Strategic Information Revelation in Fund-Raising Campaigns

Mehmet Bac* Parimal Kanti Bag†

July 28, 2000

Abstract

We consider a multi-stage game of fund-raising for a public project with a random number of potential contributors. The fund-raiser, who observes this number, decides whether to reveal or suppress the information before contributions are given. The fund-raiser's objective is to collect maximal contributions. We show that whether the public project is convex or non-convex can be the key to the fund-raiser's announcement decision. If the technology is convex, this number is always revealed. In the non-convex case the number may not be revealed at all or sometimes revealed only when it is in an intermediate range. When the number is not revealed, the fund-raiser induces largest total contributions by making a non-binding appeal that the contributors contribute a specified minimum amount. *Journal of Economic Literature* Classification Numbers: D73, H41.
Key Words: Fund-raising, free-riding, information revelation, per-head contribution, equilibrium selection

*Department of Economics, Bilkent University, Bilkent 06533, Ankara, Turkey; E-mail: bac@bilkent.edu.tr

†Department of Economics, Birkbeck College, University of London, 7-15 Gresse St., London W1P 2LL, U.K.; E-mail: pbag@econ.bbk.ac.uk

1 Introduction

Organizing potential beneficiaries of any public project and persuading them to make voluntary contributions can be a challenging task. Important information about prospective contributors must be taken into consideration in designing fund-raising schemes that are reasonably *simple.* While in the *direct contribution* models of public goods the planner has practically no role to play, in the *mechanism design* approaches the planner often employs quite sophisticated rules. We present a model of fund-raising and public good contributions intermediate between these two dominant approaches and address some new issues.

A standard working assumption by many researchers in the public goods literature is that of *complete* information by the players (or contributors) about the different characteristics of the contribution/provision game. One such characteristic, quite important for the contribution decisions of any contributor, is the *number* of potential contributors.[1] However, it is more likely that the fund-raising organizations, rather than the contributors, will be better informed about such crucial data, and therefore have the option of concealing or publicizing their information to the contributors: whether a public good game is a complete or an incomplete information game is determined endogenously.

We focus on the question of strategic information revelation by a fund-raiser who wants to collect maximal contributions for a public project. The number of potential contributors is *random.* In the early, preparations stage, the fund-raiser learns the number of potential contributors and decides whether or not to announce this number. In addition, the fund-raiser may appeal that each contributor contribute a specified minimum amount towards the project. We ask whether such non-binding appeals can possibly play a strategic role in influencing the contribution decisions. We address these questions for two types of public good production technologies – a convex technology that involves zero minimal production, and a non-convex technology that requires a positive minimal production for the project to be viable. This distinction, as we show, turns out to be crucial.

[1] Bergstrom et al. (1986), Andreoni (1988), Admati and Perry (1991), and Varian (1994a) analyzing direct contribution games, and Bagnoli and Lipman (1989), Jackson and Moulin (1992), Varian (1994b), and Bag and Winter (1999) proposing various mechanism design solutions to the free-rider problem, all assume complete information environments and, in particular, assume that the number of players is common knowledge.

Some opposing evidences and intuitions about information revelation issues motivate our work. A fund-raiser may prefer concealing the presence of a sizeable number of potential contributors to portray a less optimistic picture and create sympathy in the minds of the contributors. On the other hand, there is also the opposite view that sometimes the organizers want to assure the prospective contributors of a critical mass of committed contributors so that the public project is almost guaranteed to take off (Andreoni, 1998). Intuitively, two countervailing scenarios may play on an individual donor's mind: if he knows there are too few potential contributors, the project may not be successfully launched even with the maximum amount of contribution he is willing to make, thus discouraging him against contribution; on the other hand, if he knows there are many potential contributors, the importance of his individual contribution for the project's viability as well as his own marginal benefit become negligible, again discouraging contribution. Thus, the fund-raiser's strategy to inform or not to inform potential contributors should depend on the expected reaction of the contributors, which in turn depends, as we will argue, on whether the public good technology is convex or non-convex.

A number of recent contributions to the fund-raising literature addresses important strategic issues affecting fund-raising arrangement: provision of specialized public goods induces higher overall contributions (Bilodeau and Slivinski (1997)), donations signal contributor's wealth (Glazer and Konrad (1996)) or the quality of charity (Vesterlund (1999), Andreoni (2000)), and often bring prestige (Harbaugh (1998)), confer warm-glow and satisfy snob appeals (Romano and Yildirirm (forthcoming)).[2] Our work is closer to Andreoni (1998), who studies a two-phase fund-raising arrangement where in the first phase the fund-raiser's principal objective is to secure the promises of contributions from the government or "leaders" to remove any uncertainty for the public that the project might not take off, which then jump-start the main contributions phase from the public. We complement Andreoni's work by shifting attention to another important phenomenon, though somewhat of a contrasting flavor than the one pointed out by Andreoni, that of appeals to the public by fund-raising organizations that without their help some pro-

[2]While all of these papers are in pure public good setting, Teoh (1997) and Hermalin (1998) respectively analyze, in team environments with public good like features where the overall productivity of joint efforts depends on an underlying 'state', how mandatory disclosure rules to announce the state, and the signaling of the state through own action by an informed leader, influence the team members' efforts.

posed project(s) might not take off.[3] Implicit in such appeals is the message that the number of potential contributors is not large and therefore each contributor must offer generously to rescue the project. Thus, the worry that the project might not take off may guide a fund-raiser to choose different strategies: Andreoni highlights the need for early assurances about the project's success to the yet untapped contributors, whereas we consider the possibility of a failed project to motivate potential contributors for generous contributions; there are no "leaders" in our model. Specifically, we focus on the fund-raiser's strategic use of the private information about the number of potential contributors. For large number of potential contributors a rough intuition gained from Andreoni (1998) suggests that the fund-raiser should announce this information, whereas the intuition for free-riding exemplified in another paper by Andreoni (1988) suggests that the fund-raiser should hide the information. For small number of potential contributors, again symmetrically opposite intuitions appear puzzling.

We show the following results. If the public good production technology is *convex*, the fund-raiser will *always reveal* his information about the number of potential contributors, because failing disclosure the contributors tend to associate positive beliefs to outcomes favorable for free-riding which the fund-raiser wants to avoid. The results and their intuitions in the non-convex case are strikingly different. When the technology is *non-convex*, there can be multiple equilibria with different symmetric contributions. First, there always exists a revealing equilibrium in which the fund-raiser announces if the number of potential contributors (weakly) exceeds a threshold number and suppresses otherwise; total contributions following announcement are just enough to make the project viable, but collapse to zero following suppression. In addition, there may exist partially revealing and/or non-revealing equilibria. In a partially revealing equilibrium, the fund-raiser suppresses small turnouts to avoid the zero contributions outcome and suppresses large turnouts to avoid too much free-riding, but announces intermediate turnouts inducing just enough contributions making the project viable. In a non-revealing equilibrium, the number of potential contributors is always suppressed so that contributors with the expectation of a viable level turnouts

[3] Instances of such appeals are quite common: public radio and television broadcasting stations appeal to the public for support subscriptions to maintain key items on the program list; schools and universities appeal to their alumni for contributions towards further development of important units that may potentially benefit the alumni's children or relatives; etc.

would make positive contributions and the fund-raiser gets to collect high overall contributions in the case of large turnouts. Total contributions are *largest* in a non-revealing equilibrium, followed by a partially revealing equilibrium, and *lowest* in a revealing equilibrium. Hence, the fund-raiser prefers the non-revealing equilibrium most, while the contributors prefer the revealing equilibrium most. Because these equilibria involve suppression of different sets of number of contributors, it is not clear how the contributors can infer which equilibrium is being played when they receive no information about the number of contributors. In such situations, the suggestion of a minimum per-head contribution by the fund-raiser can signal his strategic intent, provided the contribution game also has a symmetric equilibrium with the contribution recommended by the fund-raiser. Thus, non-binding appeals requesting a minimum per-head contribution can serve the purpose of selecting the fund-raiser's most preferred equilibrium. Our results suggest that such appeals are effective only when the technology is *non-convex* and the fund-raiser finds it optimal to *conceal* the number of contributors.[4]

The suggestion of *a minimum contribution* is a *simple* enough procedure and a salient feature of many real life fund-raising campaigns. Thus, if announcement of the number of potential contributors is perhaps less observed in practice than the revealing equilibrium of our model suggests should happen, the equilibrium selection motive we highlight explains why the fund-raising organizations might be reluctant to announce the number of potential contributors and the non-revealing equilibrium is the more likely outcome in practice.

The paper is organized as follows. Section 2 presents the fund-raising games. In Section 3, we analyze these games for the case of a convex technology. Section 4 introduces non-convexity and reviews its implications for information revelation and contribution decisions. Section 5 concludes. An appendix contains the proofs of propositions 1 and 2.

2 The Fund-Raising Games

A public good project will have to be financed through private contributions. The number of potential contributors, hereafter called players, is random:

[4]The fund-raiser's per-head contribution suggestion will have no impact in the convex case because the contribution game has a unique symmetric equilibrium in which the fund-raiser reveals the number of potential contributors.

$p(k)$ is the probability that there are $k \in \{1, \ldots, N\}$ players chosen by Nature, $\sum_{k=1}^{N} p(k) = 1$, and $p(\cdot)$ is common knowledge. Given any number of players k drawn by Nature, each of the potential N players will be assumed to have an equal chance, k/N, to be included among the k-players, so that potential players' *names do not matter* for the players' or the fund-raiser's decisions.[5]

As in the fund-raising literature, we assume that the fund-raiser, hereafter referred as the "planner", maximizes total contributions.[6] The players maximize (net) expected utilities and have identical quasi-linear preferences:[7]

$$u(m_i - g_i, G) = v(G) + m_i - g_i, \quad \text{with} \quad v'(\cdot) > 0, \quad \text{and} \quad v''(\cdot) < 0,$$

where m_i is player i's endowment of a single private good, g_i is i's contribution, and G is the level of public good. The level, G, depends on an underlying technology, thus total contributions and the level of public good provided may differ.

Below we consider a basic, simpler fund-raising game in which the planner may choose to announce or conceal the number of players. In the extended version we allow the planner to also announce a suggested per-head contribution. We assume zero cost of information announcements; similar results obtain in the presence of an operational cost.

2.1 The game Γ

The basic fund-raising game, denoted Γ, consists of *two* stages. Stage 1 can be called the preparations stage, while Stage 2 is the contributions stage when the fund drive is launched.

Stage 1. The planner observes k and then decides whether to announce or suppress k.

Stage 2. Having observed the planner's announcement or no announcement, the players update their beliefs $p(k)$ and simultaneously decide on their contributions.

The strategy to learn and accordingly decide whether to announce crucial information about the contributors is an important part of campaign

[5] See Andreoni (1998) where players' names *do* matter – there contribution from the leader is the key to a project's survival.

[6] Strictly speaking, the fund-raiser is different from a traditional planner whose primary objective happens to be either efficiency or equity.

[7] Quasi-linearity simplifies the analysis. Our results hold under weaker assumptions.

management. Fund-raisers gather information about potential donors in the preparations stage and they have the option of disclosing or not the information collected to the prospective donors. In our model, because the prospective contributors are identical, the only relevant information to be disclosed is their number.[8]

The strategies in Γ are defined as follows. The announcement strategy of the planner is a map $a : \{1, \ldots, N\} \to \{1, \ldots, N\} \cup \{\emptyset\}$ such that $a(k) = \{k, \emptyset\}$. We have $a(k) = k$ if k is announced and $a(k) = \emptyset$ if k is suppressed. Announcement of k is assumed to be always truthful, as non-truthful announcements are illegal and/or seriously undermines the planner's authority. Given an announcement strategy $a(\cdot)$, the set of announced k's is denoted by $\mathcal{A}$ and the complementary set is denoted $\mathcal{C}$. Players' (common) beliefs following planner's announcement decision is a probability distribution $\mu(\cdot)$ over the set $\{1, \ldots, N\}$, derived from the priors using Bayes' rule, whenever possible. Thus, given an announcement strategy $a(\cdot)$, if the planner announces k, then $\mu(k) = 1$ and $\mu(n) = 0, n \neq k$. If the planner makes no announcement, each player revises the probability that k players are in the game conditional on his own presence, as[9]

$$\begin{aligned}\mu(k) &= \begin{cases} \text{pr}\left(\{n = k\} | \{n \in \mathcal{C}\} \cap \{\text{I'm in the game}\}\right), & \text{for } k \in \mathcal{C} \neq \emptyset \\ 0, & \text{for } k \in \mathcal{A}. \end{cases} \\ &= \begin{cases} \frac{\text{pr}\left(\{n=k\} \cap \{n \in \mathcal{C}\} \cap \{\text{I'm in the game}\}\right)}{\text{pr}\left(\{n \in \mathcal{C}\} \cap \{\text{I'm in the game}\}\right)}, & \text{for } k \in \mathcal{C} \neq \emptyset \\ 0, & \text{for } k \in \mathcal{A}. \end{cases}\end{aligned}$$

Finally, the contribution strategies of the players depend on the planner's announcement $a(\cdot)$ and beliefs $\mu(\cdot)$, that is, $g_i : \{1, \ldots, N\} \cup \{\emptyset\} \times [0,\ 1]^N \to \mathcal{R}_+$.

We require the strategies and the belief system $(a^*(\cdot), \{g_i^*(\cdot, \cdot)\}, \mu(\cdot))$ to form a *perfect Bayesian* equilibrium. The analysis will make use of the following assumptions about the equilibrium strategies.

[8] With heterogeneous preferences, the planner will announce the number as well as the identities (or preferences) of the participants, but this will complicate the analysis without altering the main message of our paper.

[9] To give an example how to calculate the posterior, for $N = 4$, $\mathcal{C} = \{1, 4\}$, $\mathcal{A} = \{2, 3\}$,

$$\mu(1) = \frac{\text{pr}\left(\{n = 1\} \cap \{\text{I'm in the game}\}\right)}{\text{pr}\left(\{n \in \{1, 4\}\} \cap \{\text{I'm in the game}\}\right)} = \frac{(1/4) \cdot p(1)}{(1/4) \cdot p(1) + (4/4) \cdot p(4)}.$$

Assumption 1 (Symmetry) *Whenever there are both a symmetric contributions equilibrium and an asymmetric contributions equilibrium yielding the same total contributions, players always play the symmetric equilibrium, so that* $g_i^* = g_j^*$, $i, j \in \{1, 2, \ldots, N\}$.

Assumption 2 (Tie-breaking) *If announcing and suppressing* k *yield the same payoff for the planner, then he suppresses* k.

Assumption 2 is a working assumption; even a slight cost of making an announcement would break the tie in favor of not making an announcement.[10] Assumption 1, which can be justified by the focal point argument, simplifies the analysis. Hereafter we shall drop the subscript i from equilibrium contribution strategies and refer to a perfect Bayesian equilibrium satisfying assumptions 1 and 2 simply as "equilibrium".

2.2 The game Γ^+

We shall also consider the following extension of the game Γ, denoted Γ^+, to investigate the potential role of contribution suggestions by the planner:

> In Stage 1 of the game Γ, along with the announcement $a(\cdot)$ of the number of players, the planner additionally announces a suggested per-head contribution g^S. All remaining aspects of Γ are preserved in Γ^+.

The recommendation of per-head contribution in Γ^+ cannot be binding on the players.

3 The Convex Technology Case

In this section, we consider a convex technology for the public project. For simplicity we assume that the level of public good is the sum total of individual contributions.[11]

[10] We could alternatively assume that the planner announces k whenever he is indifferent and the equilibria of the game would essentially remain intact, except for the bad equilibria in which each player contributes zero when no information is announced about the potential number of contributors.

[11] This is much stronger than needed for our results. Letting $f(\sum g_i) = G$ denote the public good production technology, a continuous production function f that preserves strict concavity of $v(f(x))$ in x is sufficient.

To begin with the analysis of the game Γ, suppose that the planner announces the presence of k players. In the continuation game (stage 2), the symmetric individual Nash equilibrium contribution $g^*(k)$ is determined by the first-order condition $v'(kg^*(k)) = 1$ which, defining $\overline{G}$ through $v'(\overline{G}) = 1$, can be written as

$$v'(kg^*(k)) = v'(\overline{G}). \tag{1}$$

Therefore, $g^*(k) = \overline{G}/k$ and the aggregate Nash contribution equals $\overline{G}$. The equilibrium is immune to deviations to $g_i = 0$ if

$$v(\overline{G}) - g^*(k) \geq v((k-1)g^*(k)). \tag{2}$$

Consider now a *non-revelation* policy. If the planner adopts the strategy of never announcing the number of players, whatever be this number, then in Stage 2 the symmetric Bayes' Nash equilibrium contribution of a player, $g^*(\emptyset)$, will satisfy

$$\mu(1)v'(g^*(\emptyset)) + \mu(2)v'(2g^*(\emptyset)) + \ldots + \mu(N)v'(Ng^*(\emptyset)) = v'(\overline{G}). \tag{3}$$

Clearly, $\overline{G}/N < g^*(\emptyset) < \overline{G}$. Therefore, there exists some $k_0 \leq N$ such that for all $k \geq k_0$, $g^*(\emptyset) > \overline{G}/k$, and for all $k < k_0$, $g^*(\emptyset) \leq \overline{G}/k$. The following result is implied.

Lemma 1 *Compared to the revelation policy, the policy of non-revelation leads to lower public good level ex-post when less than k_0 number of players turn up, but higher public good level ex-post when at least k_0 number of players turn up.*

The intuition behind Lemma 1 is simple. Non-revelation of the number of players creates uncertainties for an individual player whether he is among a "few" (that is, $k < k_0$) or one among "many" (that is, $k \geq k_0$). The prospect of many other contributions enhances free-riding incentives and results in a lower public good level when the number of players turns out to be small. On the other hand, the players also consider the possibility that there may be only a few of them, which induces each player not to rely too much on free-riding opportunities and hence insure himself by contributing more than what he would contribute under perfect information of a large number of players. This induces a higher public good level ex-post, when the number of players turns out to be large.[12]

However, the non-revelation policy as stated in its extreme that the planner never announces k, or only a partial revelation policy in which the planner suppresses a (non-singleton) proper subset of N potential numbers of players, cannot be an equilibrium strategy. To see this, suppose that there is a set $\mathcal{C}$ of numbers (containing at least two elements[13]) such that the planner does not announce k if $k \in \mathcal{C}$. Each player, on observing no announcement by the planner, would determine his own contribution according to condition (3), but modified to take into consideration only those k's that belong to $\mathcal{C}$. By Lemma 1, the planner will prefer revealing small turnouts from the set $\mathcal{C}$, thus will deviate from the strategy of not revealing *any* k from the set $\mathcal{C}$. We have the following result.

Proposition 1 *Suppose that the public good production technology is convex. Then in equilibrium in the game* Γ*, the number of players is always fully revealed.*

The revealing equilibrium is the unique symmetric equilibrium in which each player contributes $\overline{G}/k$, where k is the revealed number of players. The equilibrium unraveling result in Proposition 1 falls into the category of similar results on persuasion games mostly involving disclosures of a product's quality by a seller, who is privately informed about the quality, to the buyers (Milgrom (1981), Matthews and Postlewaite (1985), Milgrom and Roberts (1986), Okuno-Fujiwara et al. (1990)). All of these papers commonly exploit the idea that when multiple parties with opposed interests are asymmetrically informed about the value of an important variable, the uninformed party, by applying "due skepticism",[14] induces full revelation of information. Likewise, in our setting failure by the planner to announce the number of players is interpreted by the players as an evidence of high turnouts, inducing full revelation.

Finally, to see whether the planner's suggestion of a per-head contribution can play any role in the convex technology case, consider the extended

[12]Whether, by not revealing the number of players and raising aggregate contribution in excess of $\overline{G}$, the planner is likely to induce an aggregate level that even exceeds the socially optimal level, is not relevant in our context: our planner, who is a fund-raiser, is solely interested in maximizing contributions.

[13]If $\mathcal{C}$ is singleton, then the announcement strategy is fully revealing.

[14]By "due skepticism" the uninformed party assumes that unless the informed party announces, the realized value of the variable must be at the level which the informed party must be *most* reluctant to announce.

game Γ^+. Note that any equilibrium outcome (total contributions) of Γ can be supported as an equilibrium outcome of Γ^+. On the other hand, since the suggestion cannot be binding, the players will not contribute the suggested per-head contribution unless the game Γ has an equilibrium with contributions equal to the suggested per-head contribution. Therefore the extension of the planner's strategy space to include suggestions of per-head contributions can have an equilibrium selection role only if the original game Γ has multiple symmetric equilibria. But since Γ has a unique symmetric equilibrium, suggesting a per-head contribution amount cannot serve any purpose.[15]

4 The Non-Convex Technology Case

In this section we consider the case of a non-convex technology: any production of the public good must meet a minimum threshold quantity requirement. This is due to an initial lumpiness in the production process, for example, a minimal amount of capital investment is essential for the construction of a local public school or hospital. This case has been the main focus of Andreoni (1998).[16] We assume the same non-convex technology,

$$G = \begin{cases} \sum_{i=1}^{n} g_i, & \text{if} \quad \sum_{i=1}^{n} g_i \geq G_{min}\,; \\ 0, & \text{if} \quad \sum_{i=1}^{n} g_i < G_{min}\,, \end{cases}$$

and adopt the fixed costs interpretation of non-convexity.[17]

The occurrence of zero contributions, a likely outcome in our model under complete information while all potential contributors contribute *simultane-*

[15]Note, however, that per-head contribution suggestions can focus players' attention on the unique symmetric equilibrium in the presence of other asymmetric equilibria. For convex technology, all asymmetric equilibria will result in the same total contribution as the symmetric equilibrium.

[16]Andreoni shows that the occurrence of a zero contributions outcome due to non-convexity can be avoided by a two-phase fund-raising arrangement where the "leader" contributors give early assurances by providing "seed grants". Marx and Matthews (2000) also consider a similar non-convexity where benefits jump discontinuously after a minimal production. However, they are not concerned with the strategic issues of fund-raising; they examine, in a direct contribution game setting, whether the opportunity to make repeated contributions can eliminate inefficiencies.

[17]As Andreoni (1998) notes, non-convexity may be due to increasing returns to scale over some range of total contributions. It can also stem from the players' preferences, instead of the technology. Our results would be similar in these cases.

ously, can be avoided through strategic "hiding" of information by the planner. In this context, we focus on the impact of non-convexity on three key aspects of fund-raising: (i) the planner's strategy of whether to announce or conceal the number of potential contributors, (ii) the planner's solicitations of a minimum per-head contribution amount, and (iii) the equilibrium level of contributions. We investigate the relationship between components (i) and (ii) of the planner's strategy, and its effect on (iii).

We need to specify what happens to players' contributions if total contributions fall short of G_{min}. We assume, like Andreoni and as is the case in most fund drives, that insufficient contributions will *not* be refunded and the planner will use the funds for some other project that yields no benefits to the players.[18]

4.1 Equilibria of the fund-raising game Γ

Let us denote the equilibria of Γ by $(\tilde{a}(\cdot), \tilde{g}(\cdot,\cdot), \mu(\cdot))$. If $G_{min} < \overline{G}$ then whenever the planner reveals k the threshold level will have no bite because the players would voluntarily contribute in excess of the threshold level. So, hereafter we consider the interesting case and assume:

Assumption 3 $G_{min} > \overline{G}$.

This implies that the equilibrium derived in Proposition 1 for the convex technology, under which all k are revealed and the planner collects the aggregate contribution $\overline{G}$, is no longer an equilibrium of Γ under the non-convex technology. Now, when the planner reveals a $k \in \{1, \dots, N\}$, the contributions stage of Γ will have a symmetric equilibrium $\{\tilde{g}(k)\}$ with $k\tilde{g}(k) = G_{min}$ if and only if individual participation constraints,

$$v(G_{min}) - \tilde{g}(k) \geq v(0), \tag{4}$$

are satisfied. This is easy to check: No player has an incentive to deviate to a lower contribution, $0 \leq g_i < \tilde{g}(k)$, as each player is pivotal for the supply of the threshold level; nor would any player contribute more than $\tilde{g}(k)$ because the marginal cost exceeds the marginal benefit: $1 > v'(G_{min})$. Thus, announcing k induces the equilibrium where each player contributes

[18] Similar results can be derived if we assume $G = \lambda \sum_i g_i$, where $0 < \lambda < 1$, if $\sum_i g_i < G_{min}$. The parameter λ would capture the diminished benefits of the players due to an alternative use of the funds which fall short of G_{min}.

an equal share for a guaranteed supply of G_{min}. The following assumption ensures that such a contributions equilibrium exists only if k is sufficiently large.

Assumption 4 $G_{min} > v(G_{min}) - v(0) > G_{min}/N$.

By ruling out the case $v(G_{min}) - G_{min}/N < v(0)$, Assumption 4 rules out the uninteresting case in which the planner is stuck in a zero-contributions equilibrium for all k. Thus, we can define:

Definition 1 *$\underline{k}$ is the minimum k such that $v(G_{min}) - G_{min}/k \geq v(0)$.*

Such a $\underline{k} > 1$ exists by Assumption 4.

4.1.1 The result

The following proposition summarizes the different equilibria of Γ in the case of non-convex technology. (See the proof for the characterization.)

Proposition 2 *Suppose the planner cannot (or does not) commit ex-ante to reveal or suppress the number of players, and the public good production technology is non-convex. Then the game Γ always has a revealing equilibrium and may have one or more of two other types of equilibria. The equilibria are:*

<u>Revealing Equilibrium</u>: The planner announces all k higher than or equal to the threshold number of players, $\underline{k}$, and suppresses all $k < \underline{k}$. Each player makes the positive contribution $\tilde{g}(k) = G_{min}/k$ if $k \geq \underline{k}$ is announced, a zero contribution $\tilde{g}(k) = 0$ if either $k < \underline{k}$ is announced or if no announcement is made.

<u>Partially Revealing Equilibrium</u>: The planner announces all k in an intermediate range $\mathcal{A} = \{\underline{k}, \dots, \overline{k}\}$ and suppresses all other k's. Each player contributes $\tilde{g}(k) = G_{min}/k$ if $k \geq \underline{k}$ is announced, $\tilde{g}(k) = 0$ if $k < \underline{k}$ is announced. If the planner makes no announcement, each player makes the positive contribution $\tilde{g}(\emptyset) = G_{min}/(\overline{k} + 1)$.

<u>Non-revealing Equilibrium</u>: The planner suppresses all $k \in \{1, \dots, N\}$. If $k \geq \underline{k}$ is announced, each player contributes $\tilde{g}(k) = G_{min}/k$. If no announcement is made, each player contributes $\tilde{g}(\emptyset) = G_{min}/\hat{k}$ where $\hat{k} \leq \underline{k}$.

4.1.2 Intuitions

In all three types of equilibria, the supply of G_{min} through symmetric contributions is guaranteed if at least a threshold number of players turn up.

In a revealing equilibrium the planner is not able to collect funds by suppressing the number of players, because players interpret no announcement as evidence of sufficiently small turnouts so that positive contributions by the players to guarantee the threshold level G_{min} are no longer individually rational. Therefore only sufficiently large turnouts will be announced, leaving small turnouts to be correctly inferred.[19]

In a partially revealing equilibrium the planner suppresses large turnouts ($k > \overline{k}$) to counter pessimistic beliefs arising from the suppression of small turnouts ($k < \underline{k}$), and each player makes a symmetric positive contribution with total contributions matching, and often exceeding, the threshold level G_{min} for large turnouts, but failing to meet the threshold level for small turnouts. Intermediate turnouts ($\underline{k} \leq k \leq \overline{k}$) are announced, inducing the threshold level G_{min}.

In a non-revealing equilibrium, the planner never announces the number of players. As stated in Proposition 2, corresponding individual contributions must generate at least the required sum G_{min} whenever a minimum number $\hat{k}$ of players turn up, where $\hat{k} \leq \underline{k}$ (recall, $\underline{k}$ is the(minimum) number of players which, under complete information, will induce each player to make non-zero contributions equal to $G_{min}/\underline{k}$). The number $\hat{k}$ cannot exceed $\underline{k}$ because, if it were to exceed, the planner would announce observations of numbers between $\underline{k}$ and $\hat{k}$ and increase total contributions to G_{min}. This would upset the non-revealing equilibrium. Given $\hat{k} \leq \underline{k}$, note that it is optimal for the planner not to reveal any $k < \hat{k}$ for this would generate zero contributions, nor is it optimal to reveal any $k \geq \hat{k}$ for this can only reduce total contributions (to zero if $\hat{k} < \underline{k}$ and $k \in \{\hat{k}, \ldots, \underline{k} - 1\}$ is announced, to G_{min} if $k \geq \underline{k}$ is announced). To see why $\hat{k}$ can be strictly lower than $\underline{k}$, suppose that the players put very high probability on the event $k = \underline{k}$. Anticipating with high probability a turnout of $\underline{k}$, the players would rather be cautious and be protective of a discrete drop in utility resulting from k being close to, yet smaller than, $\underline{k}$, as opposed to a small increase in utility from k slightly exceeding $\underline{k}$. Risk-aversion of the players (by strict concavity

[19] The fact that k values below $\underline{k}$ are suppressed stems from the tie-breaking Assumption 2. The planner would announce all k if we replace Assumption 2 by the opposite tie-breaking rule, but the equilibrium contributions will be no different.

of $v(\cdot)$) would prompt each player to contribute the amount $G_{min}/\hat{k}$ which exceeds $G_{min}/\underline{k}$, realizing the project for numbers of players even lower than $\underline{k}$, the minimum number required under complete information.

The planner's payoffs or total contributions P^j in a type-j equilibrium of Γ are as follows:

Revealing: $P^R = 0$ if $k < \underline{k}$, $P^R = G_{min}$ if $k \geq \underline{k}$;

Partially Revealing: $P^P = (k/(\overline{k}+1))G_{min}$ if $k < \underline{k}$ or $k > \overline{k}$, and $P^P = G_{min}$ if $k \in \{\underline{k}, \ldots, \overline{k}\}$;

Non-revealing: $P^N = (k/\hat{k})G_{min}$ where $\hat{k} \leq \underline{k}$.

Thus, total contributions are (weakly) largest in a non-revealing equilibrium, (weakly) lowest in the revealing equilibrium.

4.1.3 The role of non-convexity

A comparison of the non-convex case with the convex case may help understand how non-convexity makes non-revealing equilibrium an attractive outcome for the planner. In both cases, the complete information equilibrium resulting from revelation of k has each player contributing some fixed amount divided by the number of players (or else contributing zero). This amount is $\overline{G}$ in the convex case, G_{min}, in the non-convex case. The difference in the results comes from the discontinuity induced by the threshold G_{min}. In both cases, if a player cuts his contribution by ϵ, he causes a discrete drop in the probability that contributions sum to the respective amounts, $\overline{G}$ and G_{min}. In the convex case where the first-order conditions do apply and players' contributions change continuously with the beliefs, this does *not* cause a discrete payoff reduction. In particular, if the planner were to suppress in equilibrium at least two different k values, then a player's contribution under incomplete information must lie between the smallest and largest complete information contributions corresponding to these k values.

In contrast, in the non-convex case, *nothing* is provided if contributions drop below G_{min}, causing a discrete drop in the players' payoffs. As a result, optimal contributions need not be continuous in beliefs. This can create a situation where a player's contribution under incomplete information lies (weakly) above all possible complete information contributions. Thus, *in the non-convex case, the planner can only benefit from the players' uncertainty concerning their number.* This feature has a nice parallel with the auctions literature: McAfee and McMillan (1987) and Matthews (1987) show that the seller of an indivisible (private) good holding a first-price sealed-bid auction will extract a higher expected price if the bidders are kept uninformed about the number of their competitors. While for their results to hold the bidders' absolute risk-aversion must be *non-increasing*, we do not impose any such restriction.

4.1.4 Examples

The following examples illustrate the different equilibria in Proposition 2.

Example 1. (*Partially Revealing Equilibrium*) Let $v(G) = 10(G)^{0.5}$, $N = 5$, and $p(1) = 0.1, p(2) = 0.2, p(3) = 0.1, p(4) = 0.3, p(5) = 0.3$. In the case of convex technology, equilibrium total contributions $\overline{G}$ equals 25. In the case of non-convex technology, which is our concern, suppose $G_{min} = 200$. Check that $\underline{k} = 2$ and the revealing equilibrium, as argued in Proposition 2, exists. Below we verify the following partially revealing equilibrium:

> The planner announces $k \in \{2, 3\}$ and suppresses $k \in \{1, 4, 5\}$, and the players contribute respectively 100 and 200/3 for announcement of $k = 2$ and $k = 3$ and contribute $g^*(\emptyset) = 50$ when there is no announcement.

When $k = 2$ is announced, contributing 100 given that the other player contributes 100 is clearly optimal, as it just ensures the threshold level and yields a net utility of 41.42136, while deviating to zero contribution yields zero net utility. Similarly, contributing 200/3 when $k = 3$ is announced is optimal for the players. We next check that in the no-announcement continuation game no player deviates to zero contribution. In the absence of any announcement under the proposed equilibrium, calculate the posteriors. We have,

$$\mu(1) = \frac{(1/5)p(1)}{(1/5)p(1) + (4/5)p(4) + (1)p(5)} = 1/28.$$

Similarly,

$$\mu(2) = \mu(3) = 0, \;\; \mu(4) = 3/7, \;\; \mu(5) = 15/28.$$

Given these posteriors, the net expected utility of a player from contributing 50 is $(3/7) \cdot 10(200)^{0.5} + (15/28) \cdot 10(250)^{0.5} - 50 = 145.313$, whereas net expected utility by deviating to zero contribution equals $(15/28) \cdot 10(200)^{0.5} = 75.761$. Thus, deviation to zero contribution will not occur. All other deviations can similarly be ruled out.

To check that the strategy of announcing if and only if $k \in \{2, 3\}$ is optimal for the planner, consider the possible deviations. If the planner suppresses $k \in \{2, 3\}$, each player contributes $g^*(\emptyset) = 50$ and total contributions fall short of 200 achieved by announcing k. If the planner announces $k = 1$, the only player contributes zero (contributing G_{min} yields $v(G_{min}) - G_{min} = -58.578644$) which is less than the contribution 50 under no announcement. If $k = 4$ or $k = 5$ is announced, total contributions equal 200, whereas total contributions under no announcement equal 200 when $k = 4$, and 250 when $k = 5$. ||

Example 2. (*Non-revealing Equilibrium*) Again let $v(G) = 10(G)^{0.5}$, $N = 5$, $G_{min} = 200$, but $p(1) = 0.1, p(2) = 0.8, p(3) = 0.05, p(4) = 0.025, p(5) = 0.025$ Recall, $\underline{k} = 2$ and the revealing equilibrium exists. Below we verify that there is also a non-revealing equilibrium.

Suppose the players believe that they are playing a non-revealing equilibrium. With no announcement under the proposed equilibrium, the updated beliefs need to be calculated. Given the priors,

$$\mu(1) = \frac{(1/5)p(1)}{(1/5)p(1) + (2/5)p(2) + (3/5)p(3) + (4/5)p(4) + (1)p(5))} = 4/83.$$

Similarly,

$$\mu(2) = 64/83, \quad \mu(3) = 6/83, \quad \mu(4) = 4/83, \quad \mu(5) = 5/83.$$

We now check that the equilibrium symmetric contribution under no announcement is $G_{min}/\underline{k} = 100$. The net expected utility from contributing $g(\emptyset) = 100$ is $(64/83) \cdot 10(200)^{0.5} + (6/83) \cdot 10(300)^{0.5} + (4/83) \cdot 10(400)^{0.5} + (5/83) \cdot 10(500)^{0.5} - 100 = 44.677$, whereas net expected utility from deviation to zero contribution equals $(6/83) \cdot 10(200)^{0.5} + (4/83) \cdot 10(300)^{0.5} + (5/83) \cdot 10(400)^{0.5} = 30.619$. Thus, a deviation to zero contribution is not beneficial. Similarly, it can be shown that other deviations are not beneficial either. For the planner to deviate to announce k is not beneficial for any k. ||

4.2 The role of a per-head contribution suggestion

Because the fund-raising game Γ always has a revealing equilibrium, any multiplicity of equilibria in the presence of a partially revealing and/or a non-revealing equilibrium raises the following question: Since in these equilibria the planner suppresses different sets of numbers of players, and since the players' responses to no announcement depends on the equilibrium being played, how should the players coordinate their contribution strategies and what belief system should they hold *when the planner makes no announcement on k*? In what follows we address this question.

Consider the game Γ^+ where the planner's strategy space is extended to include suggestions of per-head contributions, denoted g^S. The planner's suggestion of a per-head contribution g^S can have an impact on the players' strategies only if the game Γ has an equilibrium with symmetric contributions g^S; the players will simply not adopt a non-equilibrium suggested contribution strategy.[20] The interesting case is when the planner suppresses the number of players and suggests g^S. Though there may be several equilibria in which the planner suppresses different sets of numbers of players, the suggested contribution g^S may reveal information about the set $\mathcal{C}$ of suppressed k's and the associated equilibrium that the planner wants to induce. If Γ has a continuation equilibrium in which each player contributes g^S following no announcement on k, the players can infer that this equilibrium is being played and contribute g^S accordingly.

We call $(\tilde{a}(\cdot), g^S, \mu(\cdot))$ an *induced equilibrium* of Γ^+ if $(\tilde{a}(\cdot), \tilde{g}(\cdot,\cdot) = g^S, \mu(\cdot))$ is an equilibrium of Γ and the planner's payoff (total contributions) is (weakly) higher than in any other equilibrium of Γ. For example, if Γ does not have a non-revealing equilibrium but has partially revealing equilibria, the planner would like to induce the partially revealing equilibrium with highest total contributions. This would be the corresponding induced equilibrium of Γ^+ where the planner suggests the individual contribution $g^S = G_{min}/k$ for each announced k and $g^S = \tilde{g}(\emptyset) = G_{min}/(\overline{k}+1)$ whenever he does not announce k according to $\tilde{a}(\cdot)$. Then the players will contribute $\tilde{g}(\emptyset) = g^S$ because these contributions form a symmetric equilibrium, the one yielding highest total contributions when no announcement is made about the number of players. Proposition 3 follows from the definition of an induced equilibrium and the ranking of the planner's payoffs.

Proposition 3 *The game Γ^+ has a unique induced equilibrium in which*

(i) the planner suppresses all $k = \{1, \ldots, N\}$ and suggests a per-head contribution $g^S = G_{min}/\hat{k}$, if Γ has a non-revealing equilibrium;

(ii) the planner announces k only when $k \in \{\underline{k}, \ldots, \overline{k}\}$ and suggests a per-head contribution $g^S = G_{min}/(\overline{k}+1)$, if Γ does not have a non-revealing

[20] For instance, the players will contribute the suggested per-head contribution $g^S = G_{min}/k$ if the planner announces the number of players k (weakly) exceeding $\underline{k}$.

equilibrium but has a partially revealing equilibrium;

(iii) *the planner announces* k *only when* $k \geq \underline{k}$ *and while announcing* k *he also suggests a per-head contribution* $g^S = G_{min}/k$*, if the revealing equilibrium is the unique equilibrium of* Γ.[21]

Thus, the extension of Γ to Γ^+ by including a suggested per-head contribution g^S into the planner's strategy selects the planner's most preferred equilibrium as the unique induced equilibrium of Γ^+.[22] In this induced equilibrium, the two components of the planner's strategy, the decision on whether to announce the number of players and the decision on the level of suggested per-head contributions, are interrelated in an interesting way: The suggested per-head contribution is most instrumental when it is optimal to suppress the number of players, more precisely, when the original game Γ has a non-revealing, or if not, partially revealing equilibria. Suggesting a per-head contribution when the number of players is announced has no impact, in the sense that the same (symmetric) equilibrium will result whether or not the planner suggests what each player should contribute. This is purely a consequence of uniqueness of the symmetric contributions equilibrium when k is announced. Thus, Proposition 3 implies that when potential contributors are almost identical, credible planners should either announce the number of players or suggest a per-head contribution but not both (for announcing both does not give any extra payoff and yet may involve some administrative costs), if their objective is to collect maximum funds for the public project. Of course, with quite heterogeneous potential contributors, the suggestion of a per-head contribution may induce the contributors to focus on the symmetric equilibrium.

5 Concluding Remarks

The fund-raising literature has pointed out several important strategic aspects in direct contribution public good provision schemes. We further confirm this strategic view of fund-raising by showing how the decision by the fund-raiser to suitably announce (or not announce) the number of players

[21] If $k < \underline{k}$, any per-head contribution suggestion is as good as any other as all players contribute zero.

[22] Uniqueness follows from the fact that Γ has a finite number of potential equilibria and completeness of the planner's ranking of the corresponding outcomes according to the level of total contributions.

and request a minimal contribution from each participant can influence the players' contribution decisions and mitigate the free-rider problem. We chose a non-altruistic model of donations for our analysis. The main intuitions behind our results should remain valid for additional motivations of donations as well, such as warm-glow (Andreoni, 1989) and prestige (Harbaugh, 1998).

We made several assumptions in our analysis. The assumptions of symmetry of contributions equilibria, tie-breaking for the planner's announcement strategy, quasi-linear preferences of potential contributors, all serve to simplify the analysis with no qualitative impact on our results. Because we assumed symmetric potential contributors, the announcement of the information about their number makes the contributions game one of complete information. In this context, the assumption that the fund-raiser's announcement is truthful though he can keep silent and conceal his private information is important. While the assumption is descriptive of the behavior of many major charitable organizations, it can be called into question for newly established fund-raising organizations who lack a history of successful achievements. If we allow the fund-raiser to lie to the public and misrepresent the number of potential contributors, then the non-revealing equilibrium outcome becomes the most plausible equilibrium outcome of our game: the contributors determine their contributions without taking into consideration the fund-raiser's announcement, as if they received no information. However, there may exist a host of other types of equilibria, depending on the beliefs of the contributors. We leave it for future research to investigate the case of nontruthful announcements.

Appendix

Proof of Proposition 1. Suppose that Γ has an equilibrium such that for some $\mathcal{A} \subset \{1, 2, \ldots, N\}$, with $\mathcal{A}$ containing at least two elements and at most $N-2$ elements, the planner reveals k if $k \in \mathcal{A}$ and suppresses k if $k \in \mathcal{C}$. By construction, $\mathcal{C}$ is nonempty and has at least two elements. Now order the elements of $\mathcal{C}$ from the lowest to the highest, $k_1, k_2, \ldots, \hat{k}$. Given the belief system $\mu(\cdot)$ consistent with this strategy of the planner, the symmetric contribution of each player, $g^*(\emptyset)$, is determined by the first-order condition[23]:

$$\mu(k_1)v'(k_1 g^*(\emptyset)) + \mu(k_2)v'(k_2 g^*(\emptyset)) + \ldots + \mu(\hat{k})v'(\hat{k}g^*(\emptyset)) = v'(\overline{G}).^{24}$$

[23]The first-order principle applies because of the convexity of the production technology.

[24]Our assumption 1 that players always play a *symmetric* equilibrium, whenever they

Since $k_1 < k_2 < \ldots < \hat{k}$, it must be that

$$k_1 g^*(\emptyset) < \overline{G}, \text{ and } \hat{k} g^*(\emptyset) > \overline{G}.$$

But if $k = k_1$, the planner prefers revealing k, so that each player will increase his contribution from $g^*(\emptyset)$ to $g^*(k_1)$ according to

$$v'(k_1 g^*(k_1)) = v'(\overline{G}),$$

and aggregate contribution equals $\overline{G} > k_1 g^*(\emptyset)$. This contradicts the hypothesis that the planner will suppress any $k \in \mathcal{C}$.

Now suppose Γ has an equilibrium in which $\mathcal{C} = \emptyset$ and let $\{\hat{\mu}(n)\}_{n=1}^N \gg 0$ be the players' strictly positive belief system when they receive no announcement (which would be off the proposed equilibrium path). By sequential rationality, the symmetric equilibrium contributions $\hat{g}$ will satisfy the first-order condition

$$\hat{\mu}(1)v'(\hat{g}) + \ldots + \hat{\mu}(N)v'(N\hat{g}) = v'(\overline{G}) = 1.$$

Then it must be that $N\hat{g} > \overline{G}$, which implies that the planner will deviate to not announcing $k = N$. Thus, $\mathcal{C} \neq \emptyset$ in equilibrium.

We already proved that $\mathcal{C}$ cannot be empty, nor can it have two or more elements, which imply that $\mathcal{C}$ is a singleton. We now claim that $\mathcal{C} = \{N\}$. To see this, suppose on the contrary that there is an equilibrium in which $\mathcal{C} = \{n\}$ and $n < N$. Then, if no announcement is received, by Bayes' rule $\mu(n) = 1$ and $\mu(k) = 0$ for $k \neq n$, and each player will contribute $g^*(n)$. Given this, the planner will deviate to $a(k) = \emptyset$ if $k > n$ which upsets the equilibrium.

Thus, the only possibility left is $a(k) = k$ if $k \in \{1, \ldots, N-1\}$ and $a(N) = \emptyset$, which we argue is the unique equilibrium strategy for the planner. The non-announcement of $k = N$ will, however, be correctly inferred by the players. Although total contribution will be exactly the same whether the planner announces $k = N$ or not, by the (tie-breaking) Assumption 2 the planner will not announce $k = N$. **Q.E.D.**

need to make a choice between a symmetric equilibrium and any asymmetric equilibrium with the same aggregate contributions, is rather harmless. Here, what is important is that the *expected* marginal benefit (expectation taken over different levels of public goods corresponding to different number of players) of a player equals his marginal benefit at the public good level $\overline{G}$, which, it can be checked, will also be true for any asymmetric contribution equilibrium.

Proof of Proposition 2. To show that a revealing equilibrium always exists is straightforward: Given the players' contribution strategies as specified, the planner can do no better than revealing all $k \geq \underline{k}$ and suppressing all $k < \underline{k}$. Also, given the planner's announcement strategy, the players' contribution strategies form a symmetric Bayesian Nash equilibrium. The players' beliefs about k following the announcements (or no announcement) by the planner are correct in equilibrium.

To derive plausible conditions under which a partially revealing equilibrium exists, we first establish the following claim.

Claim 1. In any (partially revealing) equilibrium with $\mathcal{C} \neq \emptyset$, $\mathcal{A} \neq \emptyset$ and $\tilde{g}(\emptyset) > 0$, the set of announced k's is of the form $\mathcal{A} = \{\underline{k}, \ldots, \overline{k}\}$ where $1 < \underline{k} \leq \overline{k} < N$ and $(\overline{k}+1)\tilde{g}(\emptyset) = G_{min}$.

Proof. Any partially revealing equilibrium will have two continuation games according to whether k is announced, or k is suppressed. In any continuation equilibria following the announcement of k, the players' optimal contribution strategies are straightforward: $\tilde{g}(k) = 0$ if $k < \underline{k}$; $\tilde{g}(k) = G_{min}/k$ if $k \geq \underline{k}$.

We now show that in any equilibrium with $\mathcal{C} \neq \emptyset$, $\mathcal{A} \neq \emptyset$ and $\tilde{g}(\emptyset) > 0$, there exists $\overline{k} < N$ such that $(\overline{k}+1)\tilde{g}(\emptyset) = G_{min}$. Note that $k\tilde{g}(\emptyset) < G_{min}$ for all $k = 1, \ldots, N$ would imply $\tilde{g}(\emptyset) = 0$, contradicting the assumption $\tilde{g}(\emptyset) > 0$. Thus, there exists k such that $k\tilde{g}(\emptyset) \geq G_{min}$. Let $\overline{k}+1$ be the smallest such k. We claim that in any equilibrium with $\tilde{g}(\emptyset) > 0$, $(\overline{k}+1)\tilde{g}(\emptyset) = G_{min}$. Suppose on the contrary that $(\overline{k}+1)\tilde{g}(\emptyset) > G_{min}$, thus, $r\tilde{g}(\emptyset) < G_{min}$ for $r < \overline{k}+1$. Given the strategy $\tilde{g}(\emptyset) > 0$, under no announcement the expected payoff of a player is written as

$$V(\emptyset) = \sum_{k=1}^{\underline{k}-1} \mu(k)v(0) + \sum_{k=\overline{k}+1}^{N} \mu(k)v(k\tilde{g}(\emptyset)) + m_i - \tilde{g}(\emptyset),$$

where $\mu(k)$'s are derived using Bayes' rule and $k\tilde{g}(\emptyset) < G_{min}$ for $k < \overline{k}+1$, thus, $v(k\tilde{g}(\emptyset)) = v(0)$. Since $\sum_{k \geq \overline{k}+1} \mu(k)v'(k\tilde{g}(\emptyset)) < 1 (= v'(\overline{G}))$ and $(\overline{k}+1)\hat{g}(\emptyset) > G_{min}$, any player can unilaterally deviate to $\tilde{g}(\emptyset) - \epsilon$, ϵ arbitrarily small and positive, to increase his individual payoff above $V(\emptyset)$. This contradicts the assumption that $\tilde{g}(\emptyset)$ is an equilibrium strategy. Thus, $(\overline{k}+1)\tilde{g}(\emptyset) = G_{min}$. We next show that in any partially revealing equilibrium where $\mathcal{A} \neq \emptyset$, the planner's strategy must generate a set of announced k's of the form $\mathcal{A} = \{\underline{k}, \ldots, \overline{k}\}$ where $1 < \underline{k} \leq \overline{k} < N$. It is not optimal to announce $k \geq \overline{k}+1$ for this can only decrease total contributions from $k\tilde{g}(\emptyset)$ to

G_{min}. Suppressing any $k \in \mathcal{A} = \{\underline{k}, \ldots, \overline{k}\}$ yields contributions $k\tilde{g}(\emptyset)$ which is strictly less than G_{min}, while announcing $k < \underline{k}$ yields zero contribution instead of the positive amount $k\tilde{g}(\emptyset)$ $(< G_{min})$. Claim 2 shows that $\overline{k} = N$ is not compatible with $\tilde{g}(\emptyset) > 0$. Finally, $\underline{k} \leq \overline{k}$ ensures that $\mathcal{A}$ is nonempty. ||

The following claim is easy to check.

Claim 2. In any equilibrium, $\tilde{g}(\emptyset) = 0$ if and only if $\tilde{a}(k) = k$ for all $k \geq \underline{k}$.

Armed with the result in Claim 1 concerning the structure of the set $\mathcal{A}$ whenever $\mathcal{A} \neq \emptyset$, $\mathcal{C} \neq \emptyset$ and $\tilde{g}(\emptyset) > 0$, we focus below on the conditions for a partially revealing equilibrium where symmetric contributions are positive under no announcement. Consider first deviations to $g_i < \tilde{g}(\emptyset)$ by player i, given the planner's announcement strategy, the corresponding beliefs $\mu(\cdot)$ and the other players' contributions $\tilde{g}(\emptyset)$. If player i deviates as above, it will take at least $\overline{k} + 1$ *other* players, each contributing $\tilde{g}(\emptyset)$, to meet the threshold contributions G_{min}. Clearly the best deviation strategy among all $g_i < \tilde{g}(\emptyset)$ is $g_i = 0$, which yields the expected payoff

$$V^0 = \left(\sum_{k=1}^{\underline{k}-1} \mu(k) + \mu(\overline{k}+1) \right) v(0) + \sum_{k=\overline{k}+1}^{N-1} \mu(k+1) v(k\tilde{g}(\emptyset)) + m_i.$$

Thus

$$V(\emptyset) \geq V^0 \tag{5}$$

must hold for the prescribed strategies to constitute an equilibrium. Consider now a deviation to $g_i > \tilde{g}(\emptyset)$. Such a deviation, if not large enough, increases the potential size of the public good without affecting the probability of its positive supply. But this would not be a beneficial deviation, as shown by the marginal evaluation in the proof of Claim 1. If the deviation to $g_i > \tilde{g}(\emptyset)$ is large enough, it can increase both the probability and the potential size of a positive supply. For instance, deviating to $g^s = G_{min} - (s-1)\tilde{g}(\emptyset)$ for $s < \underline{k}$ will ensure the supply of G_{min} with s players, including the deviator. This deviation yields the expected payoff

$$V^s = \sum_{k=1}^{s-1} \mu(k) v(0) + \sum_{k \in \mathcal{C}, k \geq s} \mu(k) v((k-1)\tilde{g}(\emptyset) + g^s) + m_i - g^s.$$

In equilibrium, we require

$$V(\emptyset) \geq V^s, \quad \text{for all } s < \underline{k}. \tag{6}$$

The planner's strategy of announcing all $k \in \{\underline{k}, \ldots, \overline{k}\}$ and suppressing k otherwise is clearly optimal given the above strategy of the players. As for the determination of $\overline{k}$ and the associated contribution level $\tilde{g}(\emptyset)$, there are only a finite number of choices for $\overline{k}$. An exhaustive verification of conditions (5) and (6) will establish whether a particular pair $(\overline{k}, \tilde{g}(\emptyset))$, where $\tilde{g}(\emptyset) = G_{min}/(\overline{k}+1)$ and $\overline{k} < N$, is compatible with (5) and (6).

Finally, we consider below a non-revealing equilibrium in which the planner suppresses all k's.

Claim 3. In any equilibrium in which $\mathcal{A} = \emptyset$ (thus, $\mathcal{C} = \{1, \ldots, N\}$), $\tilde{g}(\emptyset) > 0$ and there exists $\hat{k} \leq \underline{k}$ such that $\hat{k}\tilde{g}(\emptyset) = G_{min}$.

Proof. $\tilde{g}(\emptyset) = 0$ is clearly not compatible with $\mathcal{A} = \emptyset$; the planner would announce $k \geq \underline{k}$ and collect G_{min}. Thus, $\tilde{g}(\emptyset) > 0$ in any equilibrium in which $\mathcal{A} = \emptyset$. The arguments in the proof of Claim 1 (that show the existence of $\overline{k}+1$) can be applied to show that there must exist $\hat{k}$ such that $\hat{k}\tilde{g}(\emptyset) = G_{min}$. If $\hat{k} > \underline{k}$, which means $\underline{k}\tilde{g}(\emptyset) < G_{min}$, the planner would announce $\underline{k}$ and collect G_{min}, contradicting the assumption that $\mathcal{A} = \emptyset$. ||

Given the strategy $a(k) = \emptyset$ for all $k \in \{1, \ldots, N\} = \mathcal{C}$, we have $\mu(k) = p(k)$ for all k. Thus, $\tilde{g}(\emptyset) > 0$ will be part of a non-revealing equilibrium if

$$\begin{aligned} \hat{V}(\emptyset) &\equiv \sum_{k=1}^{\hat{k}-1} \mu(k)v(0) + \sum_{k=\hat{k}}^{N} \mu(k)v(k\tilde{g}(\emptyset)) + m_i - \tilde{g}(\emptyset) \\ &\geq \sum_{k=1}^{\hat{k}} \mu(k)v(0) + \sum_{k=\hat{k}+1}^{N} \mu(k)v((k-1)\tilde{g}(\emptyset)) + m_i \qquad (7) \end{aligned}$$

(which is the analogue of (5)), and, for all $s < \hat{k}$,

$$\hat{V}(\emptyset) \geq \sum_{k=1}^{s-1} \mu(k)v(0) + \sum_{k \geq s, k \in \mathcal{C}}^{N} \mu(k)v((k-1)\tilde{g}(\emptyset) + \hat{g}^s) + m_i - \hat{g}^s, \qquad (8)$$

where $\hat{g}^s = G_{min} - (s-1)\tilde{g}(\emptyset)$. Condition (8) is the analogue of (6) in a non-revealing equilibrium: it states that an individual deviation to $\hat{g}^s$ to meet the threshold G_{min} for $s < \hat{k}$ players is not beneficial. The verification of the conditions (7) and (8) to find if an equilibrium pair of $(\hat{k}, \tilde{g}(\emptyset))$ exists, are straightforward. **Q.E.D.**

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