The right man at the right job? Increasing returns in a matching model with a continuum of job and worker types*

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Abstract

This paper describes a search model with a continuum of worker and job types, transferable utility and an IRS contact technology. We apply a second order Taylor expansion to derive an analytical solution of the equilibrium. We find that one third of the increasing returns in contacts are absorbed by firms and workers being more choosy. Hence, strongly increasing returns in contact rates are consistent with weakly increasing returns in matching. In addition, we derive and decompose the efficiency loss due to inadequate incentives and show how unemployment benefits can reduce the loss. Finally, we derive a relation between the size of the surplus due to search frictions and the degree of substitutability of worker types at given job complexity levels. Numerical simulations of the model show that our approximations are quite accurate.

Keywords: assignment, search, unemployment

JEL codes J21, J23

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1 Introduction

In recent years a flourishing literature on equilibrium search models has emerged. The most important contributions include: Diamond (1982a), Pissarides (1990), Mortensen and Pissarides (1995), and Burdett and Mortensen (1998). Those papers contributed a lot to our understanding of the mechanisms behind unemployment and job creation and their importance for the functioning of the labor market. Nevertheless, when going through most of this literature, one feels a bit uncomfortable because both workers and firms are assumed to be homogeneous. The question that immediately comes to mind is: Why would workers and firms have to spend so much time to find each other when they are all alike? A simple institution like a centralized market place seems a perfect device to handle this type of labor market.

Homogeneity assumptions are clearly not a realistic device for analyzing search equilibria. However, they might serve the goal of tractability. The relevant question is therefore whether important issues are left out by ignoring heterogeneity. We shall argue that this is the case. In the Pissarides (1990) framework, each contact between a job seeker and a vacancy results in a match. Hence, the speed of the matching process is entirely a technical matter. Under heterogeneity, the mechanical contact process and the matching process are disentangled. Both job seekers and firms have to decide whether or not they want to give up the option value of continued search and engage in an employment relation with a particular contact.

Endogenizing the matching decision affects at least one critical issue in this literature, the measurement of returns to scale in the matching function. The common wisdom is at the moment that there are no increasing returns. The thick market arguments of Diamond (1982b) for the existence of increasing returns in the matching process were however so appealing that many authors who empirically rejected this finding felt that they had to apologize. Our model suggests that the debate should be reopened. Consider for example an increase in the scale of the market. Under increasing returns, this will raise the contact rate. The first effect for a job seeker is that it becomes easier to find a matching partner which increases the option value of remaining unemployed. Since the decision whether or not to accept a match is basically a trade off between this option value and the value of the match, an increase in the option value will make workers more choosy. The Walrasian world can be viewed as the extreme case where the contact rate goes to infinity and where the set of acceptable jobs is reduced to the set of optimal jobs.

Returns to scale are typically estimated by relating the stocks of job seekers and vacancies to the number of realized matches. This procedure ignores the fact that, under increasing returns, job seekers and firms become more choosy as the scale of the market increases. Part of the effect on contacts will be absorbed by the fact that a lower fraction of contacts are acceptable and will lead to a match. Our analysis shows that when both vacancies and unemployment increase by 1 %, this leads to 2% more contacts but only to 1.66% more matches, the rest is absorbed by smaller matching sets.

Initially, the assumption of increasing returns in the contact process was motivated by the following problem that arises in models with both constant returns to scale in contacts and heterogeneity. In an integrated labor market, low skilled immigrants would find it harder to find a hamburger job when a bunch of Harvard graduates enter the market. This unpleasant feature is something one would like to avoid. The only alternative is an increasing returns to scale contact technology. Also on practical grounds, constant returns seems to be an unrealistic assumption. It is hard to believe that job search is as effective in South Dakota as it is in downtown Manhattan. Finally, it is worth mentioning that economists are in good company when making the assumption that the contact probability for workers (vacancies) only depends on the amount of vacancies (workers) on the other side of the market. The theory of the velocity of chemical reactions in gasses is based on the same premisses. Hence, increasing pressure speeds up the reaction process. Likewise, the contact rate is higher in dense Manhattan than in Wyoming.

Sattinger (1995) is to our knowledge the first who analyzed the effects of search frictions on the assignment of workers to jobs in a model with transferable utility. In his model, there are exactly as many workers as jobs and both are finite. Sattinger shows that assignment will in general not be efficient because an individual low skilled worker who matches with an intermediate job does not take into account that this makes it harder for an intermediate worker to find an intermediate job. Another step forward, is Shimer and Smith (1999) who derive, without explicitly solving the model, the weakest conditions for efficient assortative matching under search frictions to take place.[1] Finally, Burdett and Coles (1999) discuss a variety of models which are characterized by long-term partnership decisions made by heterogeneous agents. Apart from these references, little work has been done in this area. As in the Shimer and Smith paper, we will consider a continuum of worker and job types and a log-supermodular production function. Our focus differs however from theirs. Our main interest is to provide a characterization of the equilibrium. Besides this different focus, we also allow for free entry of vacancies and product market pricing.

[1] They show that a log- supermodular production function is sufficient for a positive assortative matching equilibrium to arise.

For modelling the production structure, we apply the continuous-type comparative advantage framework first developed in Sattinger (1975) and later refined in Teulings (1995, 1999a). In this structure, worker types are characterized by a single index, referred to as the skill level. Likewise, jobs are characterized by their complexity level. Both indices are continuous. High skilled workers have a comparative advantage in complex jobs, low-skilled workers in simple jobs. The continuous type comparative advantage structure is an excellent framework for search models. First, the comparative advantage structure provides a completely natural reason for search because each worker type has its own "best" type, where her comparative advantages are best utilized. The employer and the worker have a joint interest in a good match, since the better the match, the larger the surplus that remains to be distributed. In contrast, models with universally "good" jobs would not survive a free entry condition for vacancies because only "good" jobs would be created and "bad" jobs would disappear. In addition, the continuum of workers and jobs provides a much simpler framework for analyzing how matching sets change with the scale of the market than a discrete type framework, since it allows us to do comparative statics on marginal variations. One important concept that we will use to characterize the size of the surplus due to search frictions and which makes the model more operational, is the complexity dispersion parameter. This parameter measures how easy workers can be substituted in different jobs.

The disadvantage of search models with transferable utility and heterogeneity is their complexity. The Bellman equations for job seekers and vacancies types typically require the evaluation of integrals over matching sets where both the integrand and the boundaries of these integrals are endogenous functions. This prospect has frightened off many researchers of spending further effort in this direction.

An alternative interpretation of this hurdle helps understanding the avenue that we intend to travel. Compared to the complexity of search models, a Walrasian model gives the annalist a great deal of convenience to offer. First order conditions provide a solid structure to market equilibrium. All higher order conditions can be ignored and the envelope theorem allows the researcher to ignore many indirect effects. But the first step the annalist sets outside this Utopia will immediately bring him into deep trouble. Instead of a single first order effect, he has to evaluate integrals over matching sets, which in general require a multitude of higher order effects to be taken into account.

This interpretation alludes to a straightforward idea. Perhaps we can gain insight in a world with dirty search equilibria if we would add only the second order effects. It is this idea that will be investigated in the present paper. This approach allows us to derive a number of important elasticities directly from a model with a fairly general structure, which in some cases do not depend on any of the model's parameters. For example, under risk neutrality, the elasticity of equilibrium unemployment with respect to one minus the replacement rate is equal to 0.6. This result follows directly from the mathematical structure of second order effects in an equilibrium search model. The availability of an approximate analytical solution also enables a welfare analysis. We are able to derive an explicit expression for total output loss due to search frictions. This loss is caused by three factors: mismatch, forgone production by unemployed workers and the costs of keeping vacancies open. The loss can itself be decomposed in: loss due to inadequate incentives (which can potentially be neutralized by adequate policies) and loss due to search frictions. Our solution also sheds light on the issue of optimal social insurance. When a worker matches with a sub-optimal job, this is not only bad luck for this worker but also for other workers who would have formed a better match with this job. The second effect is in general not internalized by the worker. In our model, unemployment benefits have a useful economic role in the sense that they prevent workers from accepting jobs for which they are ill suited.

A potential risk in our approach is that the approximations are so bad that we really don't learn anything from them. Therefore, we confront our approximations with the numerical solution of the model. When search frictions are small, our approximations almost exactly mimic the numerical solutions. When search frictions become larger, a problem which we label the "corner problem" arises. Under perfect substitution in the goods market, the problem is largest. The problem arises because there are little incentives to open vacancies in the corners (because there are too few workers there). This results in a spike of vacancies just above the corner which will attract many low skilled workers. Given the existence of this spike, it becomes less attractive to open vacancies just above this spike. In a sense, the spike crowds out neighboring jobs. Therefore a second spike somewhere above the first one will arise and we get a process of endogenous segmentation. Burdett and Coles (1997) and Smith (1995) show that a similar process of endogenous segmentation arises in a non-transferable utility context. This process will of course not be picked up by our second order Taylor approximations. Our analytical results are therefore most precise for the middle groups and for labor markets with small search frictions.

The paper is organized as follows. Section 2 presents the assumptions underlying the model and discusses some of its implications regarding the efficiency of the bargaining process and compares the structure of the equilibrium with the Walrasian case. Section 3 discusses some general results regarding the existence of an equilibrium and the shape of the matching sets. Section 4 presents the main innovation of the paper, the use of second order Taylor expansions for the evaluation of the integrals over the matching sets. Next, Section 5 deals with the elasticities of the search surplus and unemployment with respect to a number of key parameters. In this section, we also consider the size and nature of the efficiency losses compared to the Walrasian equilibrium. Section 6 discusses some special problems that arise in the corners of the type space when search frictions are large. In Section 7, we compare the results from our Taylor expansions to full-fledged numerical solutions of the model. Finally, Section 8 discusses some relevant extensions of our approach like risk aversion.

2 Structure of the economy

2.1 Basic assumptions

In our economy, workers and jobs are characterized by a single index, referred to as skill level s and job-complexity level c respectively. Both indices vary continuously, so that there exist an infinitum of worker and job types:

$$\begin{aligned} s &\in [s^-, s^+] \\ c &\in [c^-, c^+],\ c^- > 0 \end{aligned}$$

Let $\underline{L}$ be the size of the labor force and $\underline{l}(s)$ be the density function of s; exogenous variables will be underlined throughout the paper. Furthermore, let $h(s)$ denote the number of unemployed workers of type s per unit of labor supply. Hence, $\frac{h(s)}{\underline{l}(s)}$ is the unemployment rate for workers of type s and the aggregate unemployment rate satisfies $u \equiv \int_{s^-}^{s^+} h(s)ds$. The supply of vacancies is determined by a free entry rule, which drives the asset value of a vacancy to zero in equilibrium. We will denote the number of vacancies of type c per unit of labor supply by $g(c)$. The total number of vacancies per unit of labor supply follows then from $v \equiv \int_{c^-}^{c^+} g(c)dc$ while the total number of vacancies is $\underline{L}v$. Maintaining a vacancy is costly. We assume that those costs are independent of the job type and equal to K per period. We can think of those costs as advertisement costs. Let B denote unemployment benefits or the value of leisure, although the former interpretation is not fully consistent with the model, since we ignore the funding of these benefits. Nevertheless, we shall loosely refer to the ratio of B to reservation wages as the replacement rate.

Since each job type c produces its own commodity, we have to introduce a set of commodity prices $P(c)$. The productivity of a worker, s, at a job, c, is denoted by $F(s,c)$. The gross per period value of a match between s and c is therefore equal to $P(c)F(s,c)$. Search frictions enter the model by a simple linear contact rate $\lambda_{i\to j}$ for worker (job) type i to run into job (worker) type j:

$$\begin{aligned}\lambda_{s\to c} &\equiv \lambda^* Lg(c) \\ \lambda_{c\to s} &\equiv \lambda^* Lh(s)\end{aligned} \tag{1}$$

where λ^* is a technology parameter which measures the efficiency of the matching process. For notational convenience we define $\lambda = \lambda^*\ L$. We can interpret λ then as the relevant scale of the labor market. Matches are destroyed at an exogenous rate δ. Finally, we assume for simplicity that both workers and firms are risk neutral. From these definitions the value of search for a worker and an employer respectively, can be expressed in terms of the following Bellman equations:

$$R(s) = B + \frac{\lambda}{\rho+\delta}\int_{m_c(s)} g(c)\left[W(s,c) - R(s)\right] dc \tag{2}$$

$$K = \frac{\lambda}{\rho+\delta}\int_{m_s(c)} h(s)\left[P(c)F(s,c) - W(s,c)\right] ds \tag{3}$$

where $R(s)$ is the reservation wage of worker type s, $W(s,c)$ is the wage of a worker of type s who is employed at a job type c, and ρ is the discount rate. Matching takes place when both firm and worker are better of. Since we allow for bargaining over the match surplus, any match with a value that exceeds the sum of the outside options of firm and worker is acceptable. The functions $m_c(s)$ and $m_s(c)$ define the subsets of c and s respectively with whom type s and c respectively are willing to form a match with. These subsets are determined by the condition that the match surplus is positive.

$$P(c)F(s,c) - R(s) > 0 \tag{4}$$

Wages are set by a simple Nash bargaining rule.[2] Hence:

$$W(s,c) = \beta P(c)F(s,c) + (1-\beta)R(s) \tag{5}$$

[2] This is not the only Nash bargaining solution. Binmore (1986), Wolinsky (1987) and Abbring (1998) show that when workers cannot search while bargaining, the outcome will be independent of the option values of other job opportunities. Here, we implicitely assume that the worker can continue search while bargaining.

where β denotes the workers' bargaining power. Substituting (5) in (2) and (3) yields:

$$R(s) = B + \frac{\beta\lambda}{\rho+\delta} \int_{m_c(s)} g(c) \left[P(c)F(s,c) - R(s)\right] dc \tag{6}$$

$$K = \frac{(1-\beta)\lambda}{\rho+\delta} \int_{m_s(c)} h(s) \left[P(c)F(s,c) - R(s)\right] ds \tag{7}$$

At this point, we have to be more specific about the shape of $F(s,c)$. We assume that $F(s,c)$ is log supermodular with the following functional form:[3]

$$F(s,c) \equiv e^{s\,c} \tag{8}$$

This specification of the productivity function assures that we get a positive assortative matching equilibrium, see Shimer and Smith (1999).[4] Given our assumption on the wage bargaining, we can write the matching sets for worker type, s, and job type, c, respectively as:

$$\begin{aligned} m_c(s) &= \{c\}_{\,|\,x(s,c)>0} \\ m_s(c) &= \{s\}_{\,|\,x(s,c)>0} \end{aligned} \tag{9}$$

where the log match surplus $x(s,c) \equiv p(c) + sc - r(s)$. In the sequel, lower cases denote the logarithm of the corresponding upper cases. The definition implies: $c \in m_c(s) \Leftrightarrow s \in m_s(c)$, that is: if c is in the matching set of s, then s is in the matching set of c. The condition $x(s,c) > 0$ is equivalent to the condition $P(c)F(s,c) - R(s) > 0$, that has been applied before. Furthermore, we assume that $p(c^-) + s^- c^- > \ln B$, implying that there is a positive surplus from producing, even for the least productive worker.[5]

Since we consider a stationary economy, the number of workers finding a job must equal the number loosing their job. Hence:

$$\delta\left[\underline{l}(s) - h(s)\right] = \lambda\, h(s) \int_{m_c(s)} g(c)\, dc \tag{10}$$

[3] $F(s,c)$ is supermodular if $\forall\ c' > c$ and $\forall s' > s$ the following relation holds: $F(c,s) + F(c',s') > F(c,s') + F(c',s)$, and $F(s,c)$ is log supermodular if $f(s,c) = \log F(s,c)$ is supermodular, see e.g. Topkis (1998).

[4] The results in the next sections do depend qualitatively on the log-supermodularity assumption but not on this specific functional form.

[5] Hence, we rule out structural unemployment. Obviously, the model can deal with structural unemployment by simply adding agents whose productivity will never exceed the value of leisure, but this does not generate further insights. Note that this assumption is only fully appropriate in the case that $\eta = \infty$, since otherwise $p(c^-)$ is endogenous.

Physical output per job type can be derived from a transformation of variables:

$$Y(c) = \frac{\lambda}{\delta} \int_{m_s(c)} h(s) e^{sc} ds \tag{11}$$

The outputs of each job type will be traded on commodity markets. Assume that the demand for commodities is given by a continuous type CES utility function :

$$(1-\eta)p = \ln \left[\int_{c^-}^{c^+} \exp \left[(\eta + 1) \, \underline{q} \, (c) + (1-\eta) p(c)) \right] dc \right] \tag{12}$$

where p is the log price index of consumption (or alternatively, the price of the composite consumption good), $\underline{q}(c)$ is the weight of type c in consumption and η is the elasticity of substitution. From (12) we can derive that the demand for good c satisfies:

$$y(c) - y - \underline{q}(c) = \eta[p - p(c) + \underline{q}(c)]$$

where y denotes log aggregate consumption. We will confine our attention to two special cases, perfect substitution and zero substitution. In the first case, prices are effectively exogenous: $\eta = 0 : p - p(c) + \underline{q}(c) = 0$. Let p be the numeraire. Hence, $p = 0$, implies $p(c) = \underline{q}(c)$. In the second case, the distribution of output per job type is exogenous: $\eta = 0 : \; y(c) = \underline{q}(c)$.

$$\begin{aligned} \eta &= \infty : p(c) = \underline{q}(c), \, \forall c : \underline{q}"(c) < 0, \; \underline{q}'(c^-) = s^-, \; \underline{q}'(c^+) = s^+ \\ \eta &= 0 : y(c) = \underline{q}(c), \, \forall c : \underline{q}(c) > 0, \end{aligned} \tag{13}$$

The assumption $\underline{q}(.)$ being positive for the case that $\eta = 0$ implies that there is positive demand for all job types c. The assumption $\underline{q}"(.)$ being negative has the same effect, as will be discussed in section 2.4. In that section, we will also motivate the lower and upper bound of $\underline{q}'(c)$ in the case of $\eta = \infty$. This completes the description of the structure of the economy.

2.2 Increasing returns?

It is useful to compare our matching technology with the CRS matching technology which is frequently used in the literature (e.g. Pissarides 1990). Let m be the flow of contacts between workers and firms. Then, we have by (1):

$$m = \lambda \int_{s^-}^{s^+} h(s) \int_{c^-}^{c^+} g(c) \, dc \, ds = \lambda \, u \, v \tag{14}$$

The matching elasticities with respect to u and v are both equal to one, hence their sum is two, which is twice as large as in the constant returns specification of Pissarides (1990). Hence, in our model, the contact rate of a job seeker, $m/u = \lambda v$, is independent of the number of other job seekers on the market, and mutatis mutandis the same for vacancies. However, most of the empirical evidence suggests that returns to scale are close to constant.[6] How do we square our assumption with these empirical results? We offer three arguments.

First, the empirical research refers to the number of realized matches, while our technology refers to the number of contacts between workers and firms, or even better, potential contacts. Not all contacts will yield a match however. When s is outside the matching set of c and vice versa, both firm and worker will immediately separate and continue searching for an acceptable match. In the representative agent models, all contacts automatically result in a match. The agents in our model respond to the greater efficiency of the contact process not only by reducing their search spells, but also by becoming choosier (resulting in a better match quality). Hence, increasing returns in the contact technology does not translate one-to-one in returns to scale in the realized matching technology. It could be argued that this issue can be resolved by using data on contacts, or referrals in the wording of Berman (1997). This trick will not work either since when a Wall Street stock broker sees a "help wanted" sign in a Hamburger restaurant, he is unlikely to report this event as a referral. This type of self selection avoids a lot of potential contacts to turn into referrals. This also explains why so few job offers get rejected. Disappointing as it is, there is no direct way to establish the returns to scale elasticity in the contact technology from any existing data set. An important goal of our analysis is to establish the size of the effect of an increased contact rate on the match quality and on the number of accepted matches, so that our assumptions can be tested indirectly.

Second, one has to consider: what is scale? Obviously, saying that scale matters is not the same as saying that the US labor market with 200 million inhabitants is more efficient than that of the Netherlands with only 15 million inhabitants. A more useful way to analyze the effect of scale on the

[6] Studies that estimated an aggregate matching function include, Blanchard and Diamond (1989) for the US, Jackman et al. (1989) for the UK and Belderbos, and Teulings (1988) and Van Ours (1991) for the Netherlands. Those studies could not reject the assumption of constant returns to scale. Not all the evidence rejects IRS. Yashiv (1996) finds IRS in the Israelian matching function and Shimer (1999) gives demographical evidence for the US that supports the thick market externality arguments.. Burdett et al. (1994) argue that increasing returns to scale are obscured because mismeasurement (due to aggregation and frequency problems in the data) of the true matching function.

efficiency of the search process is to interpret it as the density of the labor market. In that case, we can compare the functioning of labor markets in for example Manhattan and Wyoming. Per job opening, the number of potential candidates for filling a vacancy is much higher in Manhattan than in the mountains of Wyoming. Elsewhere, see Gautier and Teulings (1999), we develop an empirical measure of the scale of the labor market based on this notion. We plan to test in the near future whether this type of variation is consistent with the predictions of our model.

Finally, all existing estimates of the returns to scale parameter are based on cyclical variation in the number of vacancies and unemployed. This variation is affected by all kinds of out-of-equilibrium processes, which usually are not fully modelled. We suspect that this variation does therefore not adequately reflect differences in the scale (or: density) of a labor market, leading to a downwardly biased estimate of the scale elasticity.

2.3 Efficiency of the bargaining process

In a single worker-single job world, efficiency can be achieved when the Hosios (1990) condition is satisfied: the workers' share of the match surplus is equal to his marginal contribution to the matching process (which is measured by the elasticity of the matching function). In a world with increasing returns, efficiency can never be achieved because there is insufficient output to reward each factor by its marginal contribution. The issue of efficiency gets more complicated when the heterogeneity of the workforce is taken into account because in that case, the distribution of the surplus over the different worker types also becomes an issue. However, matters are simplified by the fact that the utility function (12) is homothetic, which implies that we can separate between efficiency and distribution issues.

When we assume that costless redistribution policies are available, any efficient outcome must always maximize net aggregate output. Since there are no choice variables involved in ongoing matches and since employers always get their outside option due to the free entry condition, maximizing aggregate output is equivalent to maximizing the sum of the asset values of job seekers minus the costs for maintaining vacancies. This function $S[g(c), h(s)]$ therefore satisfies:

$$
\begin{aligned}
S[g(c), h(s)] &= \int_{s^-}^{s^+} h(s) R^o(s)\, ds - \int_{c^-}^{c^+} g(c) K\, dc = \qquad (15)\\
&\int_{s^-}^{s^+} h(s) \left\{ B + \frac{\lambda}{\rho+\delta} \left[\int_{m_c^o(s)} g(c) \left[P(c) F(s,c) - R^o(s) \right] dc \right] \right\} ds
\end{aligned}
$$

$$- \int_{c^-}^{c^+} g(c) K \, dc$$

where $R^o(s)$ denotes the asset value of a job seeker and where $m_c^o(s)$ denotes the matching set that is based on this asset value. The task of a social planner would be to choose $R^o(s)$ and $g(c)$ such that $S(\cdot)$ is maximized. The first order condition for $g(c)$ reads $S_{g(c)}[g(c), h(s)] = 0$; by duality, the first order condition for $R^o(s)$ is identical to the condition $S_{h(s)}[g(c), h(s)] = R^o(s)$, where the suffices refer to the partial derivatives of the integrands for each c and s respectively.[7]

The first order conditions for this problem can be rewritten in the form of equations (6) and (7), except that we have to replace β and $(1-\beta)$ by unity in the equations for $R(s)$ and K respectively because only then do workers and firms receive the full rewards to their marginal contribution of the matching process. There is a simple intuition for this result. In Pissarides' constant returns to scale world, a job seeker entering the labor market imposes a positive externality on employers (since their contact rate goes up) and a negative externality on workers (since their contact rate goes down). In our increasing returns world, there is no negative externality, since the contact rate for workers is independent of the number of other job seekers that are wandering around. Hence, we should award workers the full surplus of the match to reward them for the positive externality they impose on employers. Mutatis mutandis the same argument applies for employers. In section 5.3, we present a simple technique which yields an approximation of the efficiency loss.

2.4 The Walrasian benchmark and complexity dispersion

As a benchmark for future reference, we reiterate some of the previous results on the frictionless continuous type comparative advantage model, see e.g. Teulings (1995, 1999), Teulings and Vieira (1998). In this simple Walrasian world, the profit maximizing strategy for an employer with a vacancy of type c is simply to minimize (log) cost per efficiency unit, that is: $sc - r(s)$. When there are no mass points in the distribution of either labor supply or product demand, this cost curve has a unique minimum for each c. Hence, the assignment of workers to jobs can be described by a one-to-

[7] The analogue of the equation for $R(s)$ follows from the first order condition for $h(s)$. For the condition for K, the following equality is applied:
$\int_s h(s) \int_{m_c^o(s)} g(c) \; dc \, ds = \int_c g(c) \int_{m_s^o(c)} h(s) \, ds \, dc$ where $m_s^o(c)$ is defined similarly to $m_c^o(s)$.

one correspondence between s and c. The first order condition of the cost minimization problem reads:

$$r'[s(c)] = c \tag{16}$$

where $s(c)$ is the optimal assignment for job c. Differentiating this first order condition to c yields: $r"[s(c)]s'(c) = 1$. Since our supermodular production function implies that $s'(c) > 0$, it must also be that $r"(s) > 0$. This is the second order condition from the cost minimization problem: the (log) costs of hiring a worker with an additional unit of skill are increasing while the log returns are constant, namely equal to the value of c for that firm.

The zero profit condition of firms reads: $p(c) + s(c)c - r[s(c)] = 0$. Since this condition applies identically for all c, its first difference must also apply. Hence (the effect via $s(c)$ drops out by the envelope theorem):

$$-p'(c) = s(c) \tag{17}$$

Differentiating a second time yields: $-p"(c) = s'(c) = 1/r"(s)$. The lower and upper support of $\underline{q}(c)$ for $\eta = \infty$ guarantee that the lowest skill types will be employed at the simplest job types and mutatis mutandis the same for the highest skill types. For the case that $\eta = \infty$, $p(c) = \underline{q}(c)$ and hence $p"(c) = \underline{q}"(c)$.

A crucial variable in this model is the second derivative of the wage function. Since $r"[s(c)] = 1/s'(c)$, it is a measure of job heterogeneity. The higher $r"(s)$, the more variation there exists in job complexity per unit of s. This explains why $r"(s)$ is the main determinant of the elasticities of substitution and complementarity between skill types. The higher $r"(s)$ is, the more heterogeneous jobs are and the less easy workers are substitutable (since their relative productivities vary too much between job types, see Teulings (1999). Nevertheless, $r"(s)$ is not an adequate parameter for summarizing the production structure. For understanding the problem, it is important to realize that the empirical implications of the model are invariant to a linear transformation of s, since we have not yet defined the units of measurement of commodities of various c types. Any multiplicative transformation of s can be absorbed by an opposite transformation of c and any additive transformation of c can be absorbed by a compensating redefinition of commodity prices $p(c)$. However, $r"(s)$ will be affected by a linear transformation of s, which makes it unsuitable as a summary statistic that can be meaningfully compared across time and space. Teulings and Vieira (1998) therefore proposed a complexity dispersion parameter which is invariant to a linear transformation of s and which reads: $r"(s)/r'(s)^2$. It turns out that this parameter is also a determining factor for the size of the search frictions in the present model. There is a simple intuition for this result. The more easy workers can be substituted from one job to another (substitution is perfect when the complexity dispersion parameter equals zero), the wider the matching sets will be. Because the workers become less choosy, the expected unemployment duration will fall. We shall return to this issue in section 7.

3 Some characteristics of the equilibrium

Equations (6), (7) and (10) immediately reveal that it is difficult to achieve analytical results. They all involve the calculation of integrals of which both the integrand and the integration boundaries are endogenous. It is precisely this complexity which has prohibited progress in this type of models. Therefore, we attack this problem by focusing on second order Taylor expansions which will be discussed in detail in the next section. The following Definition and Theorem establish a number of characteristics of the equilibrium that will be crucial for our Taylor expansions to make sense.

Definition 1 *A steady state search equilibrium is defined as a sextet* $\{r(s), g(c), h(s), p(c), m_c(s), m_s(c)\}$

The equilibrium is fully determined by the system consisting of the following 6 functional equations: (6), and (7) for $g(c)$ and $r(s)$, (12) for $p(c)$, the two equations in (9) define the matching sets $m_c(s)$ and $m_s(c)$ and the unemployment distribution $h(s)$ follows from the steady state flow condition: (10).

Since our production function is nonnegative, log supermodular, symmetric, continuous and at least twice differentiable, we refer to Shimer and Smith (1999) for an existence proof.[8]

Theorem 2 (1) *Under the assumptions of 2.1, the following conditions hold in search equilibrium:*

1. *$r(s)$, $p(c)$, and $x(s,c)$ are twice differentiable;*
2. *$r'(s) > 0$, $r"(s) > 0$, $p"(c) < 0$;*

[8] The proof is based on a contraction mapping of the agent's continuous value functions into itself. Translated to our model: Given the CES utility function (12), condition (4) maps the reservation wages into matching sets, condition (10) maps matching sets $m_c(s)$ and $m_s(c)$ into unmatched densities and conditions (6), and (7) are the composite maps of values mapping unmatched workers and vacancies into new values. A search equilibrium is a fixed point of these mappings.

3. $x_{ss}(s,c) < 0$ *and* $x_{cc}(s,c) < 0$*;*

4. *the sets* $m_c(s)$ *and* $m_s(c)$ *are connect;*

5. *let* $c^-(s)$ *be the lower bound of* $m_c(s)$ *and let* $c^+(s)$ *be the upper bound; then* $c^{-'}(s) > 0$ *and* $c^{+'}(s) > 0$*.*

The proof of part 5 requires some notation which will play a crucial role in the subsequent analysis. We introduce functions which denote the values of c and s that maximize the match surplus $x(s,c)$ for a particular value of s and c respectively:

$$\begin{aligned} c(s) &\equiv \tilde{c} | \{x(s,\tilde{c}) \geq x(s,c), \forall c\} \\ s(c) &\equiv \tilde{s} | \{x(\tilde{s},c) \geq x(s,c), \forall s\} \end{aligned} \tag{18}$$

By this definition and part 1 of Theorem 1, $x_c\,[s, c(s)] = 0$ and $x_s\,[s(c), c] = 0$. Hence: $-p'\,[c(s)] = s$, $-p"\,[c(s)]\,c'(s) = 1$, $r'\,[s(c)] = c$, and $r"\,[s(c)]\,c'(s) = 1$. For future reference, we define the inverse function of $s(c)$: $s\,[d(s)] \equiv s$. By this definition we have $r'(s) = d(s)$ and $r"(s) = d'(s)$. The proof can be found in Appendix 1.

Theorem 1 provides a number of important characteristics of the equilibrium. Part 2 states that the characteristics of the Walrasian equilibrium (16) and the second order condition $r"(s) > 0$ and the condition $p"(c) < 0$ carry over to the search equilibrium. The conditions on the second derivatives imply that the surplus $x(s,c)$ has at most a single interior maximum in c (and s) keeping s (and c) fixed (part 3 of theorem 1). Those maxima are defined above as $c\,(s)$ and $s\,(c)$ respectively. An inspection of (6) shows that this maximum must necessarily be positive, if vacancies of that c-type exist. Hence, there are at most two solutions for the equation $x(s,c) = 0$ (keeping either s or c fixed). By this feature and by the differentiability of $x(s,c)$, matching sets are connect. Their upper and lower bounds are upward sloping, so that "on average" better skilled workers are matched to more complex jobs. As mentioned before, Shimer and Smith (1999) give a more general proof for positive assortative matching to arise. They show that log-supermodularity of the production function is a sufficient general condition

Theorem 1 also implies that there is no such thing as "good" or "bad" jobs which are independent of the skills of the worker. For each worker type s there exists one perfect match $c(s)$ that maximizes the joint surplus by making optimal use of the comparative advantages of that worker type.

Moreover, the notion of one particular type of job being best, independent of s, is inconsistent with commodity market competition and the free entry condition for vacancies. If a particular type of job would be more profitable than others, a corner solution would result where only "good" vacancies would be opened. However, under product market competition the price of the output of that job would be driven down till it is no longer best.

Under the assumptions made, the best worker for a job of type c is an interior point of the matching set of c. The intuition for this is the following. Some workers in the matching set of this job will have more skills than the worker who generates the highest output. The threat point of those workers in the wage bargain is so high that the firm receives only a small share of the output. On the other hand, some workers in the job's matching set are less productive than the firm's favorite worker. Although the firm has a strong bargaining position when bargaining with such a low productivity worker, the total surplus is too small. The firm's favorite worker type lies in between the above described cases.

The situation is depicted graphically in Figure 1-3. Figure 1 shows the surplus (the area between the (log) value-added-locus and the reservation-wage-locus for a given job type c. The surplus reaches a maximum when the job is occupied by a type $s(c)$ worker. In the Walrasian case, all type c jobs will be matched with type $s(c)$ workers because only there, both loci are tangent ($r'(s) = c$ by the first order condition, and $p(c) + s(c)c = r[s(c)]$ by the zero profit condition). Figure 2 shows the surplus for a given worker type, s. When such a worker, s, meets a job type, $c^+(s)$ or $c^-(s)$, he is indifferent between this match or remaining unemployed. When he meets a job in $< c^+, c^- >$ the match value exceeds the option value of waiting for a better match plus his value of leisure B. Figure 3 shows the functions $s(c)$, $c(s)$, $c^-(s)$ and $c^+(s)$ in the s, c-space, for the economy as a whole. Obviously, the maxima $s(c)$ and $c(s)$ are in between the upper and the lower bound. The shaded area represents the surplus for worker type s_i. When c is the maximum of $c(s)$ for a particular s, then it is not necessarily true that s is the maximum $s(c)$ for that particular c. In other words, the inverse of $s(c)$, denoted $d(s)$, is not necessarily equal to $c(s)$. Here, the following result will prove to be helpful:

$$d(s) = c(s) \Leftrightarrow \frac{d\,x[s, c(s)]}{ds} = 0 \tag{19}$$

The proof of this result is simple. By the definition of $c(s)$, $x_c[s, c(s)] = 0$. Hence, the result applies if $x_s[s, c(s)] = 0$. By the definition of $s(c)$, this is the case when $s = s[c(s)]$, or equivalently, when $d(s) = c(s)$. Q.E.D. Figure 4, gives the most simple example of the case where $d(s) = c(s)$ In this example, the boundaries of the matching sets are linear.

4 Using Taylor expansions for the integrals

The main obstacle to achieve progress in the analysis of equilibrium search models, of which the model in this paper is an example, is the evaluation of the integrals of the type that show up in the right hand side of equation (6). Our innovation is that we use a second order Taylor expansion to approximate the value of this integral. For this purpose, we benefit from the characteristics of the surplus $x(s,c)$, which has a negative second derivative in both its arguments. Since the calculation of the integral requires the evaluation of $x(s,c)$ around its maximum, it is a straightforward idea to approximate this function by a parabola. This approximation requires the maximum surplus $x\left[s,c(s)\right]$ to be relatively small. In the Walrasian case, this surplus equals zero. Hence, our approximation applies as long as we do not move too far from the competitive equilibrium. Using this idea for the integral in equation (6), yields:

$$\int_{m_c(s)} g(c)\left[e^{x(s,c)}-1\right]\,dc \cong \frac{4\sqrt{2}}{3}g\left[c(s)\right]\sqrt{c'(s)\,x\left[s,c(s)\right]^3} \tag{20}$$

$$\int_{m_s(c)} h(s)\left[e^{x(s,c)}-1\right]\,dc \cong \frac{4\sqrt{2}}{3}h\left[s(c)\right]\sqrt{s'(c)\,x\left[s(c),c\right]^3} \tag{21}$$

The derivation of these relations can be found in Appendix 2. Figure 5 provides a simple intuition for this result. First, we apply an approximation for the integrand: $e^{x(s,c)}-1 \simeq x\left(s,c\right)$. Next, the maximum of the integrand of the domain of integration can be approximated by a second order Taylor expansion: $x\left[s,c(s)\right] \simeq -\frac{1}{2}x_{cc}\left[s,c\left(s\right)\right]\Delta^2$, where $\Delta = c^+(s)-c(s) = c(s) - c^-(s)$. We apply $x_{cc}\left[s,c(s)\right] = p"\left[c(s)\right] = -c'(s)^{-1}$. Hence, the surface of the rectangle in Figure 5 equals $2\Delta x\left[s,c(s)\right] = 2\sqrt{2c'(s)x\left[s,c(s)\right]^3}$. Two thirds of this surface is below the parabola, which corresponds to the integral over the matching set. The above argument implies that the average surplus of a worker of type s above her log reservation wage $r(s)$ can be approximated by $\frac{2}{3}x\left[s,c(s)\right]$, or by a complementary argument, the average loss relative to the Walrasian optimum is: $\frac{1}{3}x\left[s,c(s)\right]$.

By substituting these expansions in (6), (7), and (10), the model can be written as a system of three functional equations:

$$1-\frac{B}{R(s)} = \frac{4\sqrt{2}}{3}\frac{\lambda\beta}{\rho+\delta}g\left[c(s)\right]\sqrt{c'(s)\,x\left[s,c(s)\right]^3} \tag{22}$$

$$\frac{K}{R(s)} = \frac{4\sqrt{2}}{3}\frac{\lambda(1-\beta)}{\rho+\delta}h(s)\sqrt{d'(s)^{-1}\,x\left[s,d(s)\right]^3}$$
$$\delta\left[\underline{l}(s)-h(s)\right] = 2\sqrt{2}\lambda h(s)g\left[c(s)\right]\sqrt{c'(s)\,x\left[s,c(s)\right]}$$

where we have substituted $d(s)$ for c for and $s\left[d(s)\right]=s$ for $s(c)$ in equation (7). The third equation can be used to eliminate the function $g\left[c(s)\right]$ from the model. Solving the resulting equation for $h(s)$ yields:

$$\frac{h(s)}{\underline{l}(s)} = \frac{\delta}{\delta+\frac{3}{2}\left[1-\frac{B}{R(s)}\right]\frac{\rho+\delta}{\beta}\frac{1}{x[s,c(s)]}} \tag{23}$$

A nice feature of this relation is that λ, the parameter which is the most difficult one to quantify in our model has cancelled out. To get some feeling about the size of the surplus which is generated by the model, we can fill in some "reasonable" parameter values for the observables in (23). Let $\delta=0.15$, $\rho=0.10$, $\beta=0.40$, $B/R(s)=0.5$, and $h(s)/\underline{l}(s)=0.05$. Readers who do not like our choices regarding these parameter values can of course easily make their own calculations. Under the above assumed parameter values, $x\left[s,c(s)\right]$, the maximum surplus relative to the outside options, turns out to be 16%.

Although equation (22) is a great simplification compared to the system of equations (6), (7) and (10), the calculation of the equilibrium is still a huge task. We need to apply therefore three further approximations, which greatly simplify our system of equations. The first of these approximations has a clear justification, the next two require more trust of the reader. There is justification, but it applies only under a particular assumption. Our approach will be to discuss the approximations and their implications extensively here and to test their applicability by comparing them with the numerical equilibrium for the full model. There is no harm in revealing at this point that for small search frictions, our approximations do surprisingly well.

First, we apply the standard assumption that the unemployment rate is close to zero: $\frac{u}{1-u}\cong u$, and that this holds for each worker type, s: $\frac{h(s)}{\underline{l}(s)-h(s)}\cong h(s)$. Second, we assume that $d(s)=c(s)$. A legitimation for this assumption is that both $c(s)$ and $d(s)$ are in the set $m_c(s)$. In a frictionless labor market, this set converges to a single point, so that the assumption is no longer an approximation. This is the one-to-one correspondence result for the Walrasian version of the model which we discussed in Section 2.4. However, in the system of equations (22) both $x\left[s,c(s)\right]$ and $x\left[s,d(s)\right]$ show up. Since in the Walrasian case both $x\left[s,c(s)\right]$ and $x\left[s,d(s)\right]$ converge to zero, we have no clue about their relative magnitude. Equation (19) shows that this assumption applies if and only if $x\left[s,c(s)\right]$ does not vary with s.

The necessity of a third assumption depends on the value of η. When $\eta = \infty$, prices are fully determined by $\underline{q}(c)$ and hence $c'(s) = \underline{q}"(c)^{-1}$. In the case that $\eta = 0$, we first have to determine commodity prices for the Walrasian case. As a first approximation, we shall assume that frictions leave relative prices (and hence, by the numeraire, absolute prices) unchanged, so that $c'(s)$ can be derived from commodity prices in the absence of search frictions.[9]

Applying these simplifications, we can derive from (20)-(22) that:

$$x^*(s)^5 = \frac{81}{128} Q^2 B^*(s)^2 K^*(s)^2 \underline{l}(s)^{-2} c'(s) \tag{24}$$

$$\frac{h(s)}{\underline{l}(s)} = \frac{2}{3} \frac{\delta\beta}{\rho+\delta} B^*(s)^{-1} x^*(s) \tag{25}$$

$$g[c(s)] = \frac{2}{3} \frac{\delta(1-\beta)}{(\rho+\delta)} K^*(s)^{-1} x^*(s) c'(s)^{-1} \tag{26}$$

where $x^*(s) \equiv x[s, c(s)]$, $Q \equiv \frac{(\rho+\delta)^2}{\delta\beta(1-\beta)\lambda}$, $B^*(s) \equiv 1 - \frac{B}{R(s)}$, and $K^*(s) \equiv \frac{K}{R(s)}$. Equations (24) and (25) describe the solution of the model completely when $\eta = \infty$. The size of the surplus $x^*(s)$ depends therefore on a composite parameter Q, one minus the replacement rate $B^*(s)$, the ratio of capital costs to reservation wages $K^*(s)$, labor supply $\underline{l}(s)$, and variation in job complexity $c'(s)$. To get some feeling for the interpretation of $K^*(s)$, consider a model where K does not reflect the cost of advertising, but the capital cost for maintaining either a vacancy or an occupied job. Then, gross value added $P(c)F(s,c)$ covers the sum of labor cost $R(s)$ and capital cost K. In that case we would set $K^*(s)$ to a value of about 0.5. A similar relation applies to the unemployment rate per worker type s. The relevant elasticities do not depend on choice parameters, but are fully determined by the logic of the model, which is a remarkable feature.

5 The analysis of the equilibrium

5.1 The scale of the labor market

Equations (24) and (25) allow an analytical evaluation of the characteristics of the equilibrium. A first issue is the magnitude of the returns to scale in the matching process that is implied by the model. Our contact technology

[9] This assumption is in line with the assumption that $c(s) \cong d(s)$. The latter implies $dx^*(s)/ds \cong 0$; $x^*(s)$ is a measure of the cost of search frictions as a share of total cost. When this cost-share is approximately constant, relative prices will not be much affected by search frictions.

implies a returns to scale elasticity of 2: doubling both the number of job seekers and vacancies quadruples the number of contacts. However, part of the effect of this increase in contacts on the number of realized matches will be undone by job seekers becoming more choosy about the jobs they are willing to accept (and mutatis mutandis the same for firms). The size of this effect can be obtained by considering the implications of a change in the parameter measuring total labor supply, $\lambda = \lambda^* \underline{L}$. By equations (24) and (25), where λ enters through Q, the elasticities of the surplus $x^*(s)$ and hence the unemployment rate $h(s)$ with respect to λ are equal to $2/5 = 0.4$. A similar equation as for $h(s)$ applies to the number of vacancies per unit of labor supply, $g[c(s)]$, which therefore has the same elasticity with respect to λ of 0.4. Hence, a 1 % increase in total labor supply will increase the total number of vacancies and unemployed by (1-0.4)% = 0.6%. By the flow equilibrium and the constancy of the separation rate δ, the number of matches varies proportionally to λ (almost, up to a factor $1-u$). A 0.6 % increase in both inputs in the matching process yields therefore a 1 % increase in the number of matches. The returns to scale elasticity can then be calculated to be equal to $\frac{1}{0.6} = 1.66$. Hence, one third of the increasing returns in the contact technology are absorbed by a greater choosiness of job seekers and firms. Accounting for some downward bias in most estimates due to the mismeasurement of the scale concept and due to aggregation bias over time (see Burdett et al. (1994)), this number might well be within the confidence interval of most empirical studies of the matching function[10]

5.2 The efficiency loss due to search frictions

A second issue regarding the equilibrium is the implied efficiency loss due to search frictions. This loss as a fraction of total output consists of three parts: the lost production due to unemployment, the cost of maintaining vacancies and the cost of misallocation that arises because search frictions induce workers to lower their reservation wage so that they end up accepting jobs that are not fully optimal. The first factor can simply be calculated from the unemployment rate $\frac{h(s)}{\underline{l}(s)}$ times the cost of unemployment relative to the reservation wage.[11] These cost are equal to one minus the replacement

[10]Increases in L, independent of λ^* are rare in reality. The following example illustrates this.When the EC countries opened their borders for workers from other countries, L increased enormously. However, the probability for a worker in Lisbon to meet a vacancy in Berlin is many times smaller than to meet a vacancy in Lisbon. In other words, $\lambda = \lambda^* L$, did not change very much.

[11]Here (and below, when discussing the cost of a vacancy), the proper normalization would be to use the actual wage $W(s, c)$ instead of $R(s)$. From section 4:

rate, $B^*(s)$. The second factor can be calculated from an expression for the number of vacancies that is maintained per unit s, $g[c(s)]\, c'(s)$, multiplied by the cost of maintaining a vacancy relative to the reservation wage, $K^*(s)$. An expression for $g[c(s)]$ can be obtained from an equation similar to that for $h(s)$ in (22).[12] For the calculation of the final term, recall from section 4 that the average loss relative to the optimal Walrasian allocation is $^1/_3 x^*(s)$. Hence:

$$Loss_{Wal} \simeq \frac{1}{3} x^*(s) \left[\frac{2\delta\beta}{\rho+\delta} + \frac{2\delta(1-\beta)}{\rho+\delta} + 1 \right] \tag{27}$$

The three terms in square brackets reflect the respective losses. All losses vary therefore proportional to $x^*(s)$. Under the parameter values discussed before, the loss is $\frac{1}{3}x^*(s)\,[0.48 + 0.72 + 1]$. Remarkably, the relative importance of unemployment and vacancies in the average loss compared to the Walras equilibrium are independent of $B^*(s)$ and $K^*(s)$. The ratio between both cost types is fully determined by the bargaining power parameter β. This is due to the fact that firms keep investing in vacancies till the costs of keeping the vacancy open are equal to their expected share in the future surpluses from an employment relation. Similarly, workers adjust their reservation wages such that the share in the expected surplus from search is equal to their reservation wage. The ratio of the cost of misallocation relative to the cost of unemployment and vacancies is $\frac{2\delta}{\rho+\delta}$: the higher the separation rate δ, the more often employment relations will be separated and the more often workers will experience an unemployment spell; the higher the discount rate ρ, the more costly will be search and hence, the greater will be misallocation.

From the expression for Q, it can be seen that the surplus $x^*(s)$ (and therefore also the efficiency loss) are proportional to the inverse of $\beta(1-\beta)$; hence, both are minimized for $\beta = 0.5$.[13] This result implies that according to most estimates of the distribution of bargaining power (e.g. Abowd and Lemieux, 1991) workers bargaining power is too low! Interestingly, recent estimates suggest workers' bargaining power to be lower in continental Europe with more centralized wage bargaining regimes (Teulings, 1998; Nickell, 1998) which suggests that the European problem is not that workers have too much bargaining power. We return to this issue in section 7. From equation

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$\mathrm{E}_{m_c(s)}\left[\frac{W(s,c)}{R(s)} - 1\right] \simeq \frac{1}{3}x\,[s, c(s)]$. Hence, the proper cost are: $B^*(s) + \frac{1}{3}x\,[s, c(s)]$. Substitution in equation (27) reveals that this is a second order effect in $x\,[s, c(s)]$.

[12] Take the third equation, drop the factor $[\underline{l}(s) - h(s)]$ on the left hand side and use the second equation to substitute for $h(s)$ in the right hand side.

[13] That large surpluses yield high inefficiencies might be strange at first sight. However, surplusses are defined relative to the reservation wage of the worker. Large surplus are due to low reservation wages.

(25), unemployment is proportional to $\beta^{0.6}(1-\beta)^{-0.4}$; hence, it is minimized by letting $\beta \to 0$. However, this would from a welfare point of view not be optimal because the workers would accept jobs at which they are not very productive and employers would spend too many resources on maintaining vacancies.

5.3 Efficiency loss due to inadequate incentives

There is another way to decompose the efficiency loss: the costs of search frictions versus the costs of inadequate incentives due to the increasing returns in matching, see the discussion in Section (2.3). Theoretically, a social planner could undo this second component by letting workers and firms participate as if they would get the full match surplus. The magnitude of this second component can then be calculated from the first order conditions of the maximization of (15). Deriving the equivalent of (24) for these equations is equivalent to replacing the factor $\beta(1-\beta)$ in the denominator of Q by unity. Hence, the surplus $x^*(s)$ would go down by a factor $\beta^{0.4}(1-\beta)^{0.4}$ if a social planner could implement adequate incentives. In the same vein, unemployment and vacancies go down by respective factors: $\beta^{-0.6}(1-\beta)^{0.4}$, and $\beta^{0.4}(1-\beta)^{-0.6}$. The welfare loss of the decentralized search equilibrium compared to the social planner's search optimum reads[14]:

$$Loss_{SP} \simeq \frac{1}{3}\left[\frac{\rho+3\delta}{\rho+\delta} - \beta^{0.4}(1-\beta)^{0.4}\frac{\rho+5\delta}{\rho+\delta}\right] x^*(s) \tag{28}$$

This loss is minimized by setting β equal to one half, which mimics the conclusion of the previous paragraph. At this minimum, the expression in square brackets is equal to 4.3 %. Comparing equation (27) to equation (28) reveals that for $\beta = 0.5$ and $\delta/\rho \to 0$, the inefficiency introduced by the lack of incentives would be compensated for by quadrupling the size of the labor market, i.e. the parameter λ. For other values of β, the inefficiencies are larger. Hence, these are significant. The social planners level of unemployment is $\beta^{-0.6}(1-\beta)^{0.4}$ times the level of unemployment in the decentralized economy, which is 1.15 for $\beta = 0.5$ or 1.41 for $\beta = 0.4$. Social planner's unemployment is therefore higher than unemployment in the decentralized market equilibrium. There is too low a reward for search activities which are produced under increasing returns. Hence, job seekers accept jobs too easily.

[14]The first term within brackets is simply equal to Loss_{Wal} while the last term within the brackets is Loss_{Wal} where β and $(1-\beta)$ are set at 1 (which would reflect unemployment and the stock of vacancies when incentives were adequate).

5.4 The trade off between bargaining power and the replacement rate

The elasticity of unemployment with respect to one minus the replacement rate is -0.6 (-1 directly in the unemployment equation and 0.4 indirectly via $x^*(s)$). Meyer (1990) finds an elasticity of unemployment with respect to the benefit level of up to about minus unity for the United States. His source of variation is mainly structural variation in legislation between states. Hence, we feel comfortable to interpret his estimate as reflecting the elasticity of equilibrium unemployment. This estimate is consistent with our model when the replacement rate is 0.60, in which case a 1 % decline in B would yield a 1.5 % increase in one minus the replacement rate and hence a 0.9 % decline in unemployment. Note that the model implies that the detrimental effect on unemployment goes up with every percent further increase in B. This might explain the European problems that have arisen for a large part after the construction of an extensive social security system.

The more interesting issue is what the efficient level of unemployment benefits is. A social planner has little instruments to implement the allocation that takes away all losses embodied in (28). Unemployment benefits might be the only instrument available that can alleviate the consequences of reservation wages being below their optimal value. For an analysis of this issue, we have to drop the interpretation of B as being the value of leisure. Hence, B has to be funded. Suppose we pay unemployment benefits from an insurance premium that is proportional to earned wages. When we define $B^*(s)$ relative to the net reservation wage, this will have no further impact on the model (since the difference between the gross and the net value of $r(s)$ is a constant). However, our loss function, equation (27), has to take into account that the cost of unemployment relative to the reservation wage is no longer equal to $B^*(s)$ but to unity. In order to account for this difference, the first term between square brackets has to be divided by $B^*(s)$. Substituting equation (25) for $x^*(s)$ and minimizing the resulting expression with respect to $B^*(s)$ yields:

$$B^*(s) = \frac{3\delta\beta}{\rho + \delta + 2\delta(1-\beta)} \tag{29}$$

The optimal net replacement rate, $B/R(s) = 1 - B^*(s)$ is therefore a negative function of the bargaining power of workers. This fits the intuition: when workers have a high bargaining power, it does not make sense to further strengthen their position by providing them a luxurious outside option. Using the parameter values that have been applied before, the optimal replacement rate is equal to 58 %. [15]

5.5 The effect on (reservation)wages

The effect of search frictions on reservation wages can reasonably well be approximated by the effect on $x^*(s)$. Since $p(c)$ is known (when $\eta = 0$) or can be approximated by its value for the Walrasian equilibrium and since $c(s)$ can be solved from the equation $-p'[c(s)] = s$ and is therefore independent of the level of search frictions, all the variation in $x^*(s)$ in response to search frictions is due to variation in reservation wages:

$$\frac{d\,r(s)}{d\,\lambda} = -\frac{d\,x^*(s)}{d\,\lambda} \tag{30}$$

where we use λ as a measure for labor market frictions. Note that $x^*(s)$ is not the only channel along which $r(s)$ (through $c'(s) = 1/r''(s)$) enters in equation (24); $r(s)$ also enters via $B^*(s)$ and $K^*(s)$. However, in the limiting case close to the Walrasian equilibrium, $x^*(s) \ll B^*(s), K^*(s)$. Hence, the relative effect of a 1% decline in real wages on $x^*(s)$ is much larger than the effect on $B^*(s)$ or $K^*(s)$. We shall therefore ignore the latter two effects and identify the effect of search frictions on $r(s)$ with minus the effect on $x^*(s)$.

Apart from the effect on reservation wages, we can also analyze the effect on accepted wages. In the Walrasian case, both coincide. With frictions, actual log wages, denoted by $w(s)$, exceed $r(s)$ in almost all employment relations. This difference can be interpreted as the rents from search. The average rents can be calculated, again by a second order Taylor expansion analogous to the way we calculated the efficiency loss. This surplus, which is the complement of the efficiency loss, equals ${}^2/_3\beta\,x^*(s)$. Using the same technique of Taylor expansions, we can calculate the standard deviation of rents to be:

$$\text{Std.dev.}\,[w(s) - r(s)] = \frac{2}{3\sqrt{5}}\beta\,x^*(s)$$

For $\beta = 0.40$, this expression equals $0.12\,x^*(s)$. For reasonable values of $x^*(s)$ this is a small number, in particular when compared with the standard deviation of industry differentials, which has often been earmarked as rents. Krueger and Summers (1988) report a value of the standard deviation of industry differentials of about 0.15. When these differentials are indeed

[15] Note however that in our model, B is the same for all worker types, hence a proper interpretation of (29) is that it is the optimal replacement rate for the average worker. If the social planner can observe s, he would choose $B(s)$ according to (29).

rents, these results suggests that they cannot be attributed fully to search frictions. However, a complete analysis of the implications of search frictions for industry differentials requires a somewhat more complicated model than the one discussed here. We get back to this issue in Section 8 of this paper.

5.6 The complexity dispersion parameter

Whereas equation (24) is convenient from an analytical point of view, it is less suitable for an empirical evaluation, since we have no idea about the units of measurement of s. As discussed in section 2.4, the scale of measurement of s can even be changed by a linear transformation without changing the empirical implications of the model. It is therefore more useful to write the model in terms of an effect in log wages where we have no dispute about its unit of measurement. In addition, all issues regarding the substitutability of workers in given jobs go into the complexity dispersion parameter. Applying this transformation yields:

$$\widehat{x}(r)^5 = \frac{81}{128} Q^2 \widehat{B}^*(r)^2 \widehat{K}^*(r)^2 f(r)^{-2} \widehat{\gamma}(r) \tag{31}$$

where $\widehat{z}[r(s)] \equiv z(s), z = x, B^*, K^*$, where, $f(r)$ is the density function of the distribution of log reservation wages r (weighted by persons) and $\widehat{\gamma}(r)$ is the complexity dispersion parameter discussed in Section 2.4. In relation to the Walrasian version of the model: $\widehat{\gamma}[r(s)] \equiv \frac{r''(s)}{r'(s)^2} = \frac{c'(s)}{c(s)^2}$. This complexity dispersion parameter is free of arbitrary choices of units of measurement and can therefore be meaningfully compared across countries and across different points in time. The second equality makes clear that the complexity dispersion parameter is fully determined by the locus of $c(s)$. Hence, it is either exogenous, when $\eta = \infty$, or determined by the Walrasian equilibrium when, $\eta = 0$, at least in our approximations. Teulings (1999) shows that in the Walrasian case, this parameter plays a crucial role in the structure of substitution and complementarity between various worker types. It can be interpreted as a compression elasticity, measuring the percentage decrease in the return to human capital per percent increase in its stock. This parameter is therefore crucial in determining both the degree of substitutability between types of labor and the compression of wage differentials in course of the accumulation of human capital as well as in the magnitude of search frictions in the economy. We shall apply this concept when analyzing the distributional implications of search frictions.

5.7 Distributional implications

When analyzing the distributional consequence of search frictions, we apply our Taylor expansions beyond the area where their validity can be justified easily. The reason for this problem is easy to understand. Due to equation (30), there can only be distributional effects when $\frac{dx^*(s)}{ds} \neq 0$. This contradicts the most crucial result that underlies our approximations, namely equation (19). Nevertheless, equation (31) offers a useful device for analyzing the forces that are at work.

By equation (30), we have to analyze the derivative of $\widehat{x}(r)$. Four factors on the right hand side of (31) depend on r. We shall consider their contribution to the variation in $\widehat{x}(r)$ separately. The factors $\widehat{B}^*(r)^{0.4}\,\widehat{K}^*(r)^{0.4}$will be considered jointly. They reach a minimum for $R = 2B$, which is equivalent to a replacement rate of 50 %. In most cases, this maximum will therefore be interior to the domain of R. Next, we consider the third factor, the distribution of reservation wages. As long as the rents $\widehat{x}(r)$ are reasonably small, the distribution of r can be approximated by the distribution of observed wages, which is well described by a normal distribution with a standard deviation σ in the order of magnitude of $0.30 - 0.60$ for OECD countries. Without loss of generality, the expectation of r will be normalized to zero.[16] Hence, the variation in R has a larger impact on the third factor than on the first two factors considered jointly. The impact of the density is directly related to the increasing returns characteristic of the model: the smaller the number of workers in a particular market segment, the higher will be the cost of search frictions since contact rates are a linear function of the number of workers in a segment. Hence, search frictions are much more important in the tails of the skill distribution than around the median. Regarding the fourth factor, the complexity dispersion parameter $\widehat{\gamma}(r)$, we have little clue regarding its slope. Teulings and Vieirra (1998) report it to be upward sloping in Portugal. However, we are reluctant to take this as an established fact. Since we are not sure about the sign of the slope, this agnoticism applies a fortiori to the magnitude of the slope.

We conclude that the most important effect of search frictions on the wage distribution is through the slope of the initial density function $f(r)$. Concentrating on this effect, we have:

[16] E.g. consider the case of $B = 0.5$. Then the first two factors reach their maximum for $R = 1$, which is the same value for which the third factor reaches its minimum (due to the normalization of $\mathrm{E}[r] = 0$). Now, consider the value of both factors at $R = 2$ relative to their value at $R = 1$. The first two are at 0.89 of their maximum, the third factor is at $e^{\frac{1}{2}\frac{1}{2}\left(\frac{\ln 2}{\sigma}\right)^2} = 1.95$. This conclusion holds a fortiori for smaller values of σ or larger values of r.

$$r < 0 \Rightarrow \widehat{x}'(r) < 0$$
$$r > 0 \Rightarrow \widehat{x}'(r) > 0$$

Both the lowest and the highest wages will therefore be reduced most by search frictions, while the median is the least affected due to the large number of workers around the median. Hence, search frictions will not affect wage dispersion that much, but they will lead to a wage distribution that is skewed to the left compared to the wage distribution that would apply in a friction-free Walrasian world. Definite effects of search frictions on wage dispersion can only be expected if the complexity dispersion parameter is sloped. An upward slope will yield compression of the wage distribution due to search frictions, a downward slope will generate the opposite effect.

6 The corner problem

Burdett and Coles (1997) and Smith (1995) show in the context of a non-transferable utility (but otherwise similar) model that there will be complete segmentation. The intuition for this result is that both sides of the market are only willing to match with all types which are higher than their reservation types. Consequently, the highest types are in the position of being most selective and will therefore only match with each other. When the highest types are no longer available for the middle types, the middle types can do no better than match with each other and finally, the lowest types are left over and will also match with each other.

At first sight, this segmentation result is a peculiarity of the non-transferable utility assumption, which will not transfer to models with Nash bargaining. This view is mistaken. Though less extreme than the model of Burdett and Coles, the model in this paper contains similar problems. The problem can best be explained by a graph. Figure 6 depicts the type space $\underline{L}(s), J(c)$ ($\underline{L}(s)$ is the distribution function associated with $\underline{l}$, J is the distribution of occupied jobs in the Walrasian equilibrium), that is we transformed s and c such that every point in the graph has equal density in the Walrasian equilibrium. For simplicity, we assume that the Walrasian outcome is represented by the diagonal. The matching sets $m_c(s)$ can be plotted in the figure by their upper and lower bound, $c^+(s)$ and $c^-(s)$ respectively, which results in a band which goes from the south-west corner to the north-east corner, see panel A. As a thought experiment, consider how this band would look like when every worker has an equal probability of being matched. Since the upper and the lower bound are drawn to be parallel, the matching probabilities are constant for all s, except for the corners. Now look at the problem from the point of view of vacancies. In panel B, the most and the least complex vacancy will have a zero matching probability. Hence such a vacancy will never be opened in equilibrium. Only the vacancies in the interval (c^{--}, c^{++}) have a constant matching probability. Outside this interval, in the south-west and north-east corners of the type space, the matching probabilities decline quickly. Panel C depicts the case where each vacancy has an equal matching probability, but it is easy to see now that worker types at both ends of the support face the same problem as vacancies did before. Panel D sketches the only outcome that would give every job type and every worker type an equal matching probability, that is, Burdett and Coles complete segmentation result. By Theorem 1, this outcome can never be an equilibrium of the model in this paper, since for this, $c^{-}(s)$ and $c^{+}(s)$ must be differentiable. Nevertheless, panel D is illustrative for the type of force that is at work in this economy. We refer to this phenomenon as the corner problem.

We offer a somewhat intuitive argument on how this economy deals with the corner problem. The argument is meant mainly to facilitate the understanding of the simulation results in the next section. The way in which the economy deals with the problem depends on whether $\eta = \infty$ or $\eta = 0$. An example of the former case is sketched in Panel E. In that case, there is no necessity to open vacancies of all types. Let $c^{-,\lambda}$ be the least complex vacancy that will be opened with search frictions λ. Since the expected value of s in the matching set of this vacancy will be larger than of the least skilled worker, $\mathrm{E}\Big[s|s \in m_s(c^{-,\lambda})\Big] > s^{-}$, and since this vacancy is best adapted to its matching set if it, loosely speaking, maximizes the surplus of the expected value of its matching set. Hence $\mathrm{E}\Big[s|s \in m_s(c^{-,\lambda})\Big] > \mathrm{E}[s|s \in m_s(c^{-})]$

A similar argument applies to the north-east corner of the type-space. Search frictions will therefore truncate both the upper and the lower support of the domain of the vacancies that are opened. The density of jobs is depicted in the right quadrant of Panel E. There is a bulk at $c^{-,\lambda}$ and $c^{+,\lambda}$, to compensate for the deleted vacancies at the extremes of the support. This clustering in vacancies will lead to some clustering in the matching sets, which looks like a smoothed version of the market segmentation result of Burdett and Coles (1997). When search frictions are large, we have simulation results which show a further clustering, with a concentration of vacancies for some job types and no vacancies for neighboring job types. When $\eta = 0$, vacancies of all types are opened, since each job type is by the nature of the CES utility function a necessity in the composite consumption good. Hence, the corner problem for vacancies at both ends of the domain needs to be resolved by price adjustment. Commodity prices at both ends will be higher with search frictions than they are in a friction-free Walrasian world.

The corner problem and the implied non-linearities in $c^-(s)$ and $c^+(s)$, depicted in Panel E, are obviously not picked up by the Taylor expansions presented in the previous section. These expansions ignore completely the boundary conditions imposed by the upper and lower support of s and c and which are the cause of the corner problem. However, although the corner problem leaves its marks in the corners of the simulations, the Taylor expansions turn out to do a good job in predicting $x^*(s)$ and $h\,(s)$.

7 Simulations

7.1 Filling up the holes in the specification

Where the Taylor expansions presented in section 4 provide an excellent analytical instrument for analyzing the characteristics of market equilibrium, our trust in the conclusions of that analysis depends on the precision with which this approximation mimics the true equilibrium. This section therefore presents a number of numerical simulations of the equilibrium of the economy. A simulation of the model requires that two holes in the specification of the model till sofar, the specification of $\underline{l}(s)$ and $\underline{q}(c)$, are filled. We propose a convenient specification that gives a simple relation between the parameters of these functions on the one hand and the wage distribution and the complexity dispersion parameter in the Walrasian benchmark on the other hand.

$$\underline{q}(c) = -\frac{(1-\alpha)}{\alpha}\frac{1}{\gamma_0}\left[(\gamma_0 c)^{\frac{-\alpha}{1+\alpha}} - 1\right] - c \text{ for } \alpha \neq 0 \qquad (32)$$

$$= \frac{1}{\gamma_0}\ln(\gamma_0 c) - c \qquad \text{for } \alpha = 0 \qquad (33)$$

where $\alpha < 1$. Hence, for the Walrasian benchmark, $p'[c(s)] = [\gamma_0\, c(s)]^{-\frac{1}{1-\alpha}} - 1 = -s$, or $c(s) = \frac{1}{\gamma_0}(1-s)^{-(1-\alpha)}$. The complexity dispersion parameter, $\gamma(s) = \frac{c'(s)}{c(s)^2}$ satisfies therefore:

$$\gamma(s) = (1-\alpha)\gamma\,(1-s)^{-\alpha} \qquad (34)$$

where $\gamma(s) \equiv \hat{\gamma}\,[r(s)]$. Hence, $\alpha = 0$ corresponds to a constant complexity dispersion parameter, $\alpha > 0$ to an increasing and $\alpha < 0$ to a decreasing parameter. Integrating $r'(s) = c(s)$ and applying the zero profit constraint $p(c) + sc - r(s) = 0$ yields the locus of log reservation wages, which we will denote by $r_{Wal}(s)$ in the frictionless world:

$$
\begin{aligned}
r_{Wal}(s) &= -\frac{1}{\gamma_0 \alpha}\left[(1-s)^{\alpha}-1\right] \text{ for } \alpha \neq 0 \qquad (35)\\
&= -\frac{1}{\gamma_0}\ln(1-s) \qquad \text{for } \alpha = 0
\end{aligned}
$$

It is convenient for the presentation of the simulation results to transform both s and c: $s^* \equiv -\ln(1-s)$ and $c^* \equiv \ln c$. Applying this transformation yields a model which is almost linear:

$$
\begin{aligned}
\gamma^*(s^*) &= (1-\alpha)\gamma_0 e^{\alpha s^*} \qquad (36)\\
c^*(s^*) &= (1-\alpha)s^* - \ln \gamma_0 \qquad (37)\\
r^*_{Wal}(s^*) &= -\frac{1}{\gamma_0 \alpha}\left[e^{-\alpha s^*}-1\right] \text{ for } \alpha \neq 0\\
&= \frac{1}{\gamma_0}s^* \qquad \text{for } \alpha = 0
\end{aligned}
$$

where $c^*(s^*) \equiv c^*$ and $r^*(s^*) \equiv r$. Hence, the relation between s^* and c^* is linear and has a slope equal to unity in the case of a constant complexity dispersion parameter, $\alpha = 0$. The relation between r and s^* is only linear when the complexity dispersion parameter is constant. Its slope equals the inverse of the complexity dispersion parameter.

It is convenient to specify the distribution of s^* instead of s: $s^* \tilde{} N\left[0, (\gamma_0\sigma)^2\right]$. From equation (36), combined with the formula for the standard deviation of the log normal distribution (since $\ln(1-\gamma_0\alpha r) = \alpha s^*$ is distributed normally), we have:

$$
\begin{aligned}
\text{Std.dev.}\,[r] &= \frac{1}{\gamma_0\alpha}\sqrt{\exp\left[2\left(\gamma_0\alpha\sigma\right)^2\right]-\exp\left[\left(\gamma_0\alpha\sigma\right)^2\right]} \text{ for } \alpha \neq 0 \qquad (38)\\
&= \sigma \qquad \text{for } \alpha = 0
\end{aligned}
$$

Applying a second order Taylor expansion to the expression under the square root yields a more intelligible expression for the standard deviation:

$$
\text{Std.dev.}\,[r] \cong \sigma\sqrt{1+\frac{3}{2}(\gamma_0\alpha\sigma)^2} \qquad (39)
$$

Hence, the model has ten parameters: four parameters in Q $(\rho, \delta, \beta, \lambda)$, the elasticity of substitution η, the wage dispersion parameter σ, the parameters governing the complexity dispersion parameter (γ_0, α), and the parameters

B and K. This framework will be applied in the simulations. We specify a grid for s^* and c^* ranging from minus three till plus three times their standard deviation. We divide the domain of both variables in 100 intervals per standard deviation, yielding a matrix of 601×601. For $\eta = \infty$, the calculation requires two iteration loops. First, we take the transformed distribution of vacancies, $g^*(c^*)$, as given and solve for $r^*(s^*)$. In the second loop, we adjust $g^*(c^*)$ in order to set expected returns on a vacancy equal to K/ρ. We take as a starting value for $g^{*,\text{start}}(c^*) = 0.10\, f^* \left[\frac{c^* + \ln \gamma_0}{1-\alpha}\right]$. This iteration procedure converges quickly to an equilibrium.

When $\eta = 0$, we have a third loop. We start by taking $p^*(c^*)$ as given and proceed exactly in the same way as for $\eta = \infty$. After having solved for $r^*(s^*)$ and $g^*(c^*)$, we calculate the implied $Y^*(c^*)$ and change prices to adjust for excess supply or demand. This iteration procedure converges extremely slowly. A first hurdle is the corner problem discussed in the previous section. The iteration procedure finds it difficult to adjust prices such that there is adequate supply in the tails of the distribution. We deal with this problem by first making a separate adjustment for the tails of $p^*(c^*)$, and then solving for the whole domain.

When $\eta = 0$ multiple equilibria becomes an issue. In particular in the corners, changing prices starting from a "second loop" equilibrium tends to change reservation wages and not the distribution of output. Hence, each iteration in the third loop starts afresh from $g^{*,\text{start}}(c^*)$.

7.2 Simulation results

The simulations serve two goals. First, they allow us to confront our Taylor approximations of the equilibrium with the "true" equilibrium. Second, we are interested in the effects of changing λ on equilibrium unemployment and output for its own sake. The baseline parameter values of our simulations are: $\delta = 0.15$, $\beta = 0.40$, $\rho = 0.10$, $\theta = 4.0$, $\gamma_0 = 5.0$, $\alpha = 0.0$, $B = 0.09$, $K = 0.50$, $\sigma = 0.60$. [17] Table 1 shows aggregate outcomes for wage dispersion, output loss, and aggregate unemployment and vacancies for different values of λ. In thick markets (when λ is high), it is easy for firms and workers to find each other, resulting in lower equilibrium unemployment and vacancy rates. According to (6), this increases the reservation wage of the workers which allows them to become choosier concerning the jobs they are willing to accept. This and the higher employment rate leads to a lower output loss. For

[17] Those values are in line with what is usually found for the US. The constant complexity dispersion parameter, γ_0, was estimated to be close to 5, see Teulings (1999b) for an estimate for the US, Teulings (1995) for the Netherlands, and Teulings and Vieira (1999) for Portugal.

$\lambda = 156$, unemployment is 5.3%. This suggests that small search frictions can still result in significant unemployment. Table 2 gives the simulation results for different segments of the labor market, where $\eta = \infty$ and s^* ranges from -3 till +3 standard deviations of the skill distribution. The unemployment rate per skill category is denoted by $u(s)$. For $\lambda = 10000$, our approximations are surprisingly accurate. The maximum deviation for unemployment is 2.1% while for $x(s)$ it is 4.3% in the upper corner. For $\lambda = 2500$, our approximations still do a good job, especially for the middle groups. As the search frictions get higher, our approximations become less accurate, in particular in the corners, see the discussion in Section 6. From this we must conclude that our claims are most valid for large λ's. Figures 8 and 9 plot the matching sets for $\lambda = 10000$ and $\lambda = 39$, respectively. We clearly see that under large search frictions, the matching sets become wider. Figures 10 and 11 show the impact of the corner problem. The process of vacancy clustering which we discussed in Section 6 is clearly manifest in Figure 11. Since all goods are perfect substitutes, and firms do not want to wait forever to find workers at the most simple jobs, they avoid the most simple jobs in the corners and instead offer a cluster of jobs just above the corner. This will crowd out vacancies in the upper neighborhood of this cluster and a process of endogenous segregation results. For $\lambda = 10,000$, this only happens in the corners while for $\lambda = 39$ it happens in the middle segments as well.[18] The main reason for the fact that our approximations are less accurate for low values of λ is that our Taylor approximations do not pick up this clustering of vacancies. However, in the more realistic case with imperfect substitution, there will be less clustering because the necessity to open jobs of all types becomes stronger. In simulations with $\eta = 0$ we see that the spikes do not arise in vacancy supply but instead in output prices.

Next we can test the validity of some of the relations we derived in the previous sections. First consider (27), which captures the total loss due to search frictions. Under the assumed parameter values, this loss equals 0.73 $x(s)$, which is in line with Table 1 and Table 2. Next, consider the relation between λ and unemployment. Under small search frictions, we can consider $B^* \simeq 0.81$, then according to (25) the effect through λ on unemployment only goes through Q and has an elasticity of -0.4. Hence, every time λ is increased by 4, unemployment decreases with a factor $4^{0.4} = 1.74$. Our output is surprisingly consistent with this pattern. The same holds for vacancies. According to (25) and (26), the vu ratio in equilibrium equals: $(1 - \beta)B^*/\beta K^*$which is about 1.21. Finally, we can test whether the effect of λ on $r(s)$ is minus the effect of λ on $x^*(s)$, as implied by (30), holds. This

[18] Only under an error bound of 0.00001, this pattern arose in our simulations.

seems to be the case. Increasing λ from 2500 to 10000, decreases $x(s)$ by the same amount as it increases $r(s)$.

8 Some possible extensions

Now that we have shown that the Taylor approximations do a pretty good job in characterizing search equilibrium with heterogeneous agents, many extensions come to mind. A first extension which is both interesting and manageable with our Taylor expansion approach is the introduction of risk aversion on the side of job seekers. Suppose that we replace the dollar-for-dollar utility function applied till now by its constant-relative-risk-aversion generalization: $U = \frac{1}{1-\pi}X^{1-\pi}, \pi > 0$, where X is the monetary pay off and where π is the degree of relative risk aversion. The first order condition for the maximization of the Nash product for the wage bargaining game reads in that case:

$$\beta(1-\pi)\left[P(c)F(s,c) - W(s)\right] = (1-\beta)\left[W(s) - R(s)\left(\frac{W(s,c)}{R(s)}\right)^{\pi}\right]$$

Applying the first order Taylor expansion $\left(\frac{W(s,c)}{R(s)}\right)^{\pi} \simeq 1 + \pi\frac{W(s,c)-R(s)}{R(s)}$ makes clear that equation (5) applies approximately in the case of risk aversion. By this approximation, the equivalent of equation (6) for $R(s)$ reads:

$$R(s)^{1-\pi} = B^{1-\pi} + \frac{\beta\lambda}{\rho+\delta}\int_{m_c(s)} g(c)\left[P(s)F(s,c) - R(s)\right]^{1-\pi} dc$$

Then, the equations (24) and (25) become:

$$\begin{aligned} x^{*}(s)^{5} &= \frac{81}{128}Q^{2}B^{*}(s,\pi)^{2}K^{*}(s)^{2}\underline{l}(s)^{-2}c'(s) \\ \frac{h(s)}{\underline{l}(s)} &= \frac{2}{3}\frac{\delta\beta}{\rho+\delta}B^{*}(s,\pi)^{-1}x^{*}(s) \end{aligned}$$

where $B^{*}(s,\pi) \equiv \frac{1}{1-\pi}\left[1-\left(\frac{B}{R(s)}\right)^{1-\pi}\right] > B^{*}(s,0) = B^{*}(s)$. Risk aversion increases the surplus $x^{*}(s)$, all other parameters equal and reduces unemployment. The economic intuition behind this result is that risk aversion makes workers less patient when they are unemployed. They will sooner accept jobs for which they are not well suited than in the risk neutral case. This raises the need for an unemployment system that prevents workers from accepting jobs too easy even more than in the risk neutral case. It might also offer a solution for the small size of the rent share in wages relative to the size that has been suggested by empirical studies, like Abowd and Lemieux (1993) and Krueger and Summers (1988), and Teulings and Hartog (1998, chapter 4). Furthermore, risk aversion reduces the elasticity of unemployment and the surplus with respect to $B^*(s)$, which might explain why the measured elasticities seem to be smaller than the ones implied by the risk neutral version of the model.

A second important extension, which is not trivial, is to allow for on the job search. Since only by accident, workers will match with their favorite job, there are incentives to continue search till one finds the $c(s)$ job. In the extreme case when on the job search is costless, workers will accept any job that pays a wage which is higher than B and continue searching for better jobs, using their current wage as threat point in the bargaining. On the other hand, when on the job search is very costly, or when tenure profiles are steep, our model is a good approximation. A related extension is to endogenize the separation rate δ. One expects that the scale of the labor market is an important factor in the separation decision since the chances of finding a job are typically higher in densely populated areas. Consider for example a simple model with automatic on-the-job accumulation of skill, that is, the standard Mincer model: the skill level s goes up as the worker accumulates experience. Suppose that the worker accumulates experience, such that her skill level goes up with her tenure. Due to the comparative advantage structure, the complexity of the optimal job type will also move up. Sooner or later, the present job will no longer satisfy the reservation match constraint and the worker will separate to look for a better match. Since an increase in the scale of the labor market, λ, reduces the expected surplus in the present job, it will be optimal to separate more early in a large scale labor market. Empirical research supports the notion that separation rates are higher in central cities, see Van der Ende and Teulings (1999).

A further extension is to let capital costs or, equivalently, capital intensity vary over job types. Wages will be higher in jobs with high capital cost, because the firm gets a fixed share of the value of gross output. Firms can only recoup their high costs of investments from this fixed share if the total value of gross output is larger. This introduces a new mechanism. The most important difference is that in the comparative advantage model there is no job which is universally best. A low-skilled worker produces the highest gross value output at a simple job, while the Harvard graduate uses her comparative advantages most effectively in a complex job. In the variable capital intensity model, all workers are best off if they manage to obtain a job in the capital intensive industry. Hence, such a model fits the evidence by Krueger and Summers (1988) that some industries pay systematically higher wages than others. A similar model like the one analyzed in this paper, but extended with heterogeneous capital intensity and risk aversion (to square a low unemployment rate with high values for $x^*(s)$) might be able to provide a consistent interpretation of these phenomena. Since everybody wants to work in the capital intensive sectors, the firms in those sectors are in the position to select from a large pool of workers and they will therefore mainly hire the high $s-$types. This brings us back in the Burdett Coles (1997) world with endogenous segmentation.

Next, we can go into the implications of this model for the spacial structure of the economy. We have seen that, other things equal, search frictions have the strongest downward effect on wages in both tails of the skill distribution. Hence, the wage distribution is skewed to left relative to its Walrasian counterpart. Since search frictions are more severe in the rural areas than in the densely populated metropolitan labor markets of New York, Boston or San Francisco, wages will be most depressed in the tails of the distribution in rural areas, again, other things equal. This will stimulate mobility. Both low and high skilled workers will leave the countryside and move towards the big cities where earnings in the tails of the skill distribution are less depressed. This implication deserves testing.

Finally, it would be interesting to consider alternative ways of wage determination, like monopsony wage setting, as in Burdett and Mortensen (1998).

9 Final Remarks

Our method of applying second order Taylor expansions to evaluate the integrals over the matching sets proved to be a useful tool This approximation is quite accurate for the middle groups. Hence our analytical results are most valid outside the tales of the skill and job complexity distribution and in labor markets with small search frictions.

Previous work in this field remained highly theoretical because equilibria usually had to be calculated by numerical simulation only, which limited the insight in the forces that governed the outcome of the market process. Our method allowed us to get explicit expressions for unemployment, vacancies, the match surplus and the losses of search frictions and inadequate incentives and their relations to the scale of the labor market. Moreover, it enabled us to derive the optimal replacement rate.

Finally, we have shown that heterogeneity matters. Besides giving a rationale for search frictions, it helps us understand how significant unemployment rates and mismatch can exist even when workers and vacancies meet each other frequently.

10 Literature

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A Appendices

A.1 The proof of Theorem 1

Part 1: The continuity of $R(s)$:

By equation (6) we have:

$$\lim_{h\to 0} \Delta R(s) = \lim_{h\to 0} \frac{\lambda\beta}{\rho+\delta} \left\{ \begin{array}{c} \int_{m_c(s)} -g(c)\Delta R(s)\, dc + \\ \int_{m_c(s+h)-m_c(s)} g(c)\left[P(c)F(s+h,c) - R(s+h)\right] dc \end{array} \right\} \tag{40}$$

where $\Delta R(s) \equiv R(s+h) - R(s)$. Suppose that $\Delta R(s) \geq 0$. Then the first term on the right hand side is negative. The second term is zero, because a necessary condition for c to be in the set $m_c(s+h) - m_c(s)$ is $x(s+h,c) - x(s,c) > 0$. Since $\lim_{h\to 0} x(s+h,c) - x(s,c) = -\Delta R(s)$, this condition is never satisfied. Hence, the only way for equation (40) to apply is that $\lim_{h\to 0} \Delta R(s) = 0$. Similar arguments prove the other statements in part 1 of theorem. Q.E.D.

Part 2: The sign of the derivatives of $R(s)$ and $P(c)$:

Differentiating equation (6) yields:

$$r'(s)R(s) = \frac{\lambda\beta}{\rho+\delta} \int_{m_c(s)} g(c)\left[cP(c)F(s,c) - r'(s)R(s)\right] dc + \text{ effect via } m_c(s) \tag{41}$$

The second term on the right hand side is zero by the envelope theorem (by the definition of $m_c(s)$, $P(c)R(s,c) - R(s) = 0$ at the boundaries of the integration interval). Bringing the term with $r'(s)$ in the integrand to the right hand side of the equation shows $r'(s) > 0$. Q.E.D.

Dividing equation (41) by $R(s)$ and differentiating the result yields:

$$r''(s) = \frac{\lambda\beta}{\rho+\delta} \int_{m_c(s)} g(c)\left[c\left\{c - r'(s)\right\}\frac{P(c)F(s,c)}{R(s)} - r''(s)\right] dc + \text{ effect via } m_c(s) \tag{42}$$

The effect via the boundaries of integration is necessarily positive in this case: ignoring the effect via the boundaries yield the second derivative of the value of job search when workers would not adjust their search strategy to the increase in s. Adjusting the search strategy to the optimal strategy can only increase the value of search. Rewriting (41) as:

$$r'(s) = \frac{\lambda\beta}{\rho+\delta} \int_{m_c(s)} g(c)\left[\left\{c - r'(s)\right\}\frac{P(c)F(s,c)}{R(s)} + r'(s)\left\{\frac{P(c)F(s,c)}{R(s)} - 1\right\}\right] dc =$$

$$= \frac{\lambda\beta}{\rho+\delta}\int_{m_c(s)} g(c)\{c - r'(s)\}\frac{P(c)F(s,c)}{R(s)}dc + r'(s)\left[1 - \frac{B}{R(s)}\right] \quad (43)$$

where the second equality follows from substituting the final term by the original equation (6). Write this integral as: $Q \int_{m_c(s)} \frac{q(c)}{Q}\{c - r'(s)\}\, dc$,or Q $\mathrm{E}_q[c - r'(s)]$where: $Q = \int_{m_c(s)} q(c)\, dc$ and where E_q is the expectation operator with respect to the density function $\frac{q(c)}{Q}$. Likewise, the first term of the integral in (42) can be written as: $Q \int_{m_c(s)} \frac{q(c)}{Q} c\{c - r'(s)\}\, dc = V_q(c) + E_q(c)^2 - E_q(c)r'(s) = V_q(c) + E_q(c)\left[E_q(c) - r'(s)\right]$, where $V_q(c)$ is the variance with respect to the density function $\frac{q(c)}{Q}$. Finally note that: $Q\left\{\mathrm{V}_q[c] + \mathrm{E}_q[c - r'(s)]\right\} > Q\ [E_q c - r'(s)] > 0$. Q.E.D.

The proof of $p"(c) < 0$ follows immediately by differentiating twice equation (6) for K with respect to c. Q.E.D.

Part 3: $x_{ss}(s,c) = -r"(s) < 0$**,** $x_{cc}(s,c) = p"(c) < 0$**. Q.E.D.**

Part 4: the connectedness of $m_c(s)$ **and** $m_s(c)$**:**

$m_c(s)$ is defined by $x(s,c) > 0$. By part 1 and part 3, $x(s,c)$ has a unique maximum (either interior or exterior). If $m_c(s)$ is non-empty, this maximum has to be positive. Also by part 1 and 3, the equation $x(s,c) = 0$ for fixed s has at most two roots, one above the location of the maximum (defined as $c^+(s)$) and one below it ($c^-(s)$). All c's in between are element of $m_c(s)$, all others c's are not. A similar argument applies to $m_s(c)$. Q.E.D.

Part 5: $c^{-'}(s) > 0$ **and** $c^{+'}(s) > 0$**:**

Taking the total differential of functional equation $x[s, c^-(s)] = 0$ yields $c^{-'}(s) = -\frac{x_s[s,c^-(s)]}{x_c[s,c^-(s)]}$. Since $c^-(s) < c(s)$ and by part 3, $x_c[s, c^-(s)] > 0$. By part 1 and 2 and the definition $s(c)$, $s'(c) > 0$. Hence: $s < s[c^-(s)]$, or: $s^+[c^-(s)] = s$ (the point $c^-(s)$ is the lower bound of $m_c(s)$ but the upper bound of $m_s(c)$). Hence, by part 3, $x_s[s, c^-(s)] < 0$. Q.E.D.

A.2 The Taylor expansion of the integrals

Rewrite (6) as:

$$1 - \frac{B}{R(s)} = \frac{\beta\lambda}{\rho+\delta}\int_{m_c(s)} g(c)\left[e^{x(s,c)} - 1\right]\, dc$$

and note that $x_c[s, c(s)] = 0$ and that $x_{cc}[s, c(s)] = p''[c(s)] = -1/c'(s)$. Define $\Delta = c - c(s)$, $\Delta^- = c^-(s) - c(s)$, and $\Delta^+ = c^+(s) - c(s)$. Then:

$$\int_{m_c(s)} g(c)\left[e^{x(s,c)}-1\right] dc \tag{44}$$
$$= g\int_{\Delta^-}^{\Delta^+}\left[1+\frac{g'}{g}\Delta+o\left(\Delta\right)\right] \quad \left[E\left(1-\frac{1}{2c'}\Delta^2+o\left(\Delta^2\right)\right)-1\right]d\Delta$$

where $E\equiv e^x$ with $x\equiv x\left[s,c(s)\right]$ and where g and $g'$are evaluated at $c(s)$ and where c' denotes $c'(s)$. The arguments of these functions are suppressed for convenience. The integrand can be written as: $(E-1)\left[1+\frac{g'}{g}\Delta+o\left(\Delta\right)\right]-\frac{1}{2}c'E\left[\Delta^2+\frac{g'}{g}\Delta^3+o\left(\Delta^3\right)\right]$. Furthermore $E-1=x+o\left(x\right)$ and $x[s,c(s)]=\frac{1}{2c'}\Delta^{\#^2}+o\left(\Delta^{\#^2}\right)$, where $\Delta^{\#}\equiv c^{\#}(s)-c\left(s\right),(\#)=(-),(+)$. Hence:

$$E-1=\frac{1}{2c'}\Delta^{\#^2}+o\left(\Delta^{\#^2}\right)$$

Furthermore $-\frac{1}{2c'}\Delta^{+^2}+o(\Delta^{\#^2})=\frac{1}{2c'}\Delta^{-^2}+o(\Delta^{\#^2})$ and therefore:

$$-\frac{1}{2c'}(\Delta^++\Delta^-)(\Delta^+-\Delta^-)=o(\Delta^{\#^2})\Rightarrow\Delta^+-\Delta^-=o(\Delta^{\#})$$

Substituting these results in equation (44) yields:

$$g\int_{\Delta^-}^{\Delta^+}\left[1+\frac{g'}{g}\Delta+o\left(\Delta\right)\right]\left[E\left(1-\frac{1}{2c'}\Delta^2+o\left(\Delta^2\right)\right)-1\right]d\Delta \tag{45}$$
$$= g\left[\left(E-1\right)\left(\Delta+o(\Delta)\right)-\frac{1}{2c'}E\left(\frac{1}{3}\Delta^3+o\left(\Delta^3\right)\right)\right]_{\Delta=\Delta^-}^{\Delta^+}$$
$$= \frac{1}{2c'}g\left[\frac{2}{3}\Delta^3\right]_{\Delta=-\Delta^-}^{\Delta^+}+o\left(\Delta^{\#^3}\right)=\frac{2}{3c'}g\Delta^{\#^3}+o\left(\Delta^{\#^3}\right) \tag{46}$$

Applying $\Delta^{\#}=\sqrt{2xc'+o\left(\Delta^{\#}\right)}$ yields equation (20) in the text. Q.E.D.

B Figures

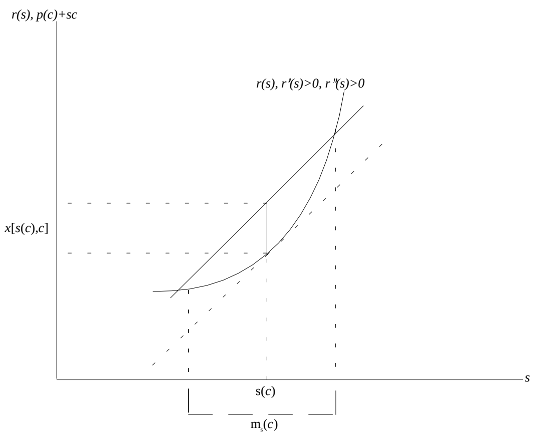

Figure 1: Walras versus search frictions

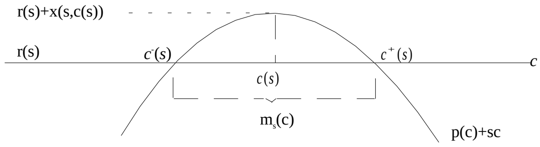

Figure 2: Approximation of the search surplus

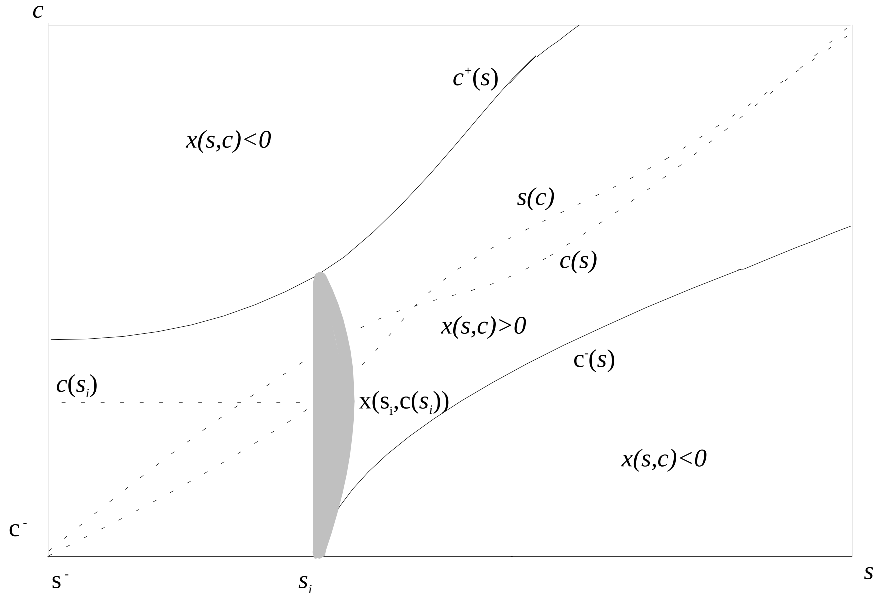

Figure 3: Matching sets, c(s), and s(c)

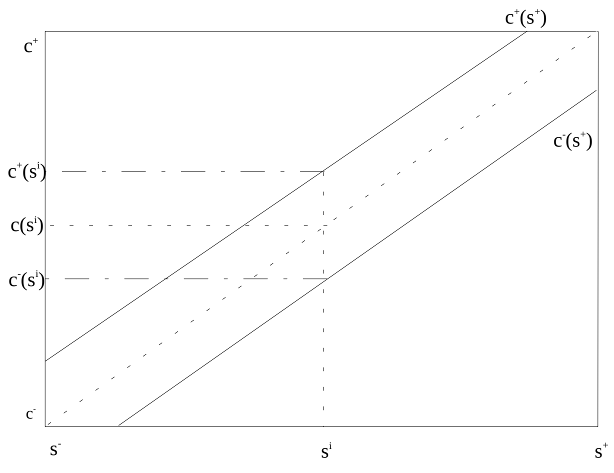

Figure 4: Lineair matching sets

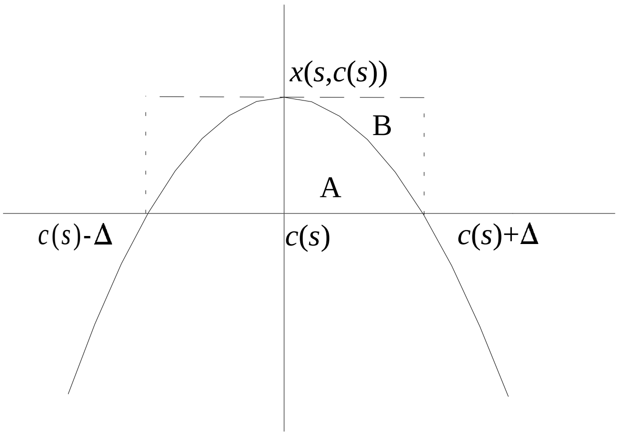

Figure 5: Taylor approximation of the surplus

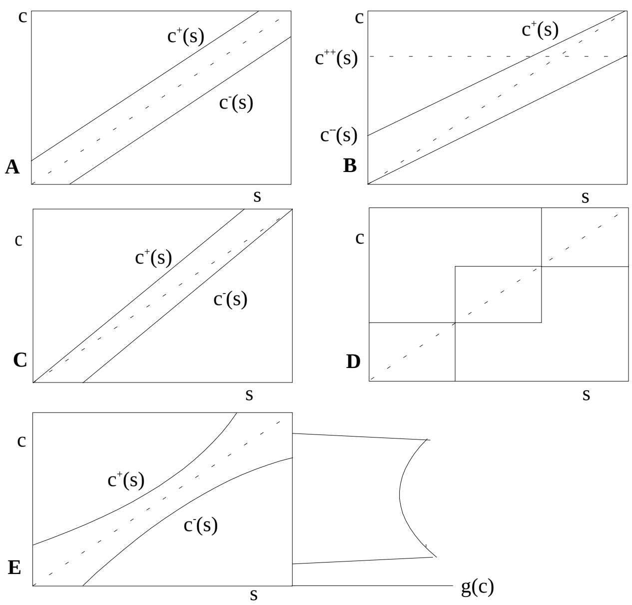

Figure 6: Illustration of the corner problem

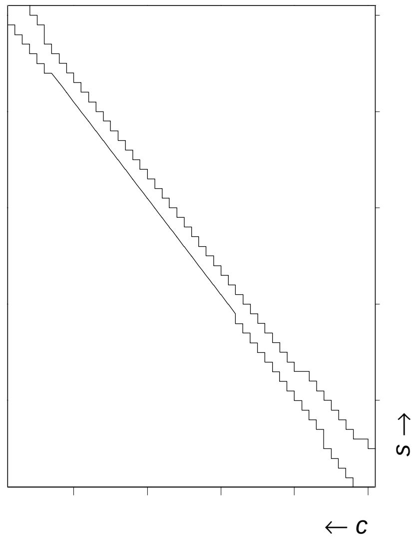

Figure 7: Matching sets for contact rate = 10000

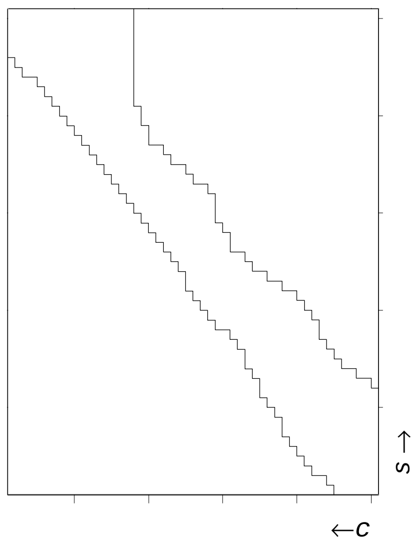

Figure 8: Matching sets fot contact rate = 39

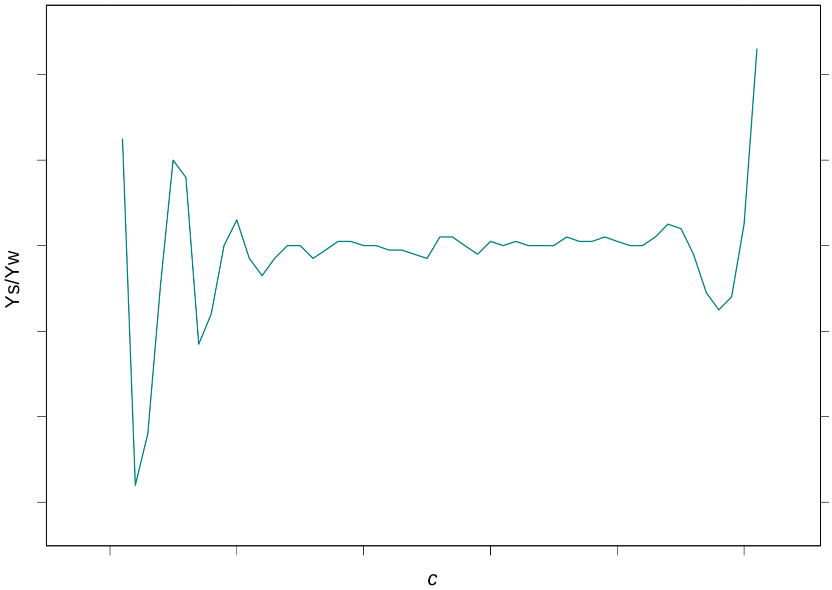

Figure 9: Output at different job types under perfect substitution for contact rate = 10000

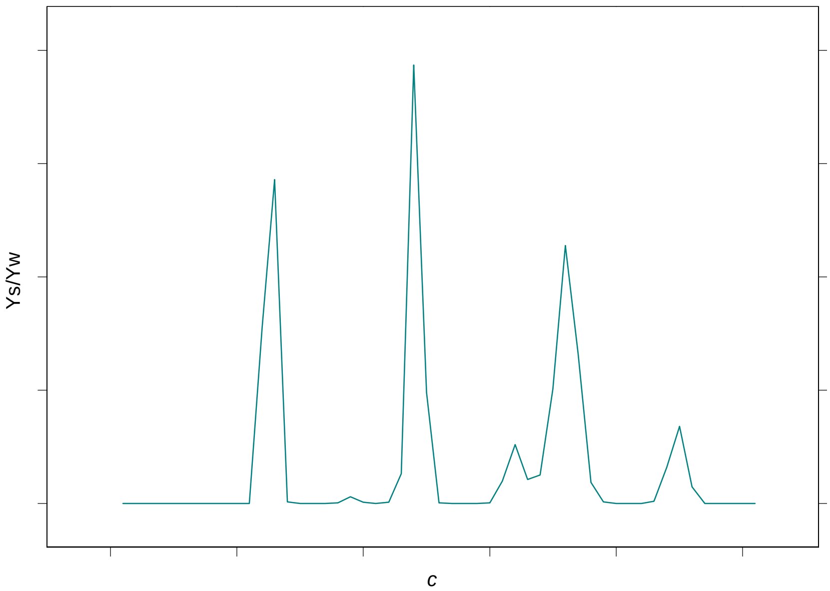

Figure 10: Output at different job types under perfect substitution for contact rate = 39

C Tables

Table 1: Aggregate outcomes for different values of λ

λ	10,000	2,500	625	156	39
stdev r	0.60	0.61	0.62	0.63	0.63
loss	0.03	0.05	0.09	0.15	0.26
aggr. $u(\%)$	0.9	1.5	2.8	5.3	9.9
aggr. $g(\%)$	2.2	3.7	6.1	10.2	14.6

Note: $\eta = \infty$, loss is output loss (in logs) due to search frictions

Table 2: Simulation results for different worker skill groups

	s^*	-6.0	-3.0	0.0	3.0	6.0
$\lambda = 10000$	$u(\%)$	2.8	1.0	0.6	0.6	0.8
	$r(s)$	-1.31	-0.65	-0.03	0.57	1.15
	$x(s)$	0.11	0.05	0.03	0.03	0.05
	error $x(s)$ %	-4.2	-1.5	-0.6	-0.4	-4.3
	error $u(s)$ %	2.1	-0.7	0.8	-0.6	-2.1
$\lambda = 2500$	$u(\%)$	4.8	1.8	1.1	1.0	1.4
	$r(s)$	-1.39	-0.69	-0.06	0.54	1.12
	$x(s)$	0.18	0.09	0.06	0.06	0.08
	error $x(s)$ %	-7.5	-3.5	-2.4	-2.3	-5.9
	error $u(s)$ %	-3.9	-1.2	-0.8	-1.1	-0.6
$\lambda = 625$	$u(\%)$	10.2	3.9	1.8	1.8	2.9
	$r(s)$	-1.47	-0.75	-0.10	0.48	1.07
	$x(s)$	0.27	0.14	0.09	0.10	0.12
	error $x(s)$ %	-22.10	-13.9	-19.8	-8.4	-23.5
	error $u(s)$ %	10.0	23.4	-4.0	1.0	15.0
$\lambda = 156$	$u(\%)$	25.3	7.6	2.7	3.0	3.1
	$r(s)$	-1.75	-0.90	-0.16	0.41	0.96
	$x(s)$	0.55	0.30	0.16	0.19	0.23
	error $x(s)$ %	-10.7	1.6	-13.6	0.37	-18.2
	error $u(s)$ %	21.8	25.2	-19.3	-7.0	-33.0
$\lambda = 39$	$u(\%)$	70.3	10.4	5.6	3.4	6.8
	$r(s)$	-2.2	-0.97	-0.25	0.25	0.72
	$x(s)$	0.94	0.36	0.23	0.34	0.25
	error $x(s)$ %	10.2	-30.2	-31.5	-3.5	-52.6
	error $u(s)$ %	-13.3	-5.4	-9.1	-42.6	-23.7

Note: $\eta = \infty$