right]$$

We have $\bar{M}^N > \bar{M}^S$ and thus

$$\frac{\bar{M}^N}{\bar{M}^S} > \frac{\xi\bar{M}^o+\bar{M}^N}{\xi\bar{M}^o+\bar{M}^S},$$

implying that the term in the square bracket of $h(b)$ is positive and $h(b) > 0$. since $h(s^N)$ increases with s^N and $h(b) > 0$, it follows that the solution to (A8), say s_g^N, must be less than b.

Next, using the definitions of η^N, η^S and b, we write (A6) and (A7) as

$$\frac{{s^N}^{1/\alpha}}{b^{1/\alpha-1}} = \frac{A(\xi + s^N)}{{K_g^o}^{1/\alpha-1}} \tag{A9}$$

$$\frac{(1-s^N)^{1/\alpha}}{(1-b)^{1/\alpha-1}} = \frac{A(\xi + 1 - s^N)}{{K_g^o}^{1/\alpha-1}}, \tag{A10}$$

where $A = \frac{\beta}{1-\beta+\alpha\beta}\left(\frac{\bar{M}^o}{1-\alpha}\right)^{1/\alpha-1}$.

Now add up (A9) and (A10):

$$\phi(s^N) \equiv \frac{{s^N}^{1/\alpha}}{b^{1/\alpha-1}} + \frac{(1-s^N)^{1/\alpha}}{(1-b)^{1/\alpha-1}} = \frac{A(2\xi+1)}{{K_g^o}^{1/\alpha-1}}. \tag{A11}$$

Upon substituting the expressions for A and ξ, it is seen that $[A(2\xi+1)]^{\alpha/(1-\alpha)} = K_l^o$. Thus (??) can be expressed as

$$\phi(s^N) = \left(\frac{K_l^o}{K_g^o}\right)^{(1-\alpha)/\alpha}. \tag{A12}$$

Note that the function $\phi(s^N)$ attains minimum at $s^N = b$ and the minimum value of $\phi(.)$ is 1. Since $s_g^N < b$, it follows that $\phi(s_g^N) > 1$, implying from (A12) that $K_g^o < K_l^o$.