The Term Structure of Real Interest Rates: Theory and Evidence from the U.K. Index-Linked Bonds*

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Abstract

This paper argues that a simple general equilibrium model can explain two of the most persistent term structure puzzles. First, Donaldson, Johnsen, and Mehra (1990) show that while in the U.S. nominal term structure the interest rates are pro-cyclical and term spreads counter-cyclical the stochastic growth model predicts that the interest rates are counter-cyclical and term spreads pro-cyclical. The resolution of this puzzle is simple. Using the data on the U.K. index-linked bonds, I show that during the sample period 1984:1–1995:8 the (ex-ante) real interest rates were counter-cyclical and term spreads pro-cyclical. Second, according to Backus, Gregory, and Zin (1989) a complete markets model can account for neither the sign nor the magnitude of average risk premiums in forward prices. This paper applies recent research by Alvarez and Jermann (1999ab) to the term premium puzzle. It is shown that the model

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produces a large risk premium with the correct sign, and unlike the complete markets model can generate enough variation in the risk premia to account for the rejections of the expectations hypothesis. In addition, when the model is calibrated to the U.K. aggregate and household data, the regressions of future spot rates and consumption growth on the term spread behave in a similar manner both in simulated and in actual data.

Keywords: Business Cycles, Term Structure of Interest Rates, General Equilibrium, Risk Premia.

1 Introduction

1.1 Background

One of the oldest problems in the economic theory is the interpretation of the term structure of interest rates. It has been long recognized that the term structure of interest rates conveys information about economic agents' expectations about future interest rates, inflation rates, and exchange rates. Actually, it is widely seen that the term structure is *the best* source of information about the economic agents' inflation expectations for one to four years ahead.[1]

Since it is usually recognized that the monetary policy can only have effect with "long and variable lags" as Friedman (1968) put it, the term structure is invaluable source of information for the monetary authorities.[2] Moreover, empirical studies[3] indicate that the term structure predicts consumption growth better than vector autoregressions or leading commercial econometric models.

The empirical success above is, unfortunately, diminished by the fact that currently available models in the literature do not seem to capture neither the basic quantitative nor qualitative features of the term structure. First, Donaldson, Johnsen, and Mehra (1990) show that while in the U.S. nominal term structure the interest rates are pro-cyclical and term spreads counter-cyclical the stochastic growth predicts that the interest rates are counter-cyclical and term spreads pro-cyclical. Therefore, while term structure predicts consumption growth both in simulated and in actual data, the coefficients in the prediction regressions have opposite signs. I call this "term spread puzzle."

Second, with risk-averse agents the term structure contains besides the expectations also the term premium. In order for the policy makers to extract information about the market expectations from the term structure they need to have a general idea about the sign and the magnitude of the term premium. But as Söderlind and Svensson (1997) note in their review

> "We have no direct measurement of this (potentially) time-varying covariance [term premium], and even ex post data is of limited use since the stochastic discount factor is not observable. It has unfortunately proved to be very hard to explain (U.S. ex post) term premia by either utility based asset pricing models or various proxies for risk."

I call this "term premium puzzle."

[1] See, e.g., Fama (1975, 1990), Mishkin (1981, 1990a, 1992) for studies about the inflation expectations and the term structure of interest rates using U.S. data. Mishkin (1991) and Jorion and Mishkin (1991) use international data. Abken (1993) and Blough (1994) provide nice surveys of the literature.

[2] Svensson (1994ab) and Söderlind and Svensson (1997) have excellent discussions about monetary policy and the role of the term structure of interest rates as a source of information.

[3] See, e.g., Harvey (1988), Chen (1991), and Estrella and Hardouvelis (1991).

The objective of this paper is to examine how much of the behavior of the term structure a simple general equilibrium model can explain. This paper concentrates on the term structure of real interest rates. The question of how to explain the behavior of the term structure of nominal interest rates and its relationship with the real term structure is left for further research.

1.2 The Term Spread Puzzle

Fama (1990) reports that

> "A stylized fact about the term structure is that interest rates are pro-cyclical. (...) [I]n every business cycle of the 1952–1988 period the one-year spot rate is lower at the business trough than at the preceding or following peak. (...) Another stylized fact is that long rates rise less than short rates during business expansions and fall less during contractions. Thus spreads of long-term over short-term yields are countercyclical. (...) [I]n every business cycle of the 1952–1988 period the five-year yield spread (the five-year yield less the one-year spot rate) is higher at the business trough than at the preceding or following peak."

Notice that this statement applies to the term structure of nominal interest rates.

Donaldson, Johnsen, and Mehra (1990) report that in a stochastic growth model with full depreciation the term structure of (ex-ante) real interest rates is rising at the top of the cycle and falling at the bottom of the cycle. In addition, at the top of the cycle the term structure lies uniformly below the term structure at the bottom of the cycle.

The economic intuition for the behavior of the interest rates in economic models is straight forward. At the top of the cycle the aggregate and individual consumption are expected to be, on average, lower in the future and thereby the agents will want to save more thereby driving the interest rates down. At the bottom of the cycle the aggregate and the individual consumption are expected to be higher in the future and the agents, consequently, have less need to save and push the interest rates up.

Donaldson, Johnsen, and Mehra justify the comparison of their theoretical results concerning the real term structure with the empirical results concerning the nominal term structure with two arguments. First, in the empirical literature (e.g., Mishkin 1981, 1990a, 1992) the real and the nominal term structure move in tandem. Second, the results for the nominal and the real term structure in the theoretical economy by Backus, Gregory, and Zin (1989) were qualitatively very similar. Both of these arguments have a flaw. With few exceptions (mentioned below), the empirical literature had used ex-post real term structure derived from the Fisher hypothesis. In addition, Labadie (1994) shows that in a monetary endowment economy the results concerning the shape of the term structure depend crucially on whether the real GDP is assumed to be trend-stationary or difference-stationary. Vigneron (1999) shows that the same is true in a real production economy, and, moreover, the degree of depreciation in capital affects the results.

The innovation in this paper is to use the data on the U.K. index-linked bonds to provide a measure of ex-ante real interest rates and to define business cycle peaks and troughs using two alternative criteria in order to investigate the robustness of results concerning the cyclical behavior of the term structure. The empirical results are consistent with earlier theoretical results by Donaldson, Johnsen, and Mehra.

1.3 The Term Premium Puzzle

The empirical research on the term structure of interest rates has concentrated on the (pure) expectations hypothesis. That is, the question has been if forward rates are unbiased predictors of future spot rates. The most popular way to test the hypothesis has been running a linear regression (error term omitted)

$$r_{t+1} - r_t = a + b(f_t - r_t),$$

where r_t is the one-period spot rate at time t and f_t is the one-period-ahead forward rate at time t. The pure expectations hypothesis implies that $a = 0$ and $b = 1$. Rejection of the first restriction, $a = 0$, gives the expectations hypothesis: term premium is nonzero but constant.

By and large the literature rejects both restrictions.[4] Rejection of the second restriction, $b = 1$, requires, under the alternative, a risk premium that varies through time and is correlated with the forward premium, $f_t - r_t$. Most studies—e.g., Fama and Bliss (1987) and Fama and French (1989)—take this to indicate the existence of time-varying risk premium. Therefore, it is interesting to ask if there are models which are capable of generating similar risk premiums to the ones observed in the real time series.

This is the question in Backus, Gregory, and Zin (1989). They use a complete markets model and their answer is that the model can account for neither the sign nor the magnitude of average risk premiums in forward prices and holding-period returns. Similar puzzles have been obtained for equity premiums by Mehra and Prescott (1985) and for holding-period yields by Grossman, Melino, and Shiller (1987).

Mehra and Prescott (1985) speculate that the most promising way to resolve the equity premium puzzle is introducing features that make certain types of intertemporal trades among agents infeasible. Usually this has meant that the markets are *exogenously incomplete*. The agents in the economy are allowed to trade only in certain types of assets, and the set of available assets is exogenously predetermined.

Heaton and Lucas (1992) use a three-period incomplete markets model to address the term premium puzzle. Their answer is that "uninsurable income shocks may help explain one of the more persistent term structure puzzles" but "the question remains whether the prediction of a relatively large forward premium will obtain in a long horizon model."

Alvarez and Jermann (1999ab) study the asset pricing implications of an endowment economy when agents can default on contracts. They show how endogenously determined solvency constraints, which prevent the agent from defaulting from his or her own contracts, help explain the equity premium and the risk free rate puzzles. For the purpose of dynamic asset pricing, this framework has three advantages over the standard incomplete markets approach described above.

First, allocations do not depend on a particular arbitrary set that is considered to be available. Second, the markets are complete and hence any security can be priced. This is particularly important when one wants to address the questions related to the term structure of interest rates. Finally, finding the solution to the incomplete markets problem involves solving a very difficult

[4] The literature is huge. Useful surveys are provided by Melino (1988), Shiller (1990), Mishkin (1990b), and Campbell, Lo, and MacKinlay (1997). The most important individual studies are probably Shiller (1979), Shiller, Campbell, and Schoenholtz (1983), Fama (1984, 1990), Fama and Bliss (1987), Froot (1989), Campbell and Shiller (1991), and Campbell (1995).

fixed point problem. In contrast, solving the model in Alvarez and Jermann (1996) can be very fast and easy to implement.

This paper applies the model of Alvarez and Jermann to the term premium puzzle. It is shown that the model produces a large risk premium with the correct sign, and unlike the complete markets model can generate enough variation in the risk premia to account for the rejections of the expectations hypothesis. In addition, when the model is calibrated to the U.K. data, the regressions of future spot rates and consumption growth on the term spread behave in a similar manner both in simulated and in actual data.

1.4 Outline of the Paper

The rest of the paper is organized as follows. Section 2 presents the used data, and analyzes the cyclical behavior of the term structure of ex-ante real interest rates. Section 3 presents the basic features of the model by Alvarez and Jermann. For details of the model the reader is referred to their paper. Section 4 explains how the allocations can be numerically solved in a simple way. Section 5 calibrates the model to match the first two moments of the risk-free rate in the U.K. given the data on aggregate consumption and invidual incomes. Section 6 presents the results related to the term structure of interest rates. Section 7 concludes.

2 The Cyclical Behavior of the Term Structure of Ex-Ante Real Interest Rates

2.1 U.K. Index-Linked Bonds

The main complication in analyzing ex-ante real interest rates is that in most economies they simply are unobservable. The most important exception is the U.K. market for index-linked debt. It constitutes a significant proportion of marketable government debt, and its daily turnover is by far the largest in the world. The U.K. market for index-linked debt was introduced in 1981 and by March 1994, it accounted approximately 15% of outstanding issues by market value.[5]

Unfortunately, the British index-linked bonds do not provide correct ex-ante real interest rates. The reason is that the nominal amounts paid by index-linked bonds do not fully compensate the holders for inflation; the used indexation operates with a lag. Both coupon and principal payments are linked to the level of the RPI (Retail Price Index) published in the month seven months prior to the payment date. In addition, the RPI number relates to a specific day in the previous month, so that the effective lag is approximately eight months. The motivation behind this system is that it allows that the nominal value of the next coupon payment is always known, and nominal accrued interest can always be calculated.

Different authors have made different assumptions in order to overcome the "indexation lag problem." Woodard (1990), Deacon and Derry (1994) and Brown and Schaefer (1994) impose Fisher hypothesis or assume that there is no "inflation risk premia." On the other hand, Kandel, Ofer, and Sarig (1996) and Barr and Campbell (1997) assume that different versions of the expectations hypothesis hold. Finally, using the properties of stochastic discount factors, Evans (1998) isolates

[5]See Brown and Schaefer (1996) for more details.

the "indexation lag problem" to a conditional covariance term between the future (maturity less the inflation lag) inflation and nominal bond prices. He then estimates this term using a VAR model and derives the real interest rates.

Evans also tests for the versions of expectations hypothesis used by Kandel, Ofer, and Sarig (1996) and Barr and Campbell (1997) and rejects both versions. Since this paper is mainly concerned on the expectations hypothesis, the methodology of Evans seems best suited for my purposes. The data presented in the next section was provided by him.

2.2 The Term Structures of Nominal and Real Interest Rates

Figures 1 and 2 present the end-of-month observations of the term structures of nominal and real interest rates in the U.K. from January 1984 until August 1995 estimated by Evans (1998). Several observations are immediate. First, while the nominal term structure is mostly upward-sloping, the real term structure contains both upward and downward-sloping pattern, with neither shape dominating. Second, the long-term of the nominal term structure is quite volatile whereas the long-term of the real term structure shows great stability. Third, the short-term of the nominal term structure is less volatile than the short-term of the real term structure. Fourth, both the short-term and the long-term of the nominal term structure seem to be more autocorrelated than corresponding maturities in the real term structure. Finally, the shapes of the real term structure are relatively simple compared to the nominal term structure. Similar observations have been obtained by Woodward (1990) and Brown and Schaefer (1994, 1996).

These observations are confirmed in Figures 3–8. Figures present the average term structures, the standard deviations of the term structures and the annual (12-month) autocorrelation of the term structures of both real and nominal interest rates.

2.3 The Cyclical Properties of the Term Structure

2.3.1 Chapman Business Cycles

In order to analyze the cyclical properties of the term structure, one needs to have definitions for the business cycle and its peak and troughs. Chapman (1997) used the following definition in his study of the cyclical properties of U.S. real term structure.[6] Business cycle expansions are defined as at least two consecutive quarters of positive growth[7] in a three quarter equally-weighted, centered moving average of the real GDP/capita ("output"). Cycle contractions are at least two consecutive quarters of negative growth in the moving average of output. A peak is the last quarter prior to the beginning of a contraction, and trough is the last quarter prior to the beginning of a expansion. Thus defined business cycle is presented in Figure 9 for the U.K. from the first quarter of 1957 until the last quarter of 1997. The quarterly observations on the GDP at 1990 prices and annual observations on the population were obtained from the CD-ROM July 1999 version of the International Monetary Fund's International Financial Statistics. The quarterly population series

[6]Since the U.S. has not had index-linked bonds before May 1996, Chapman (1997) had to estimate the real term structure by predicting expected inflation using univariate time series models. He also assumes that the inflation risk premia are zero.

[7]The growth (rate) is the difference in the logarithm of the series.

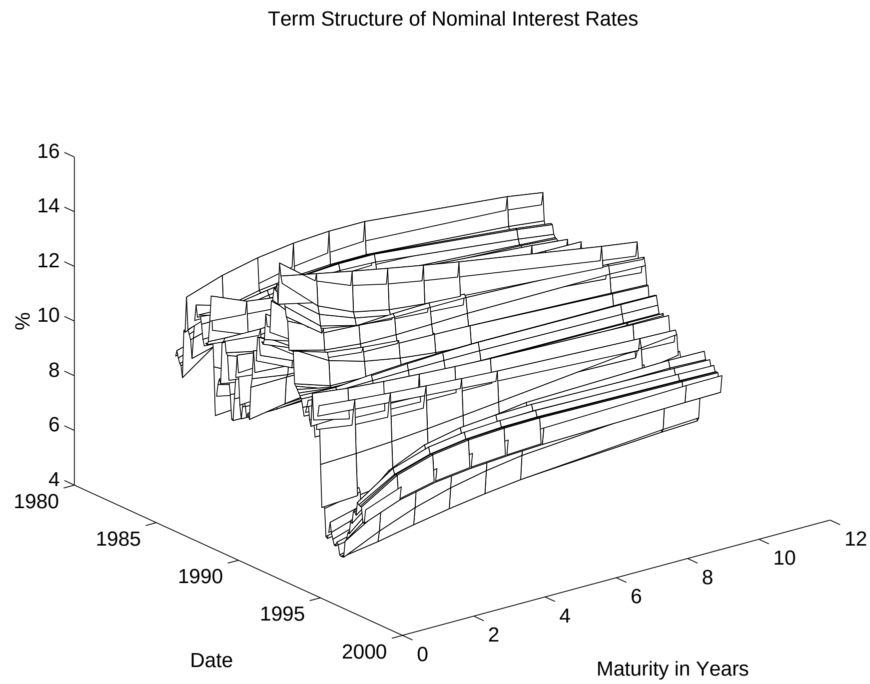

Figure 1: The term structure of nominal interest rates in the U.K. 1984:1–1995:8.

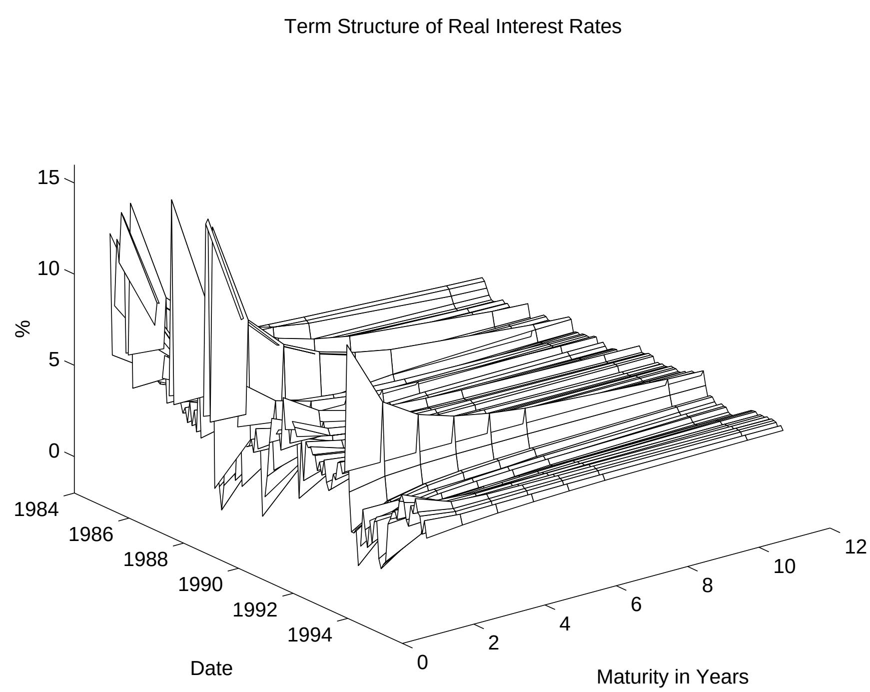

Figure 2: The term structure of real interest rates in the U.K. 1984:1–1995:8.

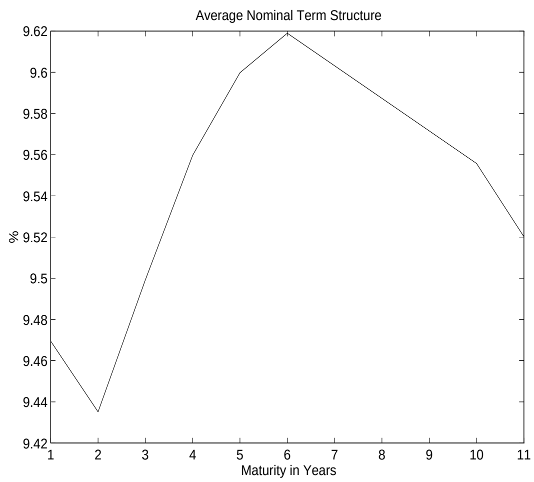

Figure 3: Average term structure of nominal interest rates in the U.K. 1984:1–1995:8.

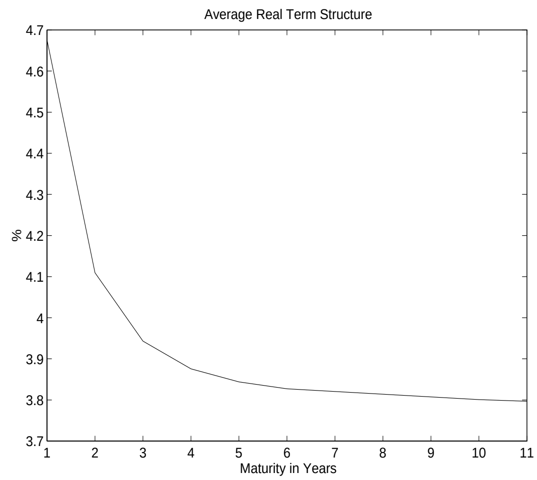

Figure 4: Average term structure of real interest rates in the U.K. 1984:1–1995:8.

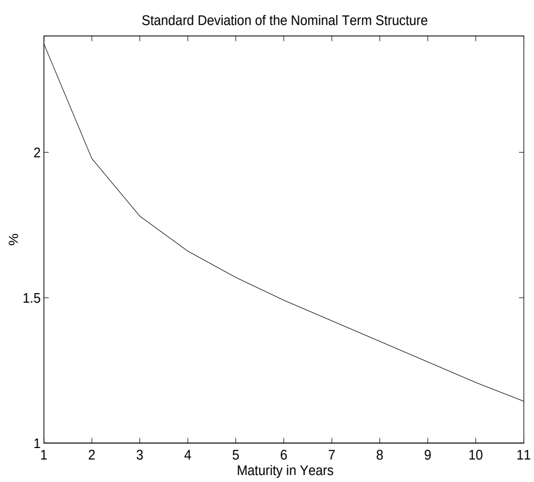

Figure 5: Standard Deviation of the term structure of nominal interest rates in the U.K. 1984:1–1995:8.

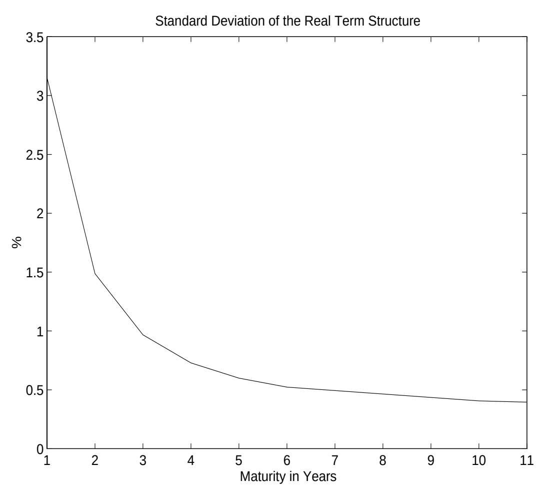

Figure 6: Standard Deviation of the term structure of real interest rates in the U.K. 1984:1–1995:8.

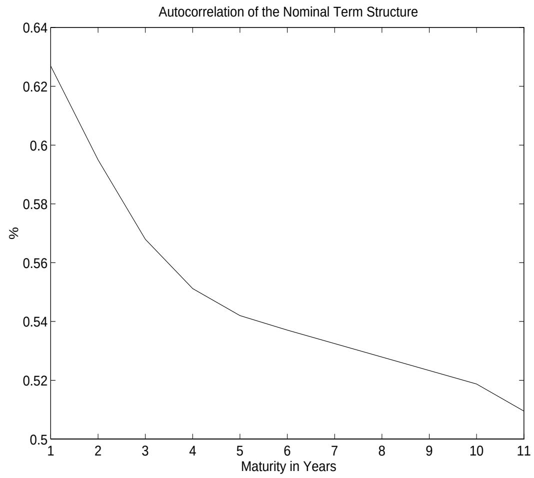

Figure 7: Autocorrelation (annual) of the term structure of nominal interest rates in the U.K. 1984:1–1995:8.

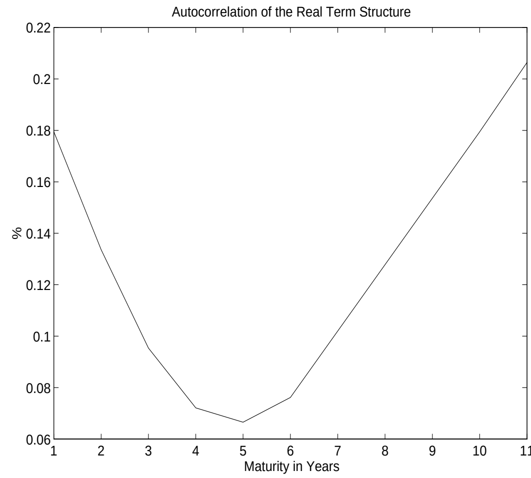

Figure 8: Autocorrelation (annual) of the term structure of real interest rates in the U.K. 1984:1–1995:8.

were constructed by assuming that population grows at constant rate within the year and that the original annual data are as of December 31 of each year.

Figures 10 and 11 show the above-defined business cycle, one-year real and nominal interest rates and the real and nominal term spread (the difference between five-year and one-year interest rates). In the sample there are only one business cycle peak and one business cycle trough. Figures 12 and 13 present the average real and nominal term structures at business cycle peaks and troughs. Peak months were January 1990 to March 1990 and troughs months were April 1992 to June 1992. Figures clearly establish that nominal interest rates are pro-cyclical while real interest rates are counter-cyclical. Recall, Donaldson, Johnsen, and Mehra (1990) report that in a stochastic growth model with full depreciation at the top of the cycle the term structure lies uniformly below the term structure at the bottom of the cycle. On the other hand, all average term structure are down-ward sloping. This may be a result of small sample bias. In the next section, I use a different definition of the business cycle.

2.3.2 Hodrick-Prescott Business Cycles

Following popular literature on real business cycle models, Chapman (1997) also provides a different definition of the business cycle based on Hodrick-Prescott filter. The business cycle is simply defined as the cyclical component of Hodrick-Prescott filtered time series. In addition, I use the same moving average filter as in the previous section. Figures 14 and 15 display both the growth and the cyclical component of Hodrick-Prescott filtered real GDP/capita with the smoothing parameter of 1600. Figures 10 and 11 show the above-defined business cycle, one-year real and nominal interest rates and the real and nominal term spread (the difference between five-year and one-year interest rates). Figures 12 and 13 present the average real and nominal term structures at business cycle peaks and troughs. Peak months were July 1988 to June 1989 and October 1994 to June 1995 and troughs months were July 1984 to June 1985 and all of 1992. Again, Figures clearly establish that

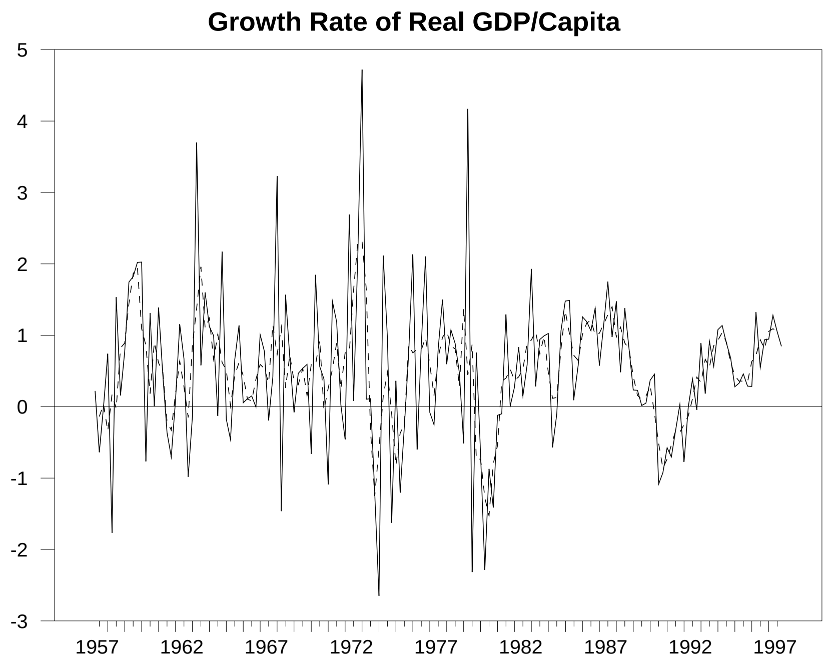

Figure 9: The growth rate of real GDP/capita in the U.K. 1957:1–1997:4 and its moving average (quarterly observations).

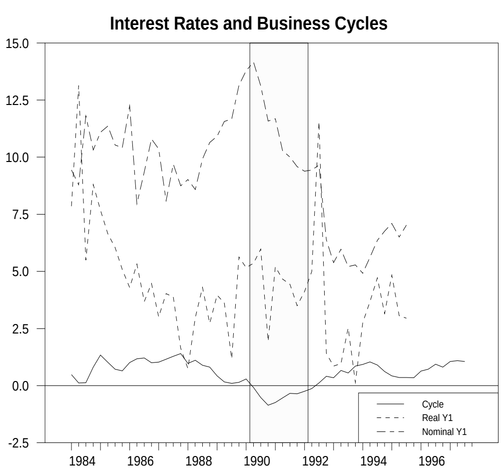

Figure 10: Chapman business cycles and interest rates in the U.K. 1984:1–1995:8.

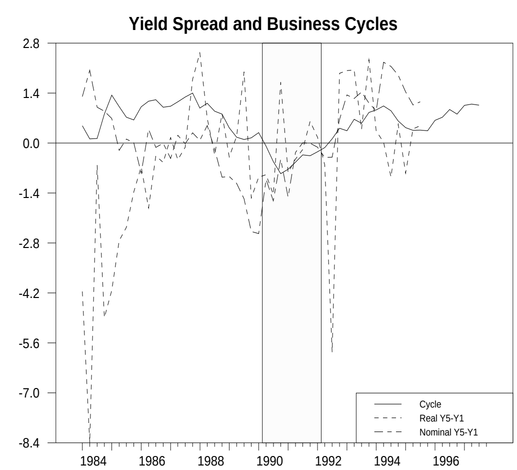

Figure 11: Chapman business cycles and term spread in the U.K. 1984:1–1995:8.

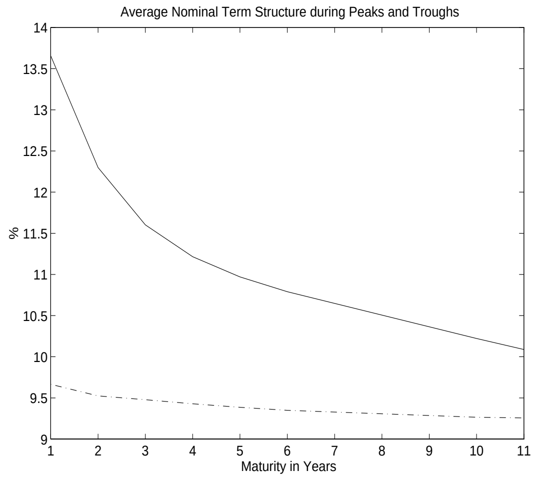

Figure 12: Average nominal term structure during Chapman business cycle peaks (solid line) and troughs.

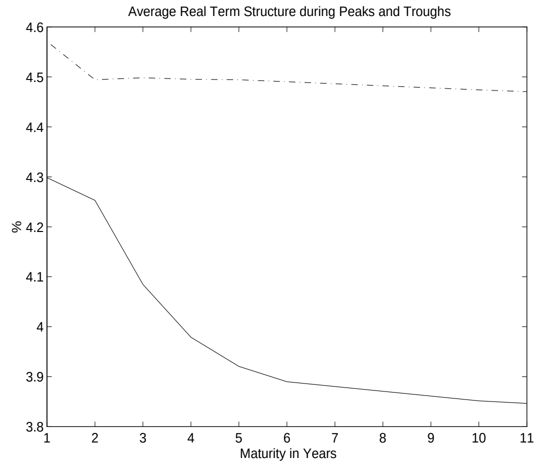

Figure 13: Average real term structure during Chapman business cycle peaks (solid line) and troughs.

real interest rates are counter-cyclical. In addition, they are consistent (with an exception of minor hump at short-term rates during peaks) with the results by Donaldson, Johnsen, and Mehra (1990) who report that in a stochastic growth model with full depreciation the term structure of real interest rates is rising at the top of the cycle and falling at the bottom of the cycle. On the other hand, the behavior of nominal interest rates is consistent neither with the theoretical results of Donaldson, Johnsen, and Mehra nor with the U.S. empirical evidence reported in Fama (1990).

3 The Alvarez-Jermann Model

3.1 The Environment

Alvarez and Jermann (1996, 1999ab) consider a pure exchange economy with two agents. Agents have identical preferences represented by the time-separable expected discounted utility. Their endowments follow a finite-state first-order Markov process. The difference between Alvarez-Jermann economy and the Mehra-Prescott economy is that the agents have an incentive to default on their contracts if honoring their contracts would leave them worse off than they would be in autarchy. This option is prevented by *endogenous participation* or *solvency constraints.*

Let $i = 1, 2$ denote each agent and $\{z_t\}$ denote a finite-state Markov process $z \in Z = \{z_1, \ldots, z_N\}$ with a transition matrix Π. z_t determines the aggregate endowment, e_t, and the individual endowments, $e_{i,t}, i = 1, 2$ as follows

$$e_{t+1} = \lambda(z_{t+1})e_t \quad \text{and} \quad e_{i,t} = \alpha_i(z_t)e_t \quad \text{for } i = 1, 2,$$

where $\lambda(z_{t+1})$ is the growth rate of aggregate endowment from period t to $t+1$ when the state in period $t+1$ is z_{t+1} and $\alpha_i(z_t)$ determines agent i's share of the aggregate endowment when the state in period t is z_t.

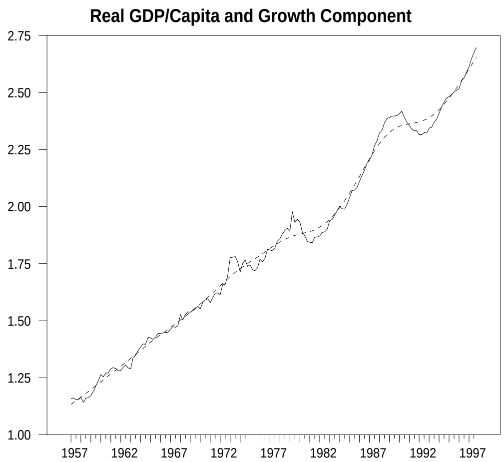

Figure 14: The real GDP/capita in the U.K. 1957:1–1997:4 and its growth component (quarterly observations).

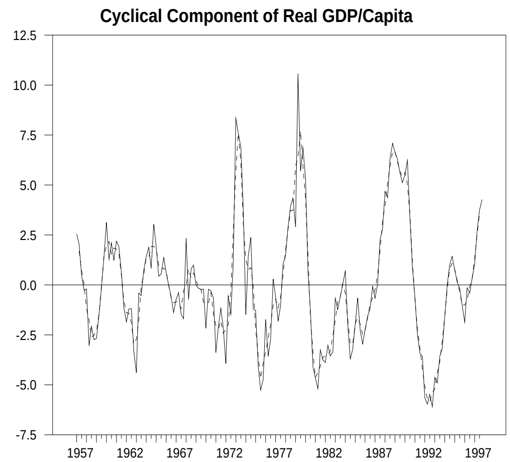

Figure 15: The cyclical component of real GDP/capita in the U.K. 1957:1–1997:4 and its moving average (quarterly observations).

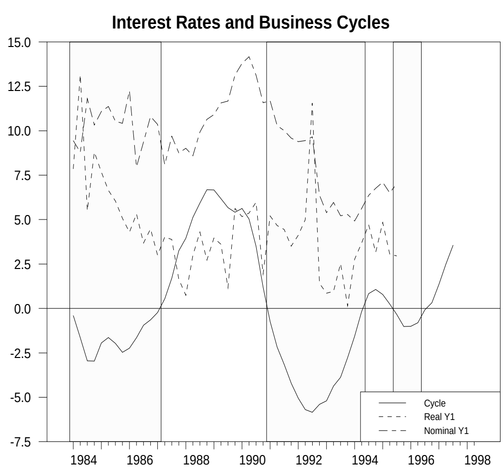

Figure 16: Hodrick-Prescott business cycles and interest rates in the U.K. 1984:1–1995:8.

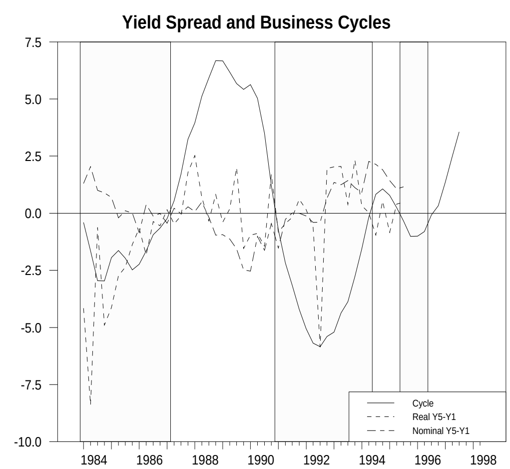

Figure 17: Hodrick-Prescott business cycles and term spread in the U.K. 1984:1–1995:8.

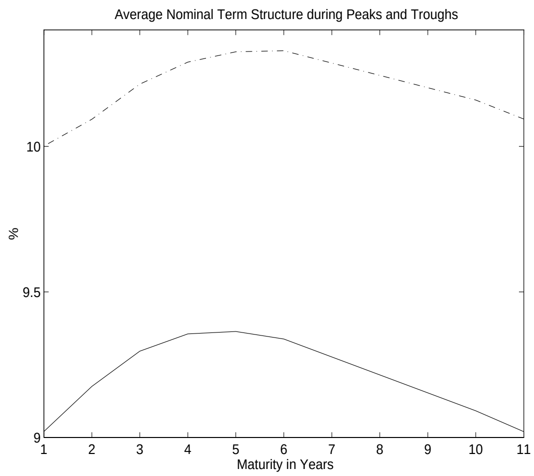

Figure 18: Average nominal term structure during Hodrick-Prescott business cycle peaks (solid line) and troughs.

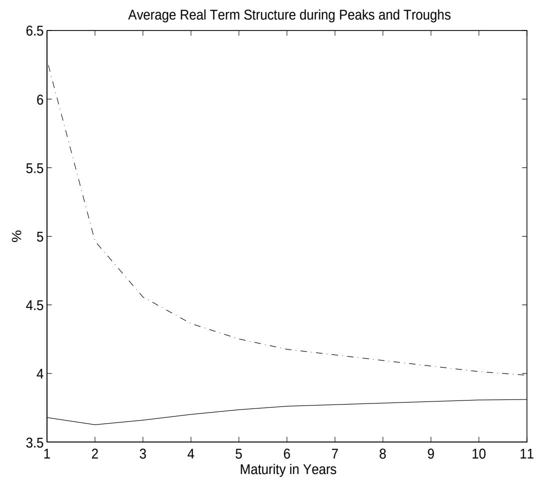

Figure 19: Average real term structure during Hodrick-Prescott business cycle peaks (solid line) and troughs.

Let $z^t = (z_1, \dots, z_t)$ denote the history of z up to period t. The matrix Π determines the conditional probabilities for all histories $\pi(z^t|z_0)$. The households care only about their consumption streams, $\{c\} = \{c_t(z^t) : \forall t \geq 0, z^t \in Z^t\}$, and rank them by their discounted expected utility

$$U(c)(z^t) = \sum_{j=0}^{\infty} \sum_{z^{t+j} \in Z^{t+j}} \beta^j u(c_{t+j}(z^{t+j}))\pi(z^{t+j}|z_t),$$

where $\beta \in (0,1)$ is a constant discount factor and the one-period utility function is of the Constant Relative Risk Aversion type

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad 1 < \gamma < \infty,$$

where γ is the agents' constant coefficient of relative risk-aversion. The participation constraints force the allocations to be such that under no history will the expected utility be lower than the one in autarchy

$$U(c_i)(z^t) \geq U(e_i)(z^t) \quad \forall t \geq 0,\ z^t \in Z^t,\ i = 1, 2.$$

3.2 Constrained Optimal Allocations

The constrained optimal allocations are defined as processes, $\{c_i\}$, $i = 1, 2$, that maximize agent 1's expected utility subject to feasibility and participations constraints at every date and every history given initial promised expected utility to the agent 2. The recursive formulation of the constrained optimal allocations is given by the following functional equation

$$TV(w, z, e) = \max_{c_1, c_2, \{w(z')\}} u(c_1) + \beta \sum_{z' \in Z} V(w(z'), z', e')\pi(z'|z)$$

subject to

$$
\begin{aligned}
c_1 + c_2 &\leq e \\
u(c_2) + \beta \sum_{z' \in Z} w(z')\pi(z'|z) &\geq w \\
V(w(z'), z', e') &\geq U^1(z', e') \quad \forall z' \in Z \qquad (1) \\
w(z') &\geq U^2(z', e') \quad \forall z' \in Z, \qquad (2)
\end{aligned}
$$

where primes denote next period values, $V(w, z, e)$ is the agent 1's value function, w is the promised utility of agent 2, and (1) and (2) are the participation constraints for the agent 1 and agent 2, respectively.

The aggregate endowment is growing over time, but the CRRA utility function implies that the value function, $V(\cdot)$, autarchy values of utility, $U^i(\cdot)$, and the policies, $\{C_1(\cdot), C_2(\cdot), W(\cdot)\}$, satisfy the following homogeneity property

Proposition 1. *For any $y > 0$ and any (w, z, e):*

$$
\begin{aligned}
V(y^{1-\gamma} w, z, ye) &= y^{1-\gamma} V(w, z, e) \\
U^i(z, ye) &= y^{1-\gamma} U^i(z, e) \quad \textit{for } i = 1, 2 \\
C_i(y^{1-\gamma} w, z, ye) &= y C_i(w, z, e) \quad \textit{for } i = 1, 2 \\
W(y^{1-\gamma} w, z, ye) &= y^{1-\gamma} W(w, z, e).
\end{aligned}
$$

Proof. See the proof of Proposition 3.9 in Alvarez and Jermann (1996). ■

Defining a new set of "hat"-variables as follows

$$
\begin{aligned}
u(c) &= e^{1-\gamma} u(\frac{c}{e}) = e^{1-\gamma} u(\hat{c}) \\
U^i(z', e') &= (e')^{1-\gamma} U^i(z', 1) = \hat{U}^i(z', 1) \quad \text{for } i = 1, 2 \\
w &= \frac{e^{1-\gamma}}{e^{1-\gamma}} w = e^{1-\gamma} \hat{w} \\
w' &= \frac{(e')^{1-\gamma}}{(e')^{1-\gamma}} w' = (e')^{1-\gamma} \hat{w}',
\end{aligned}
$$

and using the above proposition in the following way

$$
\begin{aligned}
V(w, z, e) &= V(\frac{e^{1-\gamma}}{e^{1-\gamma}} w, z, \frac{e}{e} e) = e^{1-\gamma} V(\hat{w}, z, 1) \\
V(w', z', e') &= (e')^{1-\gamma} V(\frac{w'}{(e')^{1-\gamma}}, z', 1) = (\lambda(z')e)^{1-\gamma} V(\hat{w}, z, 1),
\end{aligned}
$$

the functional equation can be rewritten with stationary variables as

$$
TV(\hat{w}, z, 1) = \max_{\hat{c}_1, \hat{c}_2, \{\hat{w}(z')\}} u(\hat{c}_1) + \beta \sum_{z' \in Z} V(\hat{w}(z'), z', 1) \lambda(z')^{1-\gamma} \pi(z'|z)
$$

subject to

$$\hat{c}_1 + \hat{c}_2 \leq 1 \tag{3}$$

$$u(\hat{c}_2) + \beta \sum_{z' \in Z} \hat{w}(z')\lambda(z')^{1-\gamma}\pi(z'|z) \geq \hat{w} \tag{4}$$

$$\begin{aligned} V(\hat{w}(z'), z', 1) &\geq \hat{U}^1(z', 1) \quad \forall z' \in Z, \\ \hat{w}(z') &\geq \hat{U}^2(z', 1) \quad \forall z' \in Z \end{aligned} \tag{5}$$

In order to guarantee the above maximization problem is well-defined it is needed to assume that

$$\max_{z \in Z} \left\{ \beta \sum_{z' \in Z} \lambda(z')^{1-\gamma}\pi(z'|z) \right\} < 1.$$

3.3 Asset Pricing

To analyze asset prices, the economy must be decentralized. The definition of equilibrium is a direct extension of the Recursive Competitive Equilibrium by Prescott and Mehra (1980). The markets are complete so that in every state z there are one-period Arrow securities to each next period states of nature z'. However, the solvency constraints prevent the agents holding so much debt in any state that they would like to default on their debt contracts. The solvency constraints affect each state differently as the relative value of autarchy compared to honoring the contract is different in every state. The following three definitions determine the Recursive Competitive Equilibrium for an economy with heterogeneous agents and solvency constraints.

Definition 1. The equilibrium objects *are*

1. *The aggregate state* $(A, z) \in \Re^2 \times Z \equiv \mathcal{A} \times Z$, *where* $A = (A_1, A_2)$ *is the list of each agent's wealth at the beginning of the period.*

2. *Individual state* $(a, A, z) \in \Re \times \Re^2 \times Z \equiv \boldsymbol{A} \times \mathcal{A} \times Z$.

3. *Law of motion for the aggregate state:* $\Psi : \mathcal{A} \times Z \to \mathcal{A} \times Z$ *so that* $\Psi_{z'}(A, z)$ *determines the asset holdings of each agent at the beginning of the next period when the next period's state is* z' *and the current period's state is* (A, z).

4. *Arrow pricing function:* $q : \mathcal{A} \times Z \to \Re^N$ *so that* $q_{z'}(A, z)$ *determines the price of a security that pays one unit of consumption good at the beginning of the next period when the next period's state is* z' *and the current period's state is* (A, z).

5. *State contingent solvency constraints:* $B : \mathcal{A} \times Z \to \mathcal{A}$ *so that* $B^i(A, z)$ *is the minimum asset position agent* i *can take in asset that pays when the state is* (A, z).

Definition 2. The household's problem *given the current state* (a, A, z) *is to maximize the expected utility*

$$H^i(a, A, z) = \max u(c) + \beta \sum_{z' \in Z} H^i(a_{z'}, A_{z'}, z')\pi(z'|z)$$

subject to solvency and budget constraints

$$a_{z'} \geq B^i(A', z') \quad \forall z' \in Z$$
$$\sum_{z' \in Z} q_{z'}(A, z) a_{z'} + c \leq a + e_i(z)$$

and the equilibrium law motion for the aggregate state

$$A_{z'} = \Psi_{z'}(A, z).$$

Let $g^i : \mathcal{A} \times Z \rightarrow \Re \times Z$ be the optimal decision rule of agent i so that $g^i_{z'}(A, z)$ determines the portfolio holdings by agent i of a security that pays one unit of consumption good at the beginning of the next period when the next period's state is z' and the current period's state is (A, z). The equilibrium can now be defined.

Definition 3. The equilibrium *is a collection of objects* $\{\Psi, B, H, g, q\}$ *such that*

1. *Given* Ψ*,* B*, and* q*, the decision rule* $g^i(a, A, z)$ *attains* $H^i(a, A, z)$ $\forall(a, A, z)$*.*
2. *The representative agent is representative:* $g^i_{z'}(A_i, A, z) = \Psi^i_{z'}(A, z)$ $\forall A, z, i$*.*
3. *The solvency constraint prevents default:*

$$a_{z'} \geq B^i(\Psi^i_{z'}(A, z), z') \implies H^i(a_{z'}, \Psi^i_{z'}(A, z), z') \geq U^i(z') \quad \forall A, z, i.$$

4. *When the solvency constraints bind the continuation utility equals autarchy utility*

$$a_{z'} = B^i(\Psi^i_{z'}(A, z), z') \implies H^i(a_{z'}, \Psi^i_{z'}(A, z), z') = U^i(z').$$

The crucial point of the model, as far as asset pricing is concerned, is that the prices of Arrow securities are given by the maximum of the marginal rates of substitution of agent 1 and 2. That is, if the current state is (A, z) the price of an Arrow security that pays one unit of consumption good at the beginning of the next period when the next period's state is z' is given by

$$q_{z'}(A, z) = \max_{i=1,2} \beta \frac{u'(c_{i,t+1})}{u'(c_{i,t})} \pi(z'|z)$$

In sections 5 and 6 I will show how to price stocks and bonds of different maturities using state-contingent Arrow securities as building blocks. The key idea is to build a portfolio of Arrow securities that exactly replicates the payoff structure of the asset in question. The same method can be applied to any asset whose payoffs is a function of z_t.

4 Algorithm

To illustrate how the model fits the data, I will solve the model for the simplest case that produces nonconstant interest rates and equity prices. This is obtained by introducing uncertainty in the growth rate of aggregate endowment while treating the agents in a symmetric fashion. The similar techniques can be applied for more complicated cases.

In particular, following Alvarez and Jermann there will be three exogenous states: z_1, z_3, and z_5.[8] The states z_1 and z_5 are associated with a recession, and following Heaton and Lucas (1996) the recessions are associated with a widening of inequality in earnings. During the boom both agents have the same endowment. The states are ordered so that

$$e_1(z_1) = e_2(z_5) \geq e_1(z_3) = e_2(z_3) \geq e_1(z_5) = e_2(z_1)$$

and the transition matrix Π preserves the symmetry between the agents.

The results in Alvarez and Jermann (1996) indicate that during the recession the allocations do not depend on the past history. However, during the expansion there are two endogenous states: z_2, where $z_t = z_3$ and $z_{t-1} = z_1$, and z_4, where $z_t = z_3$ and $z_{t-1} = z_5$. Moreover, it is possible to solve for the allocations and prices by just solving for at most two simple nonlinear equation systems. The first nonlinear system corresponds to the case where the participation constraints do not bind, and the second system corresponds to the case where the agents agents are constrained when entering the boom period. In addition to the nonlinear equation, there is a set of inequalities that determines which case is valid.

From the feasibility (3) and nonsatiation, it follows the agent 1's consumption is always aggrete endowment less the agent 2's consumption. Hence, both systems have 10 equations in 10 unknowns: the agent 2's consumption and continuation utility in each state. From now on, c_n denotes $\hat{c}_2(z_n)$ and "hats" have been dropped from other variables as well.

In both the unconstrained case and the constrained case five equations are given by (4), during the boom neither agent has a reason to trade: $c_3 = 0.5$, (3) and symmetry imply that $c_1 + c_5 = 1$, and the participation constraint (5) holds with equality when the agent 2 receives the most favorable shock: $w(z_5) = U^2(z_5)$. The two missing equations depend on if the agents are constrained when entering the expansion state. In the unconstrained case $c_2 = c_1$ and $c_4 = c_5$, and in the constrained case $w(z_2) = U^2(z_2)$ and $c_2 + c_4 = 1$.

To solve for the allocations, one needs only to solve for the unconstrained system and check if $w(z_2) \geq U^2(z_2)$. If this is not the case, the solution is given by the constrained case.

5 Calibration

5.1 Consumption and Endowment

5.1.1 Aggregate Consumption

The first step is to calibrate the law of motion for the aggregate endowment so that it matches a few facts about aggregate consumption (in the model aggregate endowment equals aggregate

[8] I am somewhat abusing the notation here. The states z_1, z_3, and z_5 correspond to the Z above, and z_2 and z_4 corresponds to the $\mathcal{A}$ above. The labeling of the states will become obvious soon. From now on, $Z \equiv Z \cup \mathcal{A}$. Also, N is the number of elements in Z and notation such as x_j is a shorthand for $x(z_j)$.

Table 1: Cross-sectional means and standard deviations of the coefficient estimates in the regression (6).

Coefficient	Cross-Sectional Mean	Cross-Sectional Standard Deviation
$\bar{\eta}^i$	-6.4331	5.2376
ρ^i	0.2035	0.6315
σ^i	0.2830	0.2853

consumption). Campbell (1998) reports that in the annual U.K. data between 1891–1995, the average growth rate of aggregate consumption equals 1.443%, the standard deviation of the growth rate of aggregate consumption equals 2.898%, and the first-order autocorrelation equals 0.281. Using the data and definitions from Section 2.3.1 the expansions are 3.8824 times more likely to occur than the recessions. I use these as the moment conditions for the aggregate endowment.

5.1.2 Individual Endowment

The next step is to calibrate the process for the individual endowments. I follow the analysis in Heaton and Lucas (1996) but applied to the U.K. data using the British Household Panel Survey (BHPS) data. The BHPS provides a panel of monthly observations of individual and household income and other variables from September 1990 until January 1998 for more than 5,000 households, making a total of approximately 10,000 individuals.[9]

I use the subset of the panel that has reported positive income in every year since 1991. This makes a total of 2,391 individuals in the panel. For households with more than one member with reported income, I constructed the individual income to be the total household income divided by the number of the members of the household with reported income. The individual annual income dynamics are assumed to follow an AR(1)-process:

$$\log(\eta_t^i) = \bar{\eta}^i + \rho^i \log(\eta_{t-1}^i) + \epsilon_t^i, \tag{6}$$

where $\eta_t^i = e_t^i / \sum_{i=1}^n e_t^i$. Table 1 reports cross-sectional means and standard deviations of the coefficient estimates in the regression (6). The relevant numbers are the first-order autocorrelation coefficient, ρ^i, and the standard deviation of the error term, $\sigma^i = \sqrt{E[(\epsilon_t^i)^2]}$. Unfortunately, it turned out not to be possible to match ρ^i, so I chose to match σ^i.

Heaton and Lucas (1996) estimate the same coefficients using a sample of 860 households in the PSID that have annual income from 1969 to 1984. Table 2 reports cross-sectional means and standard deviations of the coefficient estimates obtained by Heaton and Lucas. The estimated autocorrelation coefficient in the BHPS, 0.2035 , is significantly smaller than same coefficient in the PSID, 0.529. On the other hand, the standard deviation of the error term in the BHPS, 0.2830, is roughly the same as the coefficient in the PSID, 0.251.

[9]For more details of the BHPS, see Taylor (1998).

Table 2: Cross-sectional means and standard deviations of the coefficient estimates in the regression (6) obtained by Heaton and Lucas (1996).

Coefficient	Cross-Sectional Mean	Cross-Sectional Standard Deviation
$\bar{\eta}^i$	-3.354	2.413
ρ^i	0.529	0.332
σ^i	0.251	0.131

5.1.3 Calibrated Values

Thus, the free parameters are e_{11} for the individual incomes

$$e_1 = \begin{bmatrix} e_{11} \\ 0.5 \\ 0.5 \\ 0.5 \\ 1 - e_{11} \end{bmatrix} \quad \text{and} \quad e_2 = \begin{bmatrix} 1 - e_{11} \\ 0.5 \\ 0.5 \\ 0.5 \\ e_{11} \end{bmatrix},$$

λ_1 and λ_2 for the growth rates of aggregate endowment

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_2 \\ \lambda_2 \\ \lambda_1 \end{bmatrix},$$

and π_{11} and π_{22} for the transition matrix

$$\Pi = \begin{bmatrix} \pi_{11} & 1-\pi_{11} & 0 & 0 & 0 \\ (1-\pi_{22})/2 & 0 & \pi_{22} & 0 & (1-\pi_{22})/2 \\ (1-\pi_{22})/2 & 0 & \pi_{22} & 0 & (1-\pi_{22})/2 \\ (1-\pi_{22})/2 & 0 & \pi_{22} & 0 & (1-\pi_{22})/2 \\ 0 & 0 & 0 & 1-\pi_{11} & \pi_{11} \end{bmatrix}.$$

Solving this system of five unknowns in five equations leads to the following endowment vectors and growth rates[10]

$$e_1 = \begin{bmatrix} 0.7706 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.2294 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0.2294 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.7706 \end{bmatrix}, \quad \text{and} \quad \lambda = \begin{bmatrix} 0.9573 \\ 1.0291 \\ 1.0291 \\ 1.0291 \\ 0.9573 \end{bmatrix},$$

[10] Due to highly nonlinear nature of the model, the results are quite sensitive to the used parameter values. Therefore, I will report all the parameter values with four decimal precision.

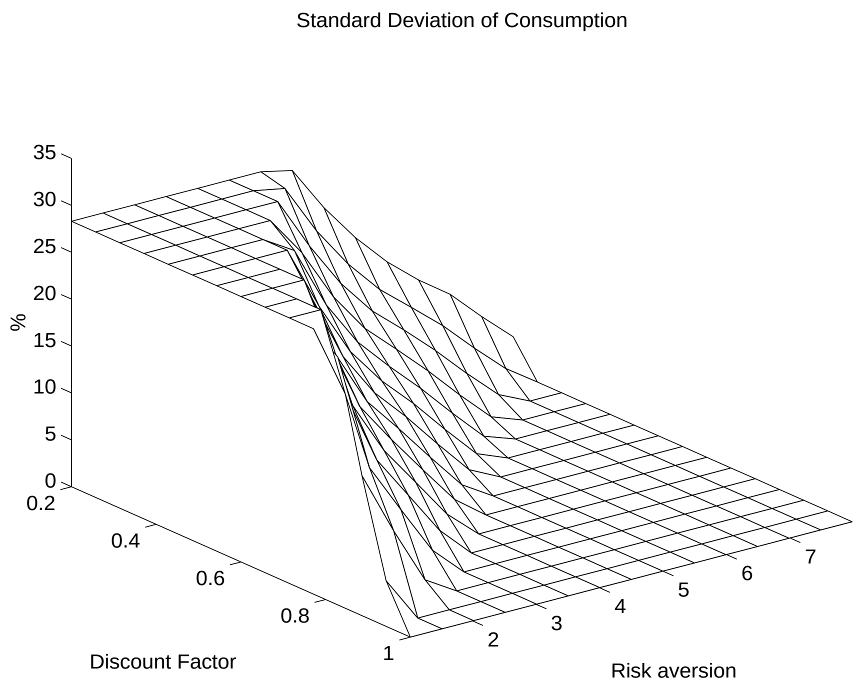

Figure 20: The standard deviation of individual consumption as a function of discount rate and risk aversion.

and the following transition matrix

$$\Pi = \begin{bmatrix} 0.4283 & 0.5717 & 0 & 0 & 0 \\ 0.0736 & 0 & 0.8527 & 0 & 0.0736 \\ 0.0736 & 0 & 0.8527 & 0 & 0.0736 \\ 0.0736 & 0 & 0.8527 & 0 & 0.0736 \\ 0 & 0 & 0 & 0.5717 & 0.4283 \end{bmatrix},$$

which implies the following stationary distribution over the states

$$\pi^* = \pi^* \Pi = \begin{bmatrix} 0.1024 & 0.0586 & 0.6781 & 0.0586 & 0.1024 \end{bmatrix}.$$

The next step is to match the agents' preference parameters, the discount rate, β, and the risk-aversion coefficient, γ, to the key statistics of the asset market data. To illustrate the features of the model, Figure 20 plots the standard deviation of individual consumption, $\text{std}(\ln(\frac{c_i}{\sum_j c_j}))$, as a function of β and γ.

The flat segment in the upper left corner of the figure corresponds to the autarchy and the flat segment in the lower right corner of the figure corresponds to the perfect risk-sharing. The parameter values which are interesting for the purpose of asset pricing are the ones which generate allocations that are between these two extremes. To accomplish this, one has to choose either relatively low risk aversion and relatively high discount factor or relatively high risk aversion and relatively low discount factor. Although I am reporting the numerical results below only for one pair of risk aversion and discount factor, I have obtained qualitatively similar results with other combinations that fall between autarchy and perfect risk-sharing.

5.2 The Risk-Free Rate and the Equity Premium

In this section I return to the earlier notation, which differentiates between "hat", and non-"hat" variables. Recall equation for the one-period Arrow security

$$q_{z'}(z) = \max_{i=1,2} \beta \frac{u'(c_{i,t+1})}{u'(c_{i,t})} \pi(z'|z) \tag{7}$$

and let m_{t+1} denote the *pricing kernel* or the *stochastic discount factor*

$$m_{t+1} \equiv \max_{i=1,2} \beta \frac{u'(c_{i,t+1})}{u'(c_{i,t})}.$$

To price a one-period bond, one composes an equal-weighted portfolio of one-period Arrow securities. The price of an one-period bond, p_t^b, is equal to the price of this portfolio given by (7)

$$p_t^b = \sum_{z^{t+1} \in Z^{t+1}} \max_{i=1,2} \beta \frac{u'(c_{i,t+1})}{u'(c_{i,t})} \pi(z^{t+1}|z_t) = E_t[m_{t+1}],$$

and the risk-free rate, r_t, is given by

$$r_t = \frac{1}{p_t^b} - 1. \tag{8}$$

Stock is a claim to an infinite stream of a consumption good. Hence, the price of equity, p_t^e, is equal to

$$p_t^e = \sum_{j=1}^{\infty} \sum_{z^{t+j} \in Z^{t+j}} q(z^{t+j}|z_t) e_{t+j} = E_t\left[\sum_{j=1}^{\infty} m_{t+j} e_{t+j}\right], \tag{9}$$

where $q(z^{t+j}|z_t)$ is the price of an Arrow security from the state z_t to the state z^{t+j} and is given by

$$q(z^{t+j}|z_t) = \prod_{k=t}^{t+j-1} q_{z_{k+1}}(z_k), \tag{10}$$

and $q_{z_{k+1}}(z_k)$ is given by (7).

To solve for the price of equity, we use a trick similar to the one in Mehra and Prescott. First, using (7), (9), and (10) notice that

$$p_t^e = \sum_{z^{t+1} \in Z^{t+1}} \max_{i=1,2} \beta \frac{u'(c_{i,t+1})}{u'(c_{i,t})} (e_{t+1} + p_{t+1}^e) \pi(z^{t+1}|z_t) = E_t[m_{t+1}(e_{t+1} + p_{t+1}^e)]. \tag{11}$$

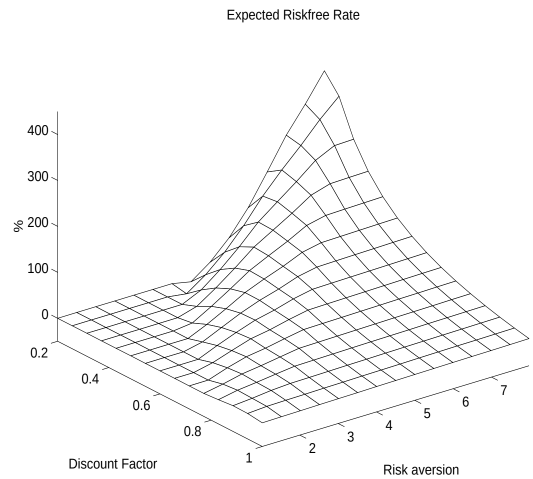

Figure 21: The expected value of the risk-free rate as a function of discount rate and risk aversion.

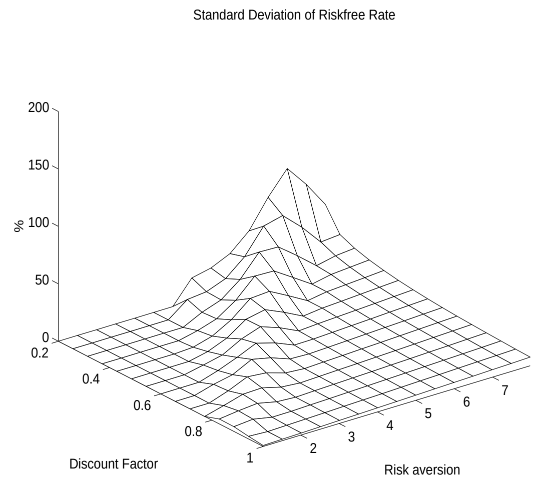

Figure 22: The standard deviation of the risk-free rate as a function of discount rate and risk aversion.

Next, noticing that p_t^e is a linear function of aggregate endowment $p_t^e(z_j) = \omega_j e_t(z_j)$, substituting this expression into (11), and remembering that $e_{t+1} = \lambda_{t+1} e_t$, gives

$$\omega_j = \sum_{k=1}^{N} \max_{i=1,2} \beta \frac{u'(c_{i,t+1})}{u'(c_{i,t})} (\lambda_k(1+\omega_k))\pi(z_k|z_j). \tag{12}$$

(12) gives a system of N unknowns in N equations. Given the solution to this system, one can calculate the rates of return. The stock pays unit of aggregate endowment next period, and hence the rate of return on equity from state j to state k, r_{jk}^e, is

$$r_{jk}^e = \frac{p_{t+1}^e(z_k) + e_{t+1}(z_k) - p_t^e(z_j)}{p_t^e(z_j)} = \frac{\lambda_k(1+\omega_k)}{\omega_j} - 1. \tag{13}$$

Using the equations (8), (12), and (13), Figures 21, 22, 23, and 24 present the expected value of the risk-free rate, the standard deviation of the risk-free rate, the expected value of the equity premium, $E_t[r_{t+1}^e] - r_t$, and the standard deviation of the rate return on equity, respectively. Notice that in autarchy no trade is allowed because one of the agents would default on his or her contract, and hence all the statistics have been set to zero.

Since I am interested in bond markets, I chose to match the expected value and the standard deviation of the risk-free rate to the values reported by Campbell (1998), 1.198 and 5.446, respectively. This lead to $\beta = 0.342$ and $\gamma = 3.482$. Table 3 summarizes the main statistics in the model given the above discount factor and risk aversion values and in the data. The values for the average equity premium and the standard deviation of the return on equity are from Campbell (1998).

The model provides relatively good fit of the asset market facts with minimal assumptions; it produces both the low average risk-free rate and the high average equity premium. Next section considers the implications of the model with the above parameter values to the term structure of interest rates.

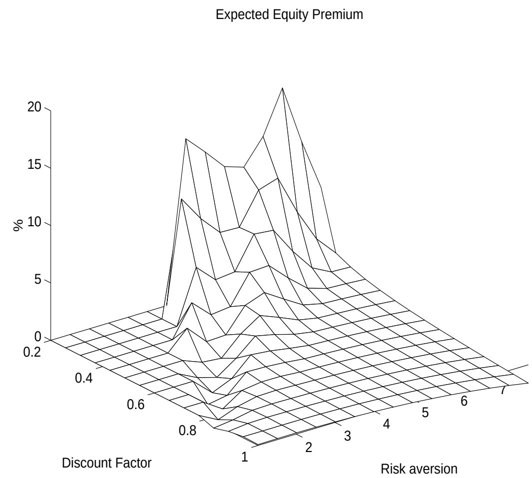

Figure 23: The expected value of the equity premium as a function of discount rate and risk aversion.

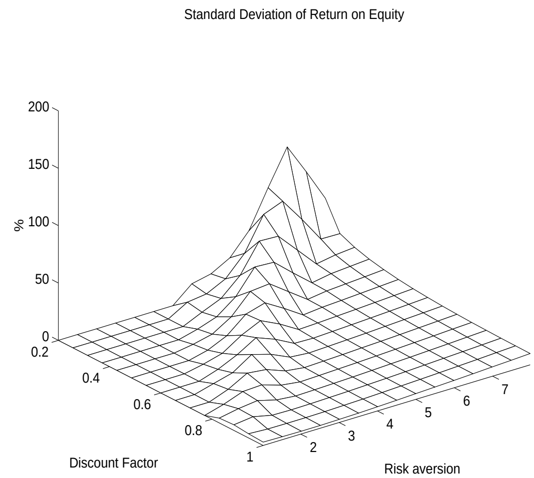

Figure 24: The standard deviation of the rate of return on equity as a function of discount rate and risk aversion.

Table 3: Selected statistics from the model when $\beta = 0.342$ and $\gamma = 3.482$ and from the data.

	Model	Data
$E[r]$ (%)	1.198	1.198
$\mathrm{std}[r]$ (%)	5.446	5.446
$E[r^e - r]$ (%)	4.993	6.116
$\mathrm{std}[r^e]$ (%)	5.148	22.675
$E(\Delta c)$ (%)	1.443	1.443
$\mathrm{std}[\Delta c]$ (%)	2.898	2.898
$\mathrm{corr}[\Delta c]$	0.281	0.281
Pr(exp.)/ Pr(rec.)	3.882	3.882
$\mathrm{std}(\ln(c_i / \sum_j (c_j)))$ (%)	26.79	—
$\mathrm{corr}(\ln(c_i / \sum_j (c_j))$	0.4329	—
$\mathrm{std}(\ln(e_i / \sum_j (e_j)))$ (%)	28.3	28.3
$\mathrm{corr}(\ln(e_i / \sum_j (e_j))$	0.4193	0.2035

6 The Term Structure of Interest Rates

6.1 Interest Rates, Forward Rates, and Risk Premium

The price of n-period zero-coupon bond is given by

$$p^b_{n,t} = \sum_{z^{t+n} \in Z^{t+n}} q(z^{t+n}|z_t) = E_t\left[\prod_{j=1}^{n} m_{t+j}\right]. \tag{14}$$

Now, using (7), (10), and the equation above notice that

$$p^b_{n,t} = \sum_{z^{t+1} \in Z^{t+1}} \max_{i=1,2} \beta \frac{u'(c_{i,t+1})}{u'(c_{i,t})} p^b_{n-1,t+1} \pi(z^{t+1}|z_t) = E_t[m_{t+1} p^b_{n-1,t+1}]. \tag{15}$$

The bond prices are invariant with respect to the time, and hence the equation (15) gives a recursive formula for pricing zero coupon bonds of any maturity.

Forward prices are defined by

$$p^f_{n,t} = \frac{p^b_{n+1,t}}{p^b_{n,t}},$$

and the above prices are related to interest rates (or yields) by

$$f_{n,t} = -\log(p^f_{n,t}) \quad \text{and} \quad r_{n,t} = -(1/n)\log(p^b_{n,t}). \tag{16}$$

To define the risk premium following Sargent (1987), write (15) for two-period bond using the conditional expectation operator and its properties

$$\begin{aligned} p^b_{2,t} &= E_t[m_{t+1} p^b_{1,t+1}] \\ &= E_t[m_{t+1}] E_t[p^b_{1,t+1}] + \mathrm{cov}_t[m_{t+1}, p^b_{1,t+1}] \\ &= p^b_{1,t} E_t[p^b_{1,t+1}] + \mathrm{cov}_t[m_{t+1}, p^b_{1,t+1}] \end{aligned}$$

which implies that

$$p^f_{1,t} = \frac{p^b_{2,t}}{p^b_{1,t}} = E_t[p^b_{1,t+1}] + \mathrm{cov}_t\left[m_{t+1}, \frac{p^b_{1,t+1}}{p^b_{1,t}}\right]. \tag{17}$$

As the conditional covariance term is zero for risk-neutral investors, I will call it the *risk premium* for the one-period forward contract, $rp_{1,t}$,

$$rp_{1,t} \equiv \mathrm{cov}_t\left[m_{t+1}, \frac{p^b_{1,t+1}}{p^b_{1,t}}\right] = p^f_{1,t} - E_t[p^b_{1,t+1}]$$

and, similarly, $rp_{n,t}$ is the risk premium for the n-period forward contract,

$$rp_{n,t} \equiv \mathrm{cov}_t\left[\prod_{j=1}^{n} m_{t+j}, \frac{p^b_{1,t+n}}{p^b_{1,t}}\right] = p^f_{n,t} - E_t[p^b_{1,t+n}].$$

In addition, I will call a difference between the one-period forward rate and the expected value of one-period interest rate next period the *term premium* for the one-period forward contract, $tp_{1,t}$,

$$tp_{1,t} \equiv f_{1,t} - E_t[r_{1,t+1}]$$

and, similarly, $tp_{n,t}$ is the term premium for the n-period forward contract,

$$tp_{n,t} \equiv f_{n,t} - E_t[r_{1,t+n}].$$

6.1.1 Analytical Results

The following two propositions and their proofs are similar to the ones in Backus, Gregory, and Zin (1989).

Proposition 2. *If either (i) both agents are risk-neutral ($\gamma = 0$) or (ii) the marginal rates of substitution are independent over time, then the risk premia are zero in all states.*

Proof. If (i) the agents are risk-neutral, then the stochastic discount factor is constant β in all states. Therefore, the bond prices and the forward prices are constant in all states, and the risk premia is zero. If (ii) the marginal rates of substitution are independent over time, as is the case when the growth rates *and* sharing rules are independent over time, (14) becomes

$$p^b_{n,t} = \sum_{z^{t+n} \in Z^{t+n}} q(z^{t+n}|z_t) = E_t\left[\prod_{j=1}^{n} m_{t+j}\right] = \prod_{j=1}^{n} E_t[m_{t+j}] = \prod_{j=1}^{n} E_t[p^b_{1,t+j-1}]$$

and the forward prices equal expected spot prices. Hence, the risk premia are zero. ∎

Proposition 3. *If the transition matrix for the economy is ergodic, then as n approaches infinity, the forward price $p^f_{n,t}$ converges to a constant.*

Proof. Let m_{ij} denote the pricing kernel between states i and j. In state i, the price of a one-period bond is $\sum_j \pi_{ij} m_{ij}$. This can be expressed as $\sum_j b_{ij}$, where b_{ij} defines a matrix B. Similarly, the price of a two-period bond is

$$\sum_j \sum_k \pi_{ij} m_{ij} \pi_{jk} m_{jk}$$

or $\sum_j b^{(2)}_{ij}$, where $b^{(2)}_{ij}$ denotes the ijth element of B^2. In general, the price of a n-period bond is given by $\sum_j b^{(n)}_{ij}$. Since the transition matrix is ergodic, the Perron-Frobenius theorem guarantees that the dominant eigenvalue of B is positive and that any positive vector operated on by powers of B will eventually approach the associated eigenvector and grow at the rate of this eigenvalue. Now remember that n-period forward price is the ratio of $(n+1)$-period bond to n-period bond. As n gets large, the ratio converges to the dominant eigenvalue of B regardless of the current state. ∎

An immediate corollary of Proposition 3 is that the risk premia will also converge. Since the transition matrix is ergodic, the expected spot price, $E_t[p^b_{1,t+n}]$, will converge and the limiting risk premium is the difference between the limiting forward price and the limiting expected spot price. Proposition 3 is related to a very general result by Dybvig, Ingersoll, and Ross (1996) that the limiting forward rate, if it exists, can never fall.

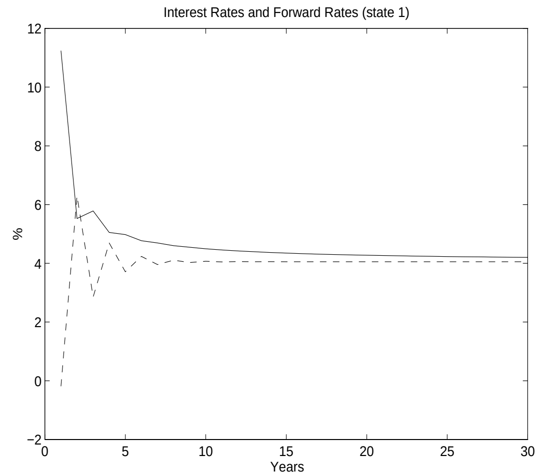

Figure 25: Interest rates (solid line) and forward rates in state 1.

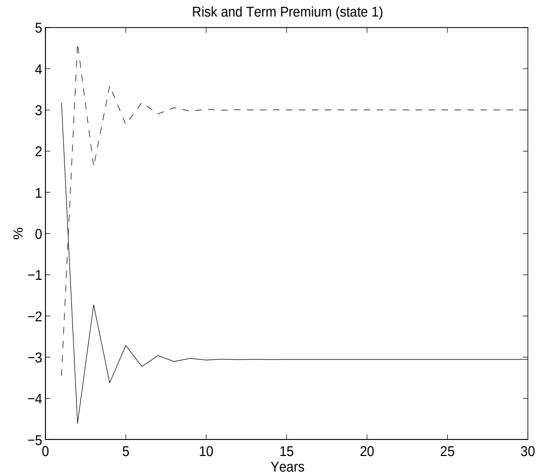

Figure 26: Risk premium (solid line) and term premium in state 1.

6.1.2 Numerical Results

Figures 25–34 present the interest rates for the maturities of 1 to 30 years, the forward rates of 1 to 30 year forward contracts, and the risk premia in the states 1–5 up to 30 years. A few things are worth noting from the figures. First, the model produces both upward and downward sloping term structures with several humps. Second, the term structure of interest rates in the states 1 and 5 corresponding to the recessions is uniformly above the one in the states 2, 3 and 4 corresponding to the expansions. Third, the term structure of interest rates is downward sloping in the recessions and upward sloping during the booms. The last two observations were also obtained by Donaldson, Johnsen, and Mehra (1990) who study the term structure of interest rates in the one good stochastic growth model.

Fama (1990) reports that "in every business cycle of the 1952–1988 period the five-year yield spread (the five-year yield less the one-year spot rate) is higher than at the business trough than at the preceding or following peak." This is clearly the opposite to what is the situation in either the stochastic growth model or the limited commitment model. The economic intuition for the behavior of the interest rates in economic models is straight forward. At the top of the cycle the aggregate and individual consumption are expected to be, on average, lower in the future and thereby the agents will want to save more thereby driving the interest rates down. At the bottom of the cycle the aggregate and the individual consumption are expected to be higher in the future and the agents, consequently, have less need to save and push the interest rates up. In Section 2, I showed that the observed cyclical behavior of the *real* interest rates, however, does not contradict the results by Donaldson, Johnsen, and Mehra (1990). Empirical results are also in line with the theoretical model presented in this Section.

Finally, Backus, Gregory, and Zin (1989) obtain a positive risk premium for all time periods and points in the cycle. The opposite is true here. This is by far the most important result of this paper: *The limited commitment model produces a large negative risk premium. The risk premium is also stable over the cycle for long maturities, but varies considerable and positively with the level*

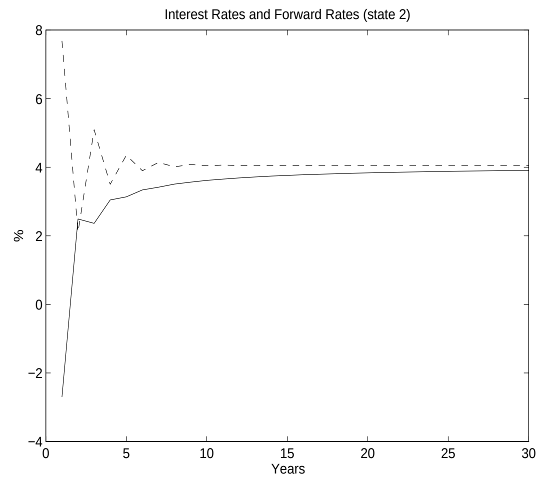

Figure 27: Interest rates (solid line) and forward rates in state 2.

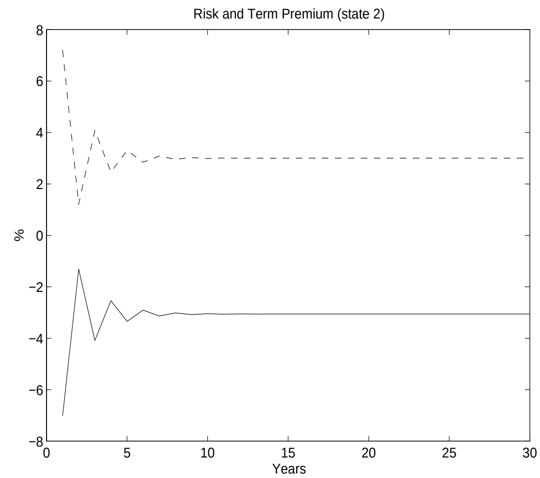

Figure 28: Risk premium (solid line) and term premium in state 2.

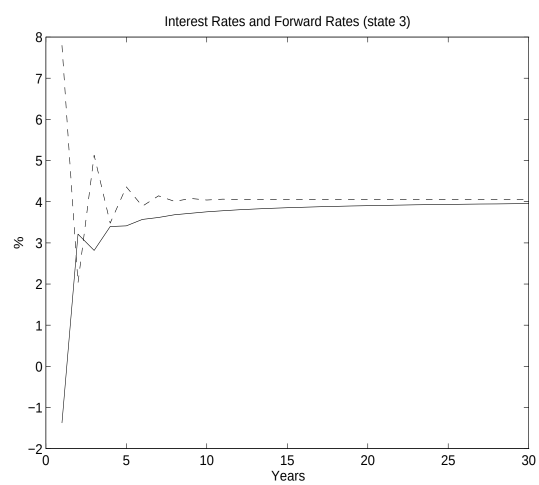

Figure 29: Interest rates (solid line) and forward rates in state 3.

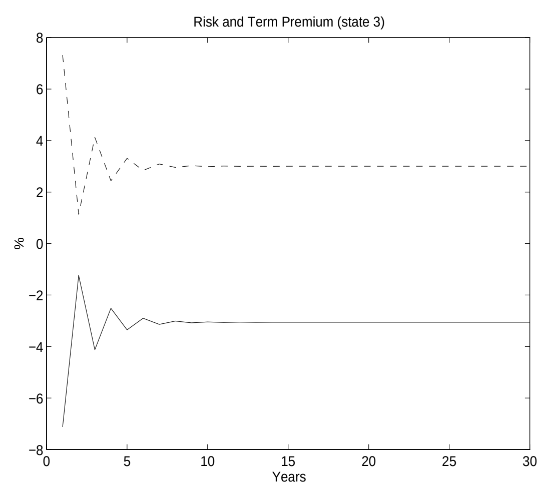

Figure 30: Risk premium (solid line) and term premium in state 3.

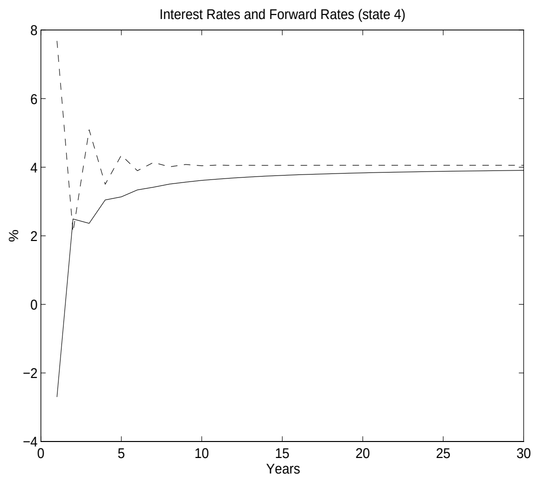

Figure 31: Interest rates (solid line) and forward rates in state 4.

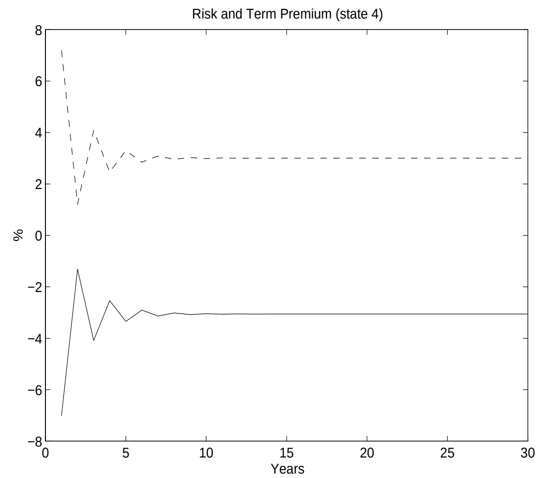

Figure 32: Risk premium (solid line) and term premium in state 4.

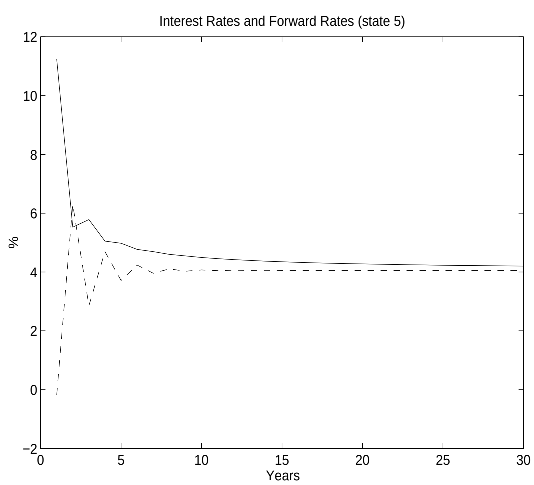

Figure 33: Interest rates (solid line) and forward rates in state 5.

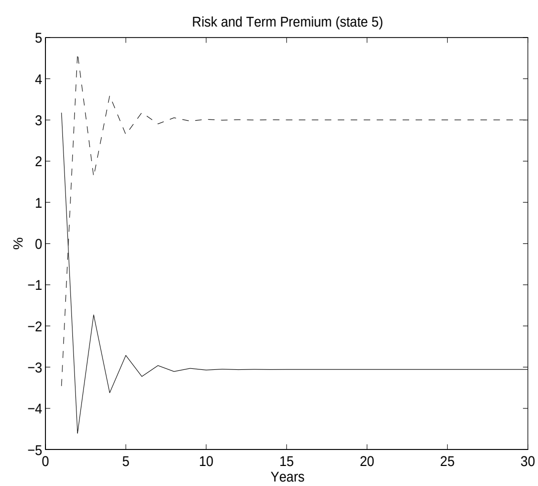

Figure 34: Risk premium (solid line) and term premium in state 5.

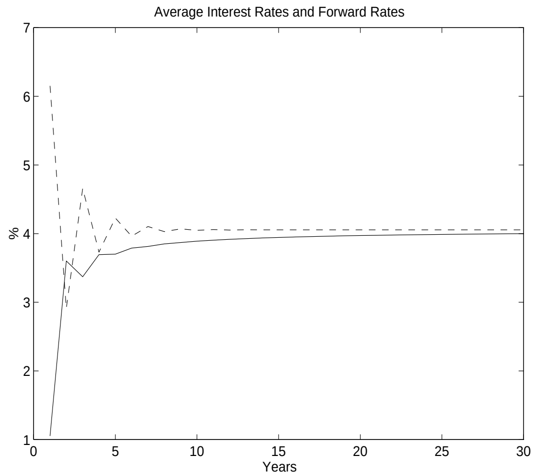

Figure 35: Average interest rates (solid line) and forward rates.

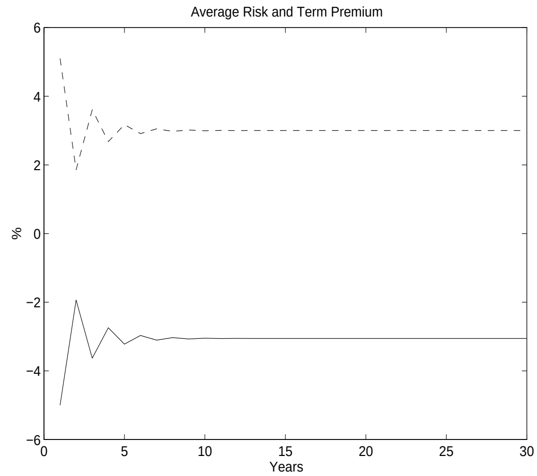

Figure 36: Average risk premium (solid line) and term premium.

of interest rates for short maturities. The premium is positive when interest rates are high (at the "bottom of the cycle") and negative when interest rates are low (at the "top of the cycle"). This result indicates that the limited commitment model may be useful in accounting for the rejections of the expectations hypothesis.

Figures 35–39 present the mean, the standard deviation, and the first-order autocorrelation of the interest rates, the forward rates, and the risk premia. The average term structure is upward sloping and the standard deviations are decreasing in the maturity as in the stochastic growth model and in the data.[11] The autocorrelations are between 0.08 and 0.2 as in the empirical term structure of real interest rates.

6.2 The Expectations Hypothesis

The oldest and simplest theory about the information content of the term structure is so called (pure) expectations hypothesis. According to the pure expectations theory forward rates are unbiased predictors of future spot rates. It is also common to modify the theory so that constant risk-premium is allowed—this is usually called the expectations hypothesis. However, it should be noted that both versions of the expectations hypothesis are always incorrect. To see this, let us assume, for a sake of an argument, that the agents are risk-neutral: $\gamma = 0$. Equation (17) reduces then into

$$p^f_{1,t} = E_t[p^b_{1,t+1}]$$

and using (8) we obtain

$$\frac{1}{1+f_{1,t}} = E_t\left[\frac{1}{1+r_{1,t+1}}\right].$$

[11] For international comparison of the term spreads (the difference between long and short rates) and the volatility of the interest rates using 1970–1993 monthly data, see Seppälä (1993).

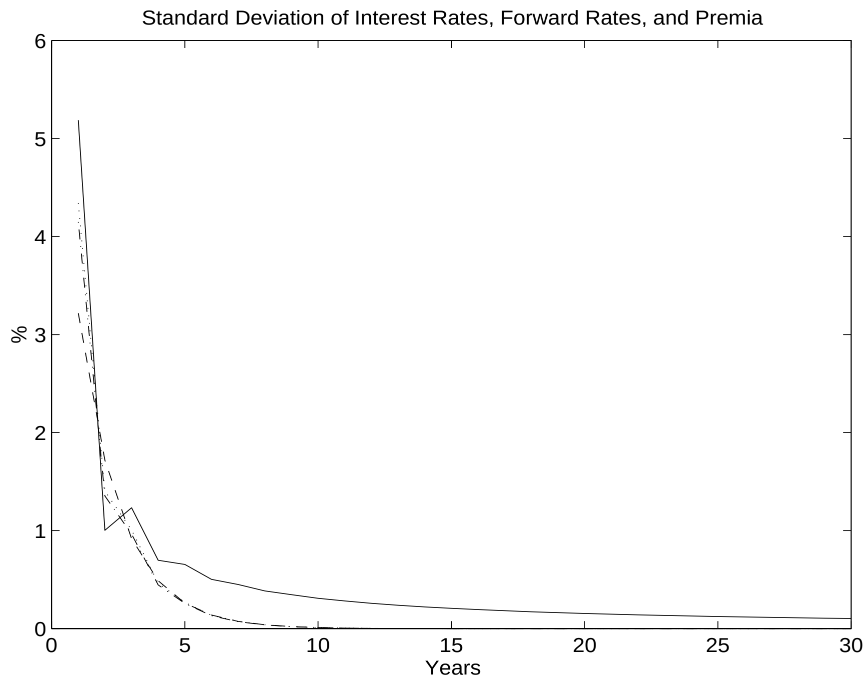

Figure 37: Standard deviation of interest rates (solid line), forward rates (dashed line), risk premium (dash-dotline), and term premium (dotted line).

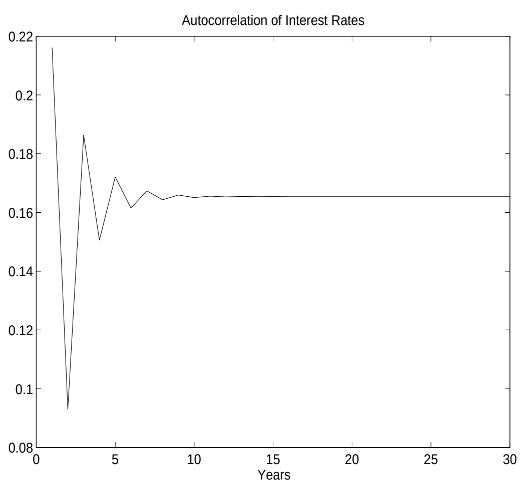

Figure 38: Autocorrelation of interest rates.

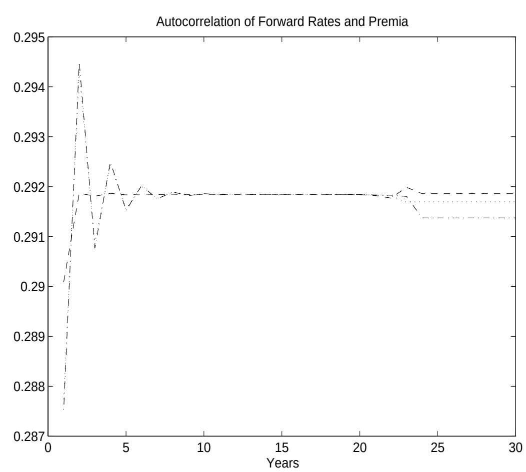

Figure 39: Autocorrelation of forward rates (dashed line), risk premium (dash-dotline), and term premium (dotted line).

Table 4: The number of rejects in each regressions in the complete markets model.

y_{t+1}	$p^b_{1,t+1} - p^f_{1,t}$	$p^b_{1,t+1} - p^f_{1,t} - rp_{1,t}$	$p^b_{1,t+1} - p^f_{1,t}$	$p^b_{1,t+1} - p^f_{1,t} - rp_{1,t}$
x_t	$p^f_{1,t} - p^b_{1,t}$	$p^f_{1,t} - p^b_{1,t}$	$p^b_{1,t} - p^f_{1,t}$	$p^b_{1,t} - p^f_{1,t}$
$Wald(a = b = 0)$	179	58	177	59
$Wald(b = 0)$	56	55	57	55
$Wald(b = -1)$	1000	1000	—	—

From the Jensen's inequality it follows that

$$f_{1,t} < E_t[r_{1,t+1}] \tag{18}$$

and the difference between the left and right hand side of (18) varies with $E_t[r_{1,t+1}]$ and $\text{var}_t[r_{1,t+1}]$. This effect is known as *convexity premium* or *bias*.

Backus, Gregory, and Zin (1989), on the other hand, tested the expectations model in the complete markets setting by starting with (17), assuming that the risk premium was constant

$$E_t[p^b_{1,t+1}] - p^f_{1,t} = a,$$

and then regressed

$$p^b_{1,t+1} - p^f_{1,t} = a + b(p^f_{1,t} - p^b_{1,t}) \tag{19}$$

to see if $b = 0$. They generated 200 observations 1000 times and used Wald test with White (1980) standard errors to check if $b = 0$ with 5% significance level. They could reject the hypothesis only roughly 50 times out of 1000 regressions which is what one would expect from chance alone. On the other hand, for all values of b except -1, the forward premium is still useful in forecasting the changes in spot prices. The hypothesis $b = -1$ was rejected every time.

Table 4 presents the number of rejections of different Wald tests in the regressions

$$y_{t+1} = a + bx_t$$

in the complete markets model. Table 5 presents the same tests for the limited commitment model. Unlike with the complete markets model, the results with the limited commitment model are consistent with empirical evidence on the expectations hypothesis. The model can generate enough variation in the risk premia to account for the rejections of the expectations hypothesis. On the hand, when the risk premium is substracted from $p^b_{1,t+1} - p^f_{1,t}$ b is equal to zero with 5% significance level.

In Table 6 the results of the regression (19) are presented for one realization of 200 observations in the completed markets model and the limited commitment model, and for the U.K. real and nominal interest rate data. The results for the complete markets model are from Backus, Gregory, and Zin. In Table 6, *Wald* rows refer to the marginal significance level of the corresponding Wald test. It is worth noticing how close are the regression coefficients in the limited commitment model

Table 5: The number of rejects in each regressions in the limited commitment model.

y_{t+1}	$p^b_{1,t+1} - p^f_{1,t}$	$p^b_{1,t+1} - p^f_{1,t} - rp_{1,t}$	$p^b_{1,t+1} - p^f_{1,t}$	$p^b_{1,t+1} - p^f_{1,t} - rp_{1,t}$
x_t	$p^f_{1,t} - p^b_{1,t}$	$p^f_{1,t} - p^b_{1,t}$	$p^b_{1,t} - p^f_{1,t}$	$p^b_{1,t} - p^f_{1,t}$
$Wald(a = b = 0)$	1000	54	1000	62
$Wald(b = 0)$	1000	65	1000	61
$Wald(b = -1)$	1000	1000	1000	1000

Table 6: The tests of the expectations hypothesis in a single regression.

Variable/Test	CM	LC	Real Data	Nominal Data
a	0.0029	0.0266	−0.0047	0.0001
se(a)	0.0017	0.0032	0.002	0.0005
b	−0.013	−0.4959	−0.4982	−0.9033
se(b)	0.053	0.0506	0.1046	0.0416
R^2	—	0.4011	0.3457	0.7218
$Wald(a = b = 0)$	0.227	2e−103	3e−7	0.0
$Wald(b = 0)$	0.801	6e−19	2e−6	0.0
$Wald(b = -1)$	1e−10	2e−19	2e−6	0.02

and in the real U.K. data. On the other hand, in the nominal term structure, the forward premium has very little power in forecasting the changes in spot prices.

Another approach for testing (another version of) the expectations hypothesis (in rates) is to run a regression

$$(n-1) * (r_{n-1,t+n} - r_{n,t}) = a + b(r_{n,t} - r_{1,t}) \quad \text{for } n = 2, 3, 4, 5, 6, 11.$$

According to the expectations hypothesis, one should find $b = 1$. Table 7 summarizes the results from this regression for the limited commitment model and real and nominal data. The expectations hypothesis is clearly rejected in all cases. However, in general, rejecting the expectations hypothesis is harder in the nominal data than rejecting it in the limited commitment model or in the real data.

6.3 Term Structure Predictions of Economic Activity

Despite the fact that the expectations hypothesis has been rejected over and over again in the empirical literature, it has also been found that the term and forward spreads forecast changes in the interest rates, consumption growth, and other economic activity. In this section, I will compare the predictions of the limited commitment model to the empirical evidence using regressions presented in three famous papers on the term structure and the future economic activity.

The first paper is by Fama and Bliss (1987) who use forward spread to predict the future changes in one-year interest rates one to four years ahead. Table 8 presents the regression results

Table 7: Expectations hypothesis regressions in rates.

Regression	a	se(a)	b	se(b)	R^2
Limited Commitment ($n = 2$)	−2.3906	0.0009	−0.0563	0.0001	0.0022
Real Data ($n = 2$)	0.2752	0.2336	0.2931	0.1448	0.0365
Nominal Data ($n = 2$)	−0.1473	0.1679	0.3570	0.2623	0.0174
Limited Commitment ($n = 3$)	−0.8379	0.0004	0.5636	0.0001	0.5553
Real Data ($n = 3$)	0.2316	0.2543	0.4033	0.1113	0.1
Nominal Data ($n = 3$)	−0.5544	0.3039	0.5518	0.3064	0.0435
Limited Commitment ($n = 4$)	−1.7303	0.0006	0.2939	0.0001	0.1171
Real Data ($n = 4$)	0.2055	0.2820	0.4199	0.1029	0.1121
Nominal Data ($n = 4$)	−0.8968	0.4275	0.6941	0.3745	0.0526
Limited Commitment ($n = 5$)	−1.2662	0.0005	0.4705	0.0001	0.3748
Real Data ($n = 5$)	0.2056	0.3135	0.3135	0.4168	0.1060
Nominal Data ($n = 5$)	−1.1834	0.5394	0.7673	0.4462	0.0539
Limited Commitment ($n = 6$)	−1.5222	0.0006	0.4001	0.0001	0.2533
Real Data ($n = 6$)	0.2225	0.3470	0.4059	0.1034	0.0942
Nominal Data ($n = 6$)	−1.4215	0.6383	0.7696	0.5111	0.0518
Limited Commitment ($n = 11$)	−2.3137	0.0006	0.6907	0.0000	0.5789
Real Data ($n = 11$)	0.3891	0.5198	0.3891	0.1336	0.0292
Nominal Data ($n = 11$)	−2.1516	0.9721	−0.1958	0.7253	0.0367

Table 8: Forward spread forecasts of future interest rate changes n periods ahead.

Regression	a	se(a)	b	se(b)	R^2
Limited Commitment ($n = 1$)	−2.3906	0.0009	0.4718	0.0001	0.3814
Real Data ($n = 1$)	0.2752	0.2336	0.6465	0.0724	0.4264
Nominal Data ($n = 1$)	−0.1472	0.1679	0.6785	0.1312	0.1273
Limited Commitment ($n = 2$)	−2.5923	0.0007	1.4067	0.0001	0.4713
Real Data ($n = 2$)	0.0130	0.2341	0.7611	0.0821	0.5269
Nominal Data ($n = 2$)	−0.6479	0.2593	1.0908	0.1269	0.2673
Limited Commitment ($n = 3$)	−3.0040	0.0007	0.8387	0.0001	0.4931
Real Data ($n = 3$)	−0.2311	0.2486	0.7742	0.1044	0.5980
Nominal Data ($n = 3$)	−0.9321	0.3089	1.4744	0.1959	0.3796
Limited Commitment ($n = 4$)	−2.9027	0.0007	1.0920	0.0001	0.4945
Real Data ($n = 4$)	−0.3615	0.2139	0.8697	0.0402	0.8045
Nominal Data ($n = 4$)	−0.9354	0.3037	1.8440	0.1833	0.5537

of equation

$$r_{1,t+n} - r_{1,t} = a + b(f_{n,t} - r_{1,t}) \quad \text{for } n = 1, 2, 3, 4$$

for the real and nominal U.K. data and the limited commitment model with 10,000 observations. The standard errors are White (1980) heteroskedasticity consistent standard errors. In all cases b is positive, very roughly around 1 on average, and predicts future interest rate changes quite well. With the exception of $n = 2$, b in the real data and in the model are very close to each other. In the data the forecast power increases with the forecast horizon and in the limited commitment model it is rougly the same regardless of the forecast horizon.

The second paper is by Fama (1990) who uses the term spread to predict the future changes in one-year real interest rates one to five years ahead. Table 9 presents the regression results of equation

$$r_{1,t+n} - r_{1,t} = a + b(r_{5,t} - r_{1,t}) \quad \text{for } n = 1, 2, 3, 4, 5$$

for the real and nominal data and the limited commitment model. The standard errors are White (1980) heteroskedasticity consistent standard errors. Again, b in the limited commitment model and in the real data are very close to each other (roughly one) in all the regression while the nominal term structure displays different dynamics. In addition, in the data the forecast power increases with the forecast horizon and in the limited commitment model it is rougly the same regardless of the forecast horizon.

The last paper is Estrella and Hardouvelis (1991) who use the term spread to predict the future changes in the log consumption growth one to four years ahead. The data are quarterly observations from 1984:1 to 1998:1 of U.K. consumption non-durables plus services per capita deflated with GNP deflator; both series were obtained from Bloomberg. Table 10 presents the regression results of equation

$$(100/n) * (\log(c_{t+n}) - \log(c_t)) = a + b(r_{10,t} - r_{1,t}) \quad \text{for } n = 1, 2, 3, 4$$

Table 9: Term spread forecasts of future interest rate changes n periods ahead.

Regression	a	se(a)	b	se(b)	R^2
Limited Commitment ($n = 1$)	-2.3014	0.0008	0.8750	0.0001	0.3823
Real Data ($n = 1$)	0.2429	0.2336	0.8218	0.0896	0.4273
Nominal Data ($n = 1$)	-0.2980	0.1703	0.6756	0.1258	0.1465
Limited Commitment ($n = 2$)	-2.8402	0.0007	1.0799	0.0001	0.4719
Real Data ($n = 2$)	-0.0052	0.2336	0.9554	0.1020	0.5298
Nominal Data ($n = 2$)	-0.6538	0.2547	1.4341	0.1581	0.2872
Limited Commitment ($n = 3$)	-2.9683	0.0007	1.1288	0.0001	0.4932
Real Data ($n = 3$)	-0.2348	0.2508	0.9569	0.1300	0.5957
Nominal Data ($n = 3$)	-0.7809	0.3120	2.0811	0.2692	0.3913
Limited Commitment ($n = 4$)	-2.9775	0.0007	1.1323	0.0001	0.4947
Real Data ($n = 4$)	-0.3375	0.2167	1.070	0.0504	0.8017
Nominal Data ($n = 4$)	-0.7699	0.3125	2.5619	0.2714	0.5333
Limited Commitment ($n = 5$)	-3.0459	0.0007	1.1584	0.0001	0.5061
Real Data ($n = 5$)	-0.3676	0.2487	0.9536	0.0584	0.7930
Nominal Data ($n = 5$)	-0.8138	0.2538	2.6099	0.1677	0.6589

for the data and the limited commitment model. The standard errors are White (1980) heteroskedasticity consistent standard errors. Recall from Section 2 that the real term structure both in the stochastic growth model and the U.K. data are countercyclical while the nominal term structure is pro-cyclical. This result is reconfirmed in Table 10.

7 Conclusions and Further Research

This paper argued that a simple general equilibrium model can explain two of the most persistent term structure puzzles. First, Donaldson, Johnsen, and Mehra (1990) show that while in the U.S. nominal term structure the interest rates are pro-cyclical and term spreads counter-cyclical the stochastic growth model predicts that the interest rates are counter-cyclical and term spreads pro-cyclical. I reexamine the results using the data on the U.K. index-linked bonds. I show that during the sample period 1984:1–1995:8 the (ex-ante) real interest rates were counter-cyclical and term spreads pro-cyclical. Therefore, "the term spread puzzle" is no puzzle after all.

Second, the empirical research on the term structure of interest rates has concentrated on the question if the forward rates are unbiased predictors of the future spot rates. By and large the literature has rejected this hypothesis. Most studies take this to indicate the existence of time-varying risk premium. Therefore, it is interesting to ask if there are models which are capable of generating similar risk premiums to the ones observed in the real time series. This is the question in Backus, Gregory, and Zin (1989). They use a complete markets model and their answer is that the model can account for neither the sign nor the magnitude of average risk premiums in forward prices.

This paper applied recent research by Alvarez and Jermann (1999ab) to this "term premium

Table 10: Term spread forecasts of future consumption growth n periods ahead.

Regression	a	se(a)	b	se(b)	R^2
Limited Commitment ($n = 1$)	1.1791	0.0004	−0.4185	0.0000	0.3486
Real Data ($n = 1$)	2.6642	0.2635	−0.1973	0.1094	0.0555
Nominal Data ($n = 1$)	2.7584	0.2946	0.1848	0.1278	0.0190
Limited Commitment ($n = 2$)	1.2158	0.0003	−0.1866	0.0000	0.1365
Real Data ($n = 2$)	2.6107	0.1891	−0.3144	0.0660	0.2322
Nominal Data ($n = 2$)	2.7554	0.2162	0.3402	0.1053	0.1057
Limited Commitment ($n = 3$)	1.2597	0.0003	−0.1206	0.0000	0.0801
Real Data ($n = 3$)	2.4517	0.1536	−0.2898	0.0696	0.2920
Nominal Data ($n = 3$)	2.6229	0.1804	0.3640	0.0818	0.1727
Limited Commitment ($n = 4$)	1.2847	0.0003	−0.0886	0.0000	0.0554
Real Data ($n = 4$)	2.3743	0.1517	−0.2607	0.0559	0.3228
Nominal Data ($n = 4$)	2.6244	0.1673	0.3952	0.0806	0.2307

puzzle". It was shown that the model produces a large risk premium with the correct sign, and unlike the complete markets model can generate enough variation in the risk premia to account for the rejections of the expectations hypothesis. In addition, when the model is calibrated to the U.K. aggregate and household data, the regressions of future spot rates and consumption growth on the term spread behave in a similar manner both in simulated and in actual data.

I conclude that the behavior of the term structure of real interest rates is no puzzle for the modern economic theory. What is needed now is a theory to explain the behavior of the term structure of nominal interest rates. Since it is usually recognized that the monetary policy can only have effect with "long and variable lags" as Friedman (1968) put it, it is crucial to understand what are the inflation expectations that drive the market. Once we understand the behavior of both nominal and real interest rates, we have correct estimates about these inflation expectations.

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