Monopolisation and Industry Structure

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Abstract

The aim of this paper is to provide empirically testable predictions regarding the relationship between market size and concentration. In a model of endogenous horizontal mergers, it is shown that *concentrated outcomes* can not be supported in a free entry equilibrium in large exogenous sunk cost industries: the *upper bound to concentration* tends to zero as market size (relative to setup costs) tends to infinity. In contrast, arbitrarily concentrated outcomes may be sustained in endogenous sunk cost industries, no matter how large the market, and even in the absence of mergers; that is, the upper bound to concentration does not decrease with market size. Using a recent equilibrium concept, which is defined not in the space of strategies, but in the space of observable outcomes, it is shown that these predictions do not depend on the details of the extensive form of the game, even allowing for side payments between firms and endogenous product choice. The results complement those of Sutton (1991) on the stability of *fragmented outcomes*.

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1 Introduction

For a long time, a major topic in the literature on industrial market structure has been to explain differences in concentration across industries by reference to a small number of explanatory variables. The agenda of this traditional strand of the literature was strengthened by the finding that the ranking of industries by concentration tends to be very similar from one country to another.[1] This regularity appeared to show that the underlying pattern of technology and tastes strongly constrains equilibrium structure. Much of the old empirical work on cross-sectional differences in concentration was rooted in Bain's (1956) structure-conduct-performance paradigm according to which structure (concentration) is determined by certain "barriers to entry". A typical study in this literature sought to explain structure by regressing observed concentration measures on proxies for barriers to entry such as scale economies, advertising and R&D intensity etc. This approach, however, was strongly criticised even prior to the game-theoretic revolution in industrial organisation (IO). In particular, researchers remarked that many of the right-hand side variables were endogenous; advertising and R&D intensity, for instance, should depend on market concentration. The econometric response to the endogeneity problem consisted in estimating a system of simultaneous equations. However, this line of research did not prove to be entirely successful; see Schmalensee (1989).

The introduction of formal game-theoretic modelling into IO increased the unease of many researchers with cross-industry studies. The dilemma of the game-theoretic approach appears to be two-fold. First, equilibrium outcomes often depend delicately on features of the model that are unobservable to an empirical researcher, and likely to vary from one industry to another. Second, even if we can pin down the specification of the game, there remains the problem that many models have multiple equilibria. The response of many researchers to this dilemma has been to focus on single-industry studies so as to rely on specific features of that industry to motivate assumptions. A quite different response of other researchers has been not to give up on cross-industry studies, but rather to seek for more robust mechanisms that hold good across a broad range of industries. An outstanding example of the latter line of research is the "bounds approach to concentration", developed by Sutton (1991) in his book *Sunk Costs and Market Structure*. The idea of the bounds approach to concentration is to divide the space of outcomes into those outcomes that can be sustained as equilibrium outcomes in a broad class of admissible models and those that can not.

Sutton (1991) applies the bounds approach to the study of the relationship between concentration and market size. Quite surprisingly, this relationship did not receive much attention in the early literature, even though the relative size of an industry appears to be exogenous (at least as a first approximation). From a theoretical viewpoint, a negative size-structure relationship was considered to be obvious: for a given level of barriers to entry, an increase in market size should raise the profitability of incumbent

[1] See, for instance, Bain (1966).

firms and thus trigger new entry, which would lead to a fall in concentration. However, the empirical evidence for a negative relationship was found to be rather weak.

Sutton shows that the alleged negative relationship between market size and concentration breaks down in certain groups of industries. In particular, he introduces the useful distinction between "exogenous" and "endogenous" sunk cost industries. In exogenous sunk cost industries, the only sunk costs involved are the exogenously given setup costs; R&D and advertising outlays are insignificant. In endogenous sunk cost industries, on the other hand, the equilibrium level of sunk costs is endogenously determined by firms' investments decisions. Roughly, these are industries in which advertising or R&D are effective in that investments in some fixed outlays raise consumers' willingness-to-pay, or reduce marginal costs of production. Sutton's predictions are that, in exogenous sunk cost industries, the lower bound to concentration (i.e. the lower bound to the set of "rationalisable" outcomes) tends to zero as the market becomes large, whereas in industries for which the endogenous sunk cost model applies the lower bound to concentration is bounded away from zero, no matter how large the market. That is, in endogenous sunk cost industries, very fragmented outcomes (in the sense of low one-firm concentration ratios) can not be supported as equilibrium outcomes in large markets; such outcomes can not be excluded in exogenous sunk cost industries.[2] An empirical test of Sutton's predictions can be found in Sutton (1991), and Robinson and Chiang (1996).

Sutton's predictions, although robust, may not appear entirely satisfactory in that they are quite "weak": they refer to the stability of fragmented outcomes in large markets only. But such a criticism would miss the point of the bounds approach to concentration. Nevertheless, the important open question, raised by Bresnahan (1992) and others, is whether or not it is possible to make tighter predictions regarding the size-structure relationship.

The aim of this paper is to investigate in what kind of industries it is possible to sustain arbitrarily *concentrated* outcomes, and in what kind of industries it is not. The question addressed in this article is thus: 'Is there an *upper bound to concentration*?' In fact, an inspection of Sutton's dataset reveals that, in the exogenous sunk cost case, the datapoints do not "fill" the space above the lower bound. That is, there appear to be limits to monopolisation of industries. Notice that identifying an upper bound does not mean providing a certain maximum concentration measure but rather finding testable comparative statics results. This obviously requires first identifying a trade off firms may face in their attempts to monopolise markets.

The economic history of the U.S. at the turn of the century provides many examples of "attempts of monopolisation". At a time when mergers were not yet scrutinised by antitrust authorities, firms attempted to monopolise industries by horizontal mergers. The most famous case is probably that of the formation, in 1901, of the United States Steel Corporation by the consolidation of twelve steel producers. In their study of the U.S.

[2]The "nonfragmentation" or "nonconvergence" result for endogenous sunk cost models has been formally shown by Shaked and Sutton (1987).

Steel case, Parsons and Ray (1975) convincingly argue that it was primarily motivated by the successful attempt to gain market power, and *not* by efficiency considerations. An important feature of the U.S. Steel case (as well as of many other merger cases) was the steady decline of U.S. Steel's market share. For instance, U.S. Steel's market share of steel ingot and casting production decreased from 65.4% in 1902 to 54% in 1911, and 38.9% in 1929.[3] According to Stigler (1950), the observed (albeit relatively slow) decline in market share was due to new entry and the expansion of fringe firms.[4] The lesson one can learn from the U.S. Steel merger case is thus the following. Firms have an incentive to monopolise the market in order to gain market power. In the absence of antitrust authorities, this can be accomplished by horizontal mergers. However, there are two mechanisms which make it difficult to persistently monopolise an industry. (1) It may not be possible to persuade all firms in the industry to merge, since a firm may be better off by deciding not to merge, given that rival firms have an incentive to merge nevertheless. (2) Even if all firms in the industry decided to merge, this may trigger new entry. Indeed, the weaker is competition in the market, the more profitable is entry. The recent game-theoretic literature on endogenous horizontal mergers (e.g. Kamien and Zang 1990) has mainly focused on the first mechanism, and we follow this tradition in the first part of the paper. In contrast, the second part of the paper explores the second mechanism.

To study the limits of monopolisation, we investigate, in the first part of the paper, the market structure that would emerge if firms were free to merge in the absence of any antitrust laws. As Stigler (1950) pointed out, the resulting market structure is not necessarily a monopoly since firms face a trade off between participation in a merger (so as to achieve a less competitive outcome) and nonparticipation (so as to take a free ride on the merging firms' effort to restrict output). We consider the linear-demand system due to Shubik and Levitan (1980), in which goods are horizontally differentiated; all products are treated symmetrically, and competition is non-localised. The degree of substitutability between goods is summarised by a one-dimensional parameter, σ. Prior to mergers, each firm is equipped with the knowledge to produce one distinct product; the product portfolio of post-merger coalitions is the collection of products offered by its members.

Following Sutton (1991), we distinguish between exogenous and endogenous sunk cost industries. In the exogenous sunk cost case, the endogenous horizontal merger game consists of three stages. At the first stage, firms decide whether or not to enter

[3] This reduction in market share was slowed down by U.S. Steel's aggressive purchase of ore deposits. The corresponding rise in market price for ore sharply reduced the profitability of entry. This case is a nice example of how backward integration reduces the profitability of new entry into the downstream market.

[4] Parsons and Ray (1975) describe a revealing example of free-riding behaviour of fringe firms. In 1930, National Steel possessed just 2.5% of the industry's ingot capacity; but in the 1930's, its ingot capacity expansion was one quarter of the industry total expansion during the 1930's. In many years, its steel production subsidiary's capacity utilisation rate was about double the industry average rate.

the industry; if a firm decides to enter it has to pay some entry fee. At the second stage, the firms that have decided to enter form "coalitions". Finally, at the third stage, the newly formed coalitions compete in prices. We show that, for a given number of firms in the industry, merger to monopoly obtains as long as products are sufficiently good substitutes. As the number of firms increases, the interval of values of σ, for which merger to monopoly obtains, shrinks and eventually vanishes in the limit. That is, for a given value of σ, monopoly does not emerge endogenously if the number of firms is sufficiently large. Moreover, in any equilibrium, the market share of the largest coalition converges to zero as the initial number of firms tends to infinity. We assume that the industry is in a long-run free-entry equilibrium; that is, the number of potential entrants is sufficiently large so that, in equilibrium, further entry is unprofitable. An increase in the size of the market relative to the level of entry costs raises the profitability of entry, and will thus lead to a larger number of entering firms. In the limit as market size (relative to setup costs) goes to infinity, the number of firms increases without bound. But this implies that it is impossible to sustain concentrated outcomes in large markets. *In exogenous sunk cost industries, the upper bound to concentration goes to zero as market size tends to infinity.*

To analyse the endogenous sunk cost case, we use the "quality-augmented" linear-demand system proposed by Sutton (1997,1998). In this case, the model consists of four stages. First, firms decide whether or not to enter the market. Second, firms form coalitions. Third, the coalitions decide how much to invest in the quality of the goods in their portfolio. Finally, the coalitions compete in prices. Even in the absence of mergers, the number of entering firms in the long-run free-entry equilibrium remains finite, no matter how large the market. This is an application of the nonconvergence result for endogenous sunk cost industries. Moreover, if products are sufficiently good substitutes (or investment in quality is sufficiently effective), then only one firm can be supported in equilibrium. Thus, not even in large markets is it in general possible to exclude arbitrarily concentrated outcomes. That is, *in endogenous sunk cost industries, the upper bound to concentration does not decrease with the size of the market.*

In the second part of the paper, we investigate the extent to which "ex-post entry" (e.g. post-merger entry) constrains the emergence of concentrated outcomes. For this purpose, we use a recent equilibrium concept by Sutton (1997), which is defined not in the space of strategies but in the space of (observable) outcomes. This equilibrium concept involves two rather weak assumptions, which are both implied by subgame perfection. Moreover, the extensive form of the game is not specified explicitly. In particular, we allow firms to make side payments and to monopolise markets not only through mergers but also through product proliferation. The key feature of the extensive form is the following: there is some penultimate stage (before firms compete in prices) at which new firms can enter the market. This formalises the notion of "ex-post entry", which is, according to Stigler (1950), an empirically powerful force preventing the monopolisation of industries, as exemplified by the U.S. Steel case. We show that, in exogenous sunk

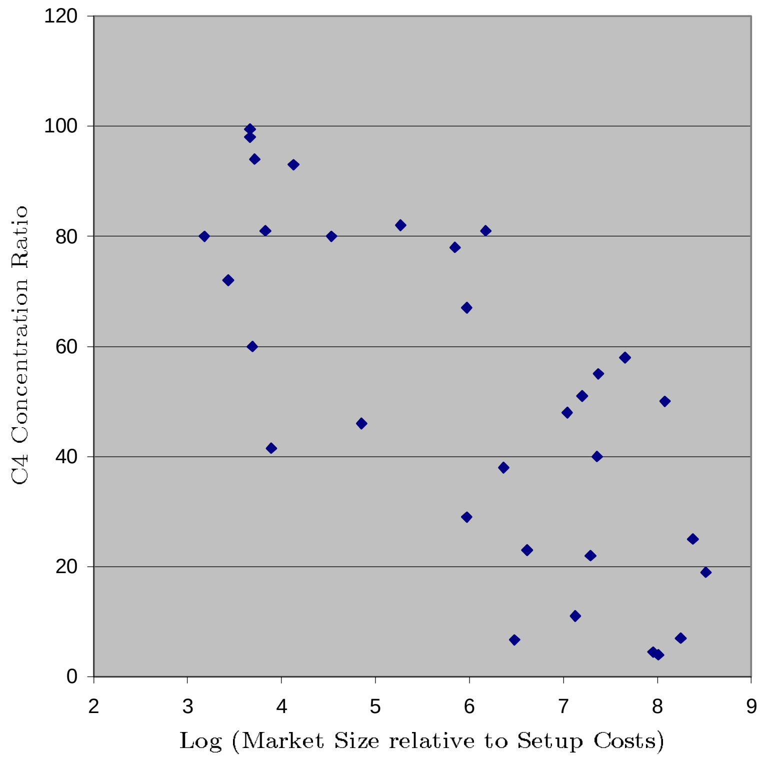

Figure 1: Exogenous Sunk Cost Industries.

cost industries, the upper bound to concentration does indeed go to zero as market size (relative to setup costs) tends to infinity. In contrast, monopoly outcomes may be sustained in endogenous sunk cost industries, no matter how large the market. Hence, *allowing for ex-post entry, the predictions of this paper hold independently of any details regarding coalition formation or product selection.*

Figures 1 and 2 reproduce Sutton's (1991) data set of the food and drink sector. The scattered points in the (concentration, market size)-space are indeed at least suggestive of the predicted relationships between the upper bound to concentration and market size (relative to setup costs) in exogenous and endogenous sunk cost industries, respectively.

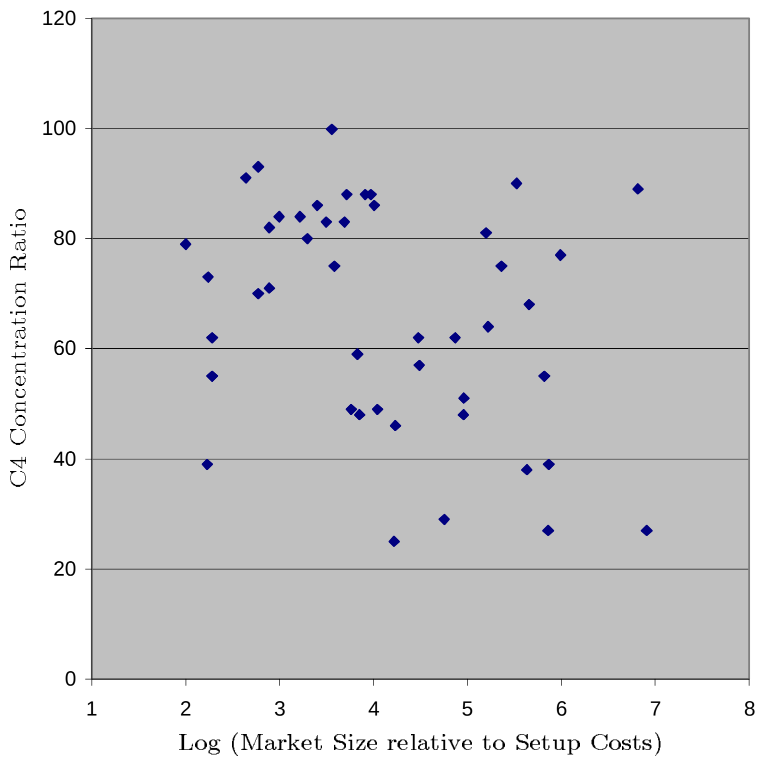

Figure 2: Endogenous Sunk Cost Industries.

2 Related Literature

This paper is closely related to several strands in the IO literature. First, it is part of the game-theoretic literature on industrial market structure, and the literature on the relationship between market size and concentration in particular. The seminal works in this literature are Sutton's books *Sunk Costs and Market Structure* (Sutton 1991) and *Technology and Market Structure* (Sutton 1998), dealing with advertising-intensive and R&D-intensive industries, respectively, and combining theory, econometric tests and case studies.[5] Much of Sutton's work is concerned with the stability of fragmented outcomes in large markets. The instability of such outcomes in models of pure vertical product differentiation was first shown by Shaked and Sutton (1983), extending earlier work by Gabszewicz and Thisse (1980). Shaked and Sutton (1987) later showed that this result generalises to industries in which firms can effectively raise consumers' willingness-to-pay, by investing in some fixed outlays. In these papers, nonconvergence is analysed in the context of static stage games. The robustness of these results to the existence of collusive underinvestment equilibria in dynamic investment games has been shown by Nocke (1998). The present article develops this literature further in that it investigates the stability of concentrated outcomes in large markets. It follows Sutton (1991) in distinguishing between exogenous and endogenous sunk cost industries, and builds upon the insights of this earlier work: whether or not merger to monopoly obtains depends upon the number of firms the market could support in the absence of mergers; it is this number that lies at the heart of the analysis in Sutton's book.

Second, in this paper, firms attempt to monopolise markets by horizontal mergers. The IO literature on horizontal mergers can roughly be divided into two strands: exogenous and endogenous mergers. The first strand is mainly concerned with the profitability and welfare consequences of a given horizontal merger by two or more firms; important papers include Salant, Switzer and Reynolds (1983), Perry and Porter (1985), Deneckere and Davidson (1985), Levin (1990), and Farrell and Shapiro (1990).[6] Most of these papers analyse mergers in a homogenous goods industry under Cournot competition. The important contribution by Salant, Switzer and Reynolds (1983) was to show that mergers tend to be unprofitable in such a setting, provided there are no efficiency gains and the merger does not lead to an almost complete monopolisation. Deneckere and Davidson (1985) showed that this result does no longer hold under Bertrand competition (with differentiated products). In fact, the first part of the present paper uses essentially the same multiproduct demand system as Deneckere and Davidson. The advantage of such a setting is that mergers are conceptually well defined; in contrast, in a homogenous goods model with constant returns-to-scale technology, as in Salant, Switzer and Reynolds (1983) and Kamien and Zang (1990), a merger by m firms is equivalent to a reduction

[5]Important empirical articles are Bresnahan and Reiss (1990), and Berry (1992).

[6]There are also some papers on the effects of exogenous horizontal mergers on collusion; see Davidson and Deneckere (1984), and Compte, Jenny and Rey (1996).

in the number of players by $m-1$.

The present paper is even more closely connected to the still underdeveloped literature on endogenous horizontal mergers, which starts from Stigler's (1950) insight that firms may not want to participate in a merger since they may prefer to take a free ride on the merging firms' effort to restrict output. Hence, the profitability of a given merger (relative to no merger at all) is in general not sufficient for a merger to occur in a noncooperative equilibrium. Using a homogenous goods Cournot model with constant returns-to-scale technology, Kamien and Zang (1990) were the first to formally analyse endogenous horizontal mergers.[7] In such a setting, clearly, merger to monopoly, although maximising industry profits, does not emerge in equilibrium if the number of firms, n, is sufficiently large. To see this, notice that, by not participating in the merger of its $n-1$ rivals, a firm can ensure itself the duopoly profit. If the monopoly profit is k times the duopoly profit, then merger to monopoly obtains only if $n \leq [k]$, where $[k]$ is the integer part of k. From Salant, Switzer and Reynolds' (1983) analysis, it is well-known that a merger that falls (significantly) short of monopoly is not profitable. Not surprisingly, then, for n sufficiently large, mergers do not occur at all in equilibrium.[8] The main differences between Kamien and Zang (1990) and the present article are the following. First, we analyse a multiproduct demand system in which mergers are not merely a reduction in the number of players. Second, we consider the case of price competition in which *any* merger is profitable in that it increases the joint profit of the participating firms; see Deneckere and Davidson (1985). Indeed, we show that mergers will occur in equilibrium, even in the limit as n tends to infinity. The open question, addressed in this paper, is whether concentrated outcomes can endogenously occur in equilibrium. Third, and most importantly, we introduce several ingredients which allow us to make empirically testable predictions: the degree of horizontal product differentiation (σ), the distinction between exogenous and endogenous sunk cost industries, and market size relative to setup costs (by assuming free but costly entry). Finally, in the second part of the paper, we show that our predictions do not depend on the details of the coalition formation game if we introduce the notion of ex-post entry. In fact, the effects of post-merger entry have been very much neglected in the literature.

Some of the insights of the endogenous horizontal merger literature have been anticipated by the literature on cartel stability and explicit collusion; see, for instance, Selten (1973), d'Aspremont et al. (1983), and Nocke (1999). In fact, these models are formally equivalent to endogenous horizontal merger models. Conversely, the results of the present article may also be of significance for the literature on cartel stability. Furthermore, this paper is related to the literatures on endogenous (noncooperative) coalition

[7] Earlier, Deneckere and Davidson (1985) discussed firms' incentives to merge, but did not formally analyse coalition formation. Later work includes Kamien and Zang (1991,1993) and Gowrisankaran (1999) .

[8] Kamien and Zang (1990) show that if merged entities are allowed to partly "demerge" after the merger (by forming independent subunits), then mergers tend to be more profitable. But again, merger to monopoly will not obtain if n is sufficiently large.

formation (e.g. Hart and Kurz 1983, Bloch 1996, Yi 1997) and multiproduct oligopoly (e.g. Champsaur and Rochet 1989 , Shaked and Sutton 1990). Finally, the second part of the paper applies the recent equilibrium concept by Sutton (1997), which allows a very elegant formalisation of post-merger entry.

3 Endogenous Horizontal Mergers and the Upper Bound to Concentration

In this section, we investigate whether concentrated outcomes can be sustained as equilibrium outcomes in large markets, using a model of endogenous horizontal mergers. Coalition formation is modelled as a noncooperative open membership game; post-merger entry is not considered. The robustness of the predictions is analysed in the next section, where we allow for ex-post entry, endogenous product selection, and more general models of coalition formation. In the following, we distinguish between exogenous and endogenous sunk cost industries.

3.1 Monopolisation in Exogenous Sunk Cost Industries

We first analyse the limits of monopolisation in exogenous sunk cost industries, where R&D and advertising outlays are insignificant; the only kind of sunk costs involved are exogenously given by firms' setup costs.

3.1.1 The Model

We consider an industry offering a potentially infinite number of substitute goods to S identical consumers. A consumer's utility is given by

$$U(\mathbf{x}; M) = \sum_{k=1}^{\infty} \left(x_k - x_k^2\right) - 2\sigma \sum_{k=1}^{\infty} \sum_{l<k} x_k x_l + M, \tag{1}$$

where x_k is consumption of substitute good k, and M is consumption of the outside good whose price is normalised to one. Let Y denote income and p_k the price of good k. Then, $M = Y - \sum_k p_k x_k$. The utility function is taken from Sutton (1997,1998), and can also be found, albeit in slightly different form, in Shubik and Levitan (1980), Deneckere and Davidson (1985) , and Shaked and Sutton (1990).[9] It defines utility over the domain of $\mathbf{x}$ for which all the marginal utilities $\partial U(\mathbf{x}; M)/\partial x_k$ are nonnegative. The form of the utility function ensures that a consumer's inverse demand for any good is linear. Income Y is assumed to be sufficiently large, $Y > 1/8\sigma$, so that $Y > \sum_k p_k x_k$ in equilibrium.

[9] Shubik and Levitan (1980), Deneckere and Davidson (1985), and Shaked and Sutton (1990) all consider price competition, whereas Sutton (1997,1998) analyses quantity competition using these preferences.

The parameter σ, $\sigma \in (0,1)$, measures the degree of substitutability between products. In the limit as $\sigma \to 1$, goods become perfect substitutes; in the limit as $\sigma \to 0$, products become independent. All goods (other than the outside good) are treated symmetrically.

We consider the following three-stage game. There are n_0 potential entrants, each of which has the knowledge to produce one distinct substitute good. At the first stage, these n_0 firms decide (either simultaneously or sequentially) whether or not to enter the industry. Entry costs in the industry are given by ϵ, $\epsilon > 0$. Since we want to confine attention to free-entry equilibria, we assume that n_0 is sufficiently large, $n_0 > [S - 8\epsilon(1-\sigma)]/(8\epsilon\sigma)$, so that in any equilibrium of the game there is at least one nonentering firm. At the second stage, the firms that have decided to enter at the previous stage simultaneously decide which "coalition" to join. All firms that have decided to join the same coalition then merge. Formally, firm k selects $z_k = i$, $i \in Z$, if it decides to join coalition M_i. A coalition structure $\left\{(M_i)_{i\in Z}\right\}$ is thus an endogenous partition of the set of entrants, induced by the vector of participation decisions, $\mathbf{z}$. Since there is an infinite number of coalitions firms can join (almost all of which will be empty in equilibrium), arbitrary coalition structures are allowed to emerge in equilibrium. This coalition formation game is sometimes called an open membership game; see Yi (1997). At the third and final stage, the newly formed coalitions, each offering the products of its "members", compete simultaneously in prices. Each coalition faces a constant marginal cost of production, c; to simplify notation, we assume $c = 0$.

Each of the n_0 firms is assumed to act so as to maximise its profit; the same applies to the merged entities at the price-setting stage of the game. The members of each coalition share the coalition's profit. Since firms are symmetric, we assume, for simplicity, that profit is shared equally.

3.1.2 Equilibrium Analysis

We now seek the pure strategy subgame perfect equilibrium (SPE) of the three-stage merger game.

Price-Setting Stage. We solve for the SPE of the merger game by inducing backwards. Let us, therefore, start with the third stage at which the merged entities simultaneously compete in prices. Suppose that n firms have entered the industry at the first stage, relabel these firms as firms 1 to n, and let $N \equiv \{1, ..., n\}$. We take firms' merger decisions, described by the vector $\mathbf{z} = (z_1, ..., z_n)$, as given. Let m_i denote the number of members of coalition M_i, i.e. $m_i \equiv \#\{k \mid z_k = i\}$. The following lemma summarises equilibrium behaviour at the price-setting stage; the proof can be found in the appendix.

Lemma 1 *For any vector of merger decisions* $\mathbf{z}$*, there exists a unique Nash equilibrium in prices. In equilibrium, each coalition sets the same price for all of its products. Coalition* M_i*'s equilibrium price is given by*

$$p^*_{M_i} = \frac{1-\sigma}{[2(1-\sigma) + (2n - m_i)\sigma]\left[1 - \sigma \sum_{j\in I} \frac{m_j}{2(1-\sigma)+(2n-m_j)\sigma}\right]}, \tag{2}$$

while its equilibrium profit per product is

$$S\pi^*_{M_i}(\mathbf{z}) = S\frac{(1-\sigma)\left[1-\sigma+(n-m_i)\sigma\right]}{2\left[1-\sigma+n\sigma\right]\left[2(1-\sigma)+(2n-m_i)\sigma\right]^2\left[1-\sigma\sum_{j\in I}\frac{m_j}{2(1-\sigma)+(2n-m_j)\sigma}\right]^2}. \quad (3)$$

Regarding price, several interesting comparative statics results can be obtained. First, for a given coalition structure, the equilibrium price is strictly increasing in the size of the coalition. The reason is that an increase in the price of a product exerts a positive externality on the demand of all other products. Each coalition internalises the externality of a price change on its own products; the more products a merged entity possesses, the higher is, therefore, the price of its products. Second, for a given number of own products, the coalition's price is strictly increasing in industry concentration in the following sense. Consider any two coalitions, M_j and M_l, say, where $m_j \geq m_l$. Then, any increase in $m_j - m_l$ that leaves $m_j + m_l$ unchanged, raises the equilibrium price of coalition M_i, $i \neq j, l$. To see this, observe that the function $\xi(m) \equiv m/[2(1-\sigma)+(2n-m)\sigma]$ is strictly convex. The intuition is that an increase in the average price of rival goods, due to an increase in concentration, induces a coalition to raise its own price as well since prices are strategic complements. Third, if $\sigma \to 1$, then, for all coalition structures other than monopoly, each coalition's price converges to the competitive price, which is equal to zero; this limit case is the famous "Bertrand paradox". If $\sigma \to 0$, then goods become independent, and the price converges to the monopoly price of $1/2$.

As to equilibrium profit, there are two important observations to make. The first is that a coalition's profit per product is decreasing in the number of its products, holding the coalition structure fixed. The second is that a coalition's profit per product is increasing in industry concentration for a given size of the coalition. The argument for the first observation comes in two parts. Consider the price-setting for a given product, fixing the prices of the $n-1$ other products at their equilibrium values. The first part of the argument then goes as follows. A one-product coalition sets price so as to maximise the profit of this product, whereas a multiproduct coalition sets a higher price since this imposes a positive externality on the coalition's other products; clearly, the larger is a coalition's product portfolio, the larger is the upward bias in pricing. The second and final part of the argument consists in noting that the larger is the coalition owning the product under consideration, the lower is the average equilibrium price of the $n-1$ other products; this follows from the fact that price is increasing in coalition size.

Merger Stage. Let us now induce backwards and analyse the second stage of the game. A vector of merger decisions, $\mathbf{z}^*$, can be sustained in an SPE of the subgame in which n firms enter if and only if

$$\pi^*_{M_{z_k^*}}(z_k^*, \mathbf{z}^*_{-k}) \geq \pi^*_{M_{z_k}}(z_k, \mathbf{z}^*_{-k}) \text{ for all } z_k \in Z,\ k \in \{1, ..., n\}. \quad (4)$$

We do not attempt here to give a general characterisation of firms' merger decisions. Instead, we focus on the conditions under which concentrated outcomes can emerge in equilibrium. For the proof of the following result, the interested reader is referred to the appendix.

Proposition 1 *If $n \in \{2,3\}$, then merger to monopoly can be supported in an SPE for all $\sigma \in (0,1)$. If $n \geq 4$, then there exists a $\widehat{\sigma}(n)$, $\widehat{\sigma}(n) \in (0,1)$, such that merger to monopoly is sustainable if and only if $\sigma \in [\widehat{\sigma}(n),1)$, where $\widehat{\sigma}(n)$ is given by*

$$\widehat{\sigma}(n) = \frac{2\left(n^2-6n+7\right)+2\sqrt{n^4-8n^3+27n^2-44n+28}}{(n-1)(4n-7)}. \tag{5}$$

To understand proposition 1, notice first that merger to monopoly from duopoly is always an equilibrium outcome: a monopolist can make at least the same profit per product as a duopolist (by mimicking the duopolists' pricing decisions), and strictly more whenever products are not independent (by raising the price slightly in order to internalise the externalities). This argument breaks down when there are more than two firms (products). Clearly, profit per product is higher under merger to monopoly than under the completely fragmented market structure where each firm offers one product only. However, if a firm deviates unilaterally, it can take a free ride on the $n-1$ merging rivals. The profit of such a free rider is strictly higher than the profit (per product) prior to the merger game, and may be higher or lower than profit under monopoly. Merger to monopoly will occur if and only if products are sufficiently good substitutes since, in this case, price competition is sufficiently tough so as to drive profits down whenever firms do not merge to monopoly.[10]

The higher is the number of entrants, n, the more "difficult" is merger to monopoly. The reason is that each firm's merger decision becomes less "decisive" as n increases; since, for a given market structure, a free-riding firm is always better off than a merging firm, firms have less incentives to merge for higher n.[11] Indeed, it is possible to show that $\widehat{\sigma}'(n) > 0$ for $n \geq 4$. Moreover, we get the following result.

Corollary 1 *For any $\sigma \in (0,1)$, there exists a finite threshold value $\widehat{n}(\sigma)$ such that merger to monopoly is sustainable in an SPE if and only if $n < \widehat{n}(\sigma)$.*

Proof. All we need to show is that $\lim_{n\to\infty} \widehat{\sigma}(n) = 1$. Since $\widehat{\sigma}'(n) > 0$ and $\widehat{\sigma}(n) \in (0,1)$ for all $n \geq 4$, it follows that $\lim_{n\to\infty} \widehat{\sigma}(n) \leq 1$. Suppose the assertion is false. Then there exists a $k \in (0,1)$ such that $\lim_{n\to\infty} \widehat{\sigma}(n) < k$. Using (5), this can be shown to lead to a contradiction. ■

The corollary shows that it is impossible to sustain merger to monopoly in markets with a sufficiently large number of firms; this holds for any degree of substitutability

[10] Monopoly profit is also decreasing in the degree of substitutability, σ, since consumers value variety. In the limit as $\sigma \to 1$, industry profit under monopoly converges to to the (standard) monopoly profit of a homogenous goods industry; under all other market structures, industry profit goes to zero as $\sigma \to 1$.

[11] With a continuum of firms, no firm would want to merge since its own decision does not affect market price, and a firm is clearly worse off by reducing output.

between products. In the following proposition, we further tighten our predictions; the proof is relegated to the appendix.

Proposition 2 *For any $(\sigma, \gamma) \in (0,1)^2$, there exists a finite $n(\sigma; \gamma)$ such that, for all $n \geq n(\sigma; \gamma)$, the market share of each coalition is bounded above by γ in any equilibrium of the merger game.*

Proposition 2 may suggest that mergers will not occur in the limit as the number of firms tends to infinity. This is not true, however, as the following proposition shows.

Proposition 3 *In equilibrium, there can be at most one single-product coalition. Hence, if $n \geq 2$, then mergers will occur in any equilibrium.*

Proof. This is essentially a corollary of Theorem 1 in Deneckere and Davidson (1985). All we need to show is that if, after relabelling, $z_k = k$ for $k \in \{1, 2\}$, and $z_k \geq 3$ for all $k \in \{3, ..., n\}$, then firm 1, say, can profitably deviate by joining coalition M_2. This deviation clearly increases the prices of all products in the industry. Decompose the price effect of the proposed deviation into two steps. First, let the outsiders (all coalitions M_k, $k \geq 3$) raise their prices to their new equilibrium values. This will benefit the members of M_2 since it raises the demand for M_2's products. Second, let coalition M_2 raise the prices of its two products to their new equilibrium values. By definition, this price increase must be profitable for the members of M_2. We have thus shown that the proposed deviation by firm 1 is profitable. ■

It is worth pointing out that proposition 3 obtains under the maintained hypothesis of no post-merger entry. A very similar proof can be used to show that, in the present setting, *any* merger is profitable in that it increases the profit of all firms involved in the merger. Nevertheless, as we have shown, concentrated outcomes will not emerge in equilibrium if the number of firms is large.

Entry Stage. We now turn to the determination of n, the number of pre-merger entrants, as a function of market size, S, and entry costs, ϵ. Since we are unable to characterise equilibrium in all ensuing subgames, it is impossible to determine the equilibrium number of entering firms, $n^*(S/\epsilon)$. It is, however, possible and useful to compute a lower bound on this number, and to study the limit behaviour of this bound as market size relative to setup costs tends to infinity. For this purpose, it is important to notice that, for a given number n of entering firms, the equilibrium profit of any entrant is bounded below by the profit in the absence of mergers, $S\underline{\pi}(n)$. The reason is that, for given participation decisions of the other $n-1$ firms, an entrant can always choose to form a coalition on its own, i.e. not to merge with other firms; this yields a lower bound on its profit. Moreover, the profit of a single-product coalition is minimal if none of the other $n-1$ firms decide to merge; see the discussion of comparative statics after lemma

1.[12] From (3), the lower bound on profits is, therefore, given by

$$S\underline{\pi}(n) = S \cdot \frac{(1-\sigma)\left[1-\sigma+(n-1)\sigma\right]}{2\left[1-\sigma+n\sigma\right]\left[2(1-\sigma)+(n-1)\sigma\right]^2}.$$

To calculate the lower bound on the number of entrants, observe that $S\underline{\pi}(n)$ is strictly decreasing in n, and strictly increasing in S. Hence, the number of entering firms in the absence of mergers, $\underline{n}(S/\epsilon)$, is the maximum integer n such that $S\underline{\pi}(n) \geq \epsilon$. Moreover, $\underline{n}(S/\epsilon)$ is strictly increasing in S/ϵ, and $\lim_{S/\epsilon\to\infty} \underline{n}(S/\epsilon) = \infty$. Since $n^*(S/\epsilon) \geq \underline{n}(S/\epsilon)$, it follows that $\lim_{S/\epsilon\to\infty} n^*(S/\epsilon) = \infty$. This, in conjunction with propositions 2 and 3, establishes the following proposition.

Proposition 4 (1.) *For any $(\sigma,\gamma) \in (0,1)^2$, there exists a finite $(S/\epsilon)(\sigma;\gamma)$ such that, for all $S/\epsilon \geq (S/\epsilon)(\sigma;\gamma)$, the market share of each coalition is bounded above by γ in any equilibrium of the game.* (2.) *For any S/ϵ sufficiently large, mergers occur in any equilibrium.*

Proposition 4 states the central prediction for exogenous sunk cost industries. It is impossible to sustain very concentrated outcomes (in the sense of high concentration ratios) in large exogenous sunk cost industries. More precisely, the *upper bound to concentration* converges to zero as market size goes to infinity.

3.2 Monopolisation in Endogenous Sunk Cost Industries

We now turn to the analysis of the limits of monopolisation in endogenous sunk cost industries, where the level of sunk costs is endogenously determined by firms' investment decisions.

3.2.1 The Model

The endogenous sunk cost model differs from the exogenous sunk cost model analysed in the previous subsection in that firms can invest in R&D or advertising so as to increase the consumers' willingness-to-pay for their products. As before, there are n_0 potential entrants, each equipped with the know how to produce one distinct substitute good, and S identical consumers. Using our previous notation, the utility function is now given by

$$U(\mathbf{x}; M; \mathbf{u}) = \sum_{k=1}^{\infty}\left(x_k - \frac{x_k^2}{u_k^2}\right) - 2\sigma\sum_{k=1}^{\infty}\sum_{l<k}\frac{x_k}{u_k}\frac{x_l}{u_l} + M, \qquad (6)$$

[12]Similarly, an upper bound on the equilibrium number of entrants can easily be found by observing that the equilibrium profit of the worst-off entrant is bounded above by the profit per product under merger to monopoly, $S\overline{\pi}(n) = S/\left(8\left[1-\sigma+n\sigma\right]\right)$, holding fixed the number n of entrants. Hence, the equilibrium number of entering firms is bounded above by $\left[S - 8\varepsilon(1-\sigma)\right]/(8\varepsilon\sigma)$.

where u_k, $u_k \in [1, \infty)$, is the perceived quality of substitute good k. If $u_k = 1$ for all k, then (6) reduces to the utility function of the exogenous sunk cost model, (1). It is easy to verify that an increase in u_k strictly increases utility whenever $x_k > 0$; that is, consumers value quality.

The timing of the game is as follows. At the first stage, the n_0 potential entrants decide whether to enter the market; the entry cost is denoted by ϵ. Again, we assume n_0 to be sufficiently large. At the second stage, the firms that have decided to enter at the previous stage play the same simultaneous-move coalition formation game as in the exogenous sunk cost case. At the third stage, the newly formed coalitions simultaneously choose the qualities of their products by investing in fixed R&D or advertising outlays. The cost of achieving quality u_k, $u_k \in [1, \infty)$, for good k is given by

$$F(u_k) = F_o \left(u_k^\beta - 1 \right), \tag{7}$$

where the parameter β is the elasticity of the investment cost function; we assume $\beta > 2$. Hence, if a firm decides to offer the basic quality of the good only ($u_k = 1$), it has not to pay further investment costs. At the final stage, the coalitions simultaneously compete in prices; production costs are assumed to be zero. Notice that the present model would coincide with the exogenous sunk cost model if we imposed the additional constraint $u_k = 1$ for all k.

3.2.2 Equilibrium Analysis

We now seek to characterise the pure strategy subgame perfect equilibrium (SPE) of the four-stage game. Let us first consider the final price-setting stage. We take as given firms' previous entry, merger and quality decisions.

Let $y_k(\mathbf{p}; \mathbf{u}) \equiv d_k(\mathbf{p}; \mathbf{u})/u_k$ be normalised demand for good k; similarly, $q_k \equiv p_k u_k$ denotes the normalised price of k. Equilibrium behaviour at the price-setting stage can now be characterised as follows.

Lemma 2 *Deleting all products with zero sales in equilibrium, coalition M_i's average normalised equilibrium price is given by*[13]

$$\overline{q}^*_{M_i} = \frac{(1-\sigma)\overline{u}_{M_i} + \sigma \sum_j \overline{m}_j (\overline{u}_{M_i} - \overline{u}_{M_j}) - (1 - \sigma + \overline{n}\sigma) \sum_j \frac{\sigma \overline{m}_j (\overline{u}_{M_i} - \overline{u}_{M_j})}{2(1-\sigma)+(2\overline{n}-\overline{m}_j)\sigma}}{[2(1-\sigma) + (2\overline{n} - \overline{m}_i)\sigma] \left[1 - \sum_j \frac{\sigma \overline{m}_j}{2(1-\sigma)+(2\overline{n}-\overline{m}_j)\sigma} \right]}, \tag{8}$$

where the $\overline{m}_j$'s and $\overline{n}$ are the numbers of products with positive sales in equilibrium, and $\overline{u}_{M_i}$ is the average over the u_k's with positive sales, $k \in M_i$. Equilibrium price of good k is

$$q_k^* = \overline{q}^*_{M_i} + \frac{u_k - \overline{u}_{M_i}}{2}, \tag{9}$$

provided sales are positive.

[13] Strictly speaking, equation (8) only holds if there are no products, which make *just* zero sales but constrain equilibrium.

The proof of lemma 2 is rather lengthy and can be found in the appendix. Indeed, in the endogenous sunk cost model, characterisation of equilibrium is more difficult than in the exogenous sunk cost model. The reason is that products of sufficiently low quality make zero sales in equilibrium; the equilibrium price of these products is not uniquely defined (except possibly for those goods that make *just* zero sales). Moreover, it is possible that a good of a certain quality is not produced in equilibrium while a good of a lower quality, owned by a different coalition, makes positive sales; this is due to "portfolio effects".

Our aim is to investigate whether it is possible to sustain concentrated outcomes in large endogenous sunk cost industries. Unfortunately, it is very hard to solve for equilibria at the investment stage; the following problems arise.

- A multiproduct coalition may find it optimal to invest in a subset of its products only. The equilibrium number of products it offers depends on the substitutability of products (σ), the effectiveness of R&D and advertising (β), and on the details of the coalition structure. For instance, a monopolist may offer less products than a coalition that has less members but faces rivals.

- Even for a fixed number of products, it is not possible to solve the first-order conditions for quality explicitly, unless the coalition structure is symmetric. Moreover, the first-order conditions are not sufficient conditions for a global profit maximum; boundary solutions are endemic.

- Multiple equilibria at the investment stage may arise even under symmetric coalition structures.

In contrast to the exogenous sunk cost case, the size of a coalition and the sizes of rival coalitions are no longer sufficient to summarise a coalition's equilibrium profit in the ensuing subgame: the profits of coalitions of equal size may differ since equilibria at the investment stage are often asymmetric.

Instead of solving the game, we therefore seek to find a lower bound on the one-firm concentration ratio that can arise in equilibrium. Clearly, any equilibrium market structure can not be more fragmented (in terms of the market share of the largest coalition) than the market structure that would arise in the absence of mergers.[14]

[14]Denote by $\widehat{n}$ the equilibrium number of entrants in the constrained game (in which mergers are not allowed), and suppose that, in the equilibrium of the unconstrained game (in which mergers are allowed), the largest coalition has $\widehat{m}$, $\widehat{m} \geq 2$, active members. Consider, for simplicity, the relative number of products as the measure of a coalition's market share. Then, for the one-firm concentration ratio in the unconstrained game to be lower than $1/\widehat{n}$, the equilibrium number of entrants would have to be larger than $\widehat{m}\widehat{n}$. It is possible to show that entry of more than $\widehat{m}\widehat{n}$ can not be profitable. To see this, notice that the profit of the worst-off entrant is maximal (under the constraint that concentration is not more than $1/\widehat{n}$) if the $\widehat{m}\widehat{n}$ single-product entrants formed $\widehat{n}$ coalitions of size $\widehat{m}$ each. But the post-entry profit per product of any of these coalitions is lower than the post-entry profit of any of the $\widehat{n}$ single-product entrants in the constrained game.

Most-Fragmented Market Structure. To find a lower bound on concentration, we assume that firms are not allowed to merge; that is, each entrant k is constrained to set $z_k = k$ at the second stage. Let us now start by considering the pricing stage, where each firm offers one product. We take the vector of qualities, $\mathbf{u}$, as given. Suppose $\mathbf{q}^*$ forms a Nash equilibrium in (normalised) prices. Then, if product k makes strictly positive sales, then any product l with $u_l \geq u_k$ makes positive sales as well. To see this, notice that firm l, offering quality $u_l \geq u_k$, can ensure itself positive sales and, hence, positive profits by setting price q_l such that $u_l - q_l = u_k - q_k^*$. Relabel firms in decreasing order of quality, i.e. $u_k \geq u_{k+1}$ for all $k \in \{1, ..., n-1\}$. Suppose there are $\overline{n}$ products with positive sales. From (8), the equilibrium price of good k, $k \leq \overline{n}$, is given by[15]

$$q_k^*(\overline{n}) = \frac{(1-\sigma)\left[2(1-\sigma)+(2\overline{n}-1)\sigma\right]u_k + \overline{n}\sigma\left[1-\sigma+(\overline{n}-1)\sigma\right](u_k - \overline{u}_N)}{\left[2(1-\sigma)+(\overline{n}-1)\sigma\right]\left[2(1-\sigma)+(2\overline{n}-1)\sigma\right]},$$

where $\overline{u}_N \equiv (1/\overline{n})\sum_{l=1}^{\overline{n}} u_l$ is the average quality of products with positive sales. Equilibrium output can be computed as

$$y_k^*(\overline{n}) = \frac{\left[1-\sigma+(\overline{n}-1)\sigma\right]}{2(1-\sigma)\left[1-\sigma+\overline{n}\sigma\right]} \cdot q_k^*(\overline{n}).$$

The equilibrium number of products with positive sales, $\overline{n}^*$, is uniquely defined as the maximum integer $\overline{n}$, $\overline{n} \leq n$, such that $q_{\overline{n}}^*(\overline{n}) > 0$. Except for the prices of products with zero sales, the equilibrium is unique.

At the third stage, firms (coalitions) simultaneously decide how much to invest in quality. Due to symmetry, any equilibrium is such that $u_k \in \{0, \overline{u}\}$ for all k. That is, each firm spends the same amount on quality, given that it invests at all. Suppose $\tilde{n}$, $\tilde{n} \leq n$, firms invest in quality. The equilibrium quality level, $\overline{u}(\tilde{n})$, is then implicitly defined by the first-order condition

$$S\frac{\left[1-\sigma+(\tilde{n}-1)\sigma\right]\left[2+3(\tilde{n}-2)\sigma+(\tilde{n}^2-5\tilde{n}+5)\sigma^2\right]}{\left[1-\sigma+\tilde{n}\sigma\right]\left[2(1-\sigma)+(2\tilde{n}-1)\sigma\right]\left[2(1-\sigma)+(\tilde{n}-1)\sigma\right]^2}\overline{u}(\tilde{n}) - \beta F_0 \overline{u}^{\beta-1}(\tilde{n}) = 0,$$

which yields

$$\overline{u}(\tilde{n}) = \left(\frac{S\left[1-\sigma+(\tilde{n}-1)\sigma\right]\left[2+3(\tilde{n}-2)\sigma+(\tilde{n}^2-5\tilde{n}+5)\sigma^2\right]}{\beta F_0\left[1-\sigma+\tilde{n}\sigma\right]\left[2(1-\sigma)+(2\tilde{n}-1)\sigma\right]\left[2(1-\sigma)+(\tilde{n}-1)\sigma\right]^2}\right)^{\frac{1}{\beta-2}}. \quad (10)$$

It is possible to show that, for the same number of products with positive quality, firms invest more in quality in the completely fragmented market structure than under monopoly; this is due to the "business stealing effect" of investment, which is internalised

[15] Again, this equation implicitly assumes that there is no product with zero sales that constrains equilibrium.

by a monopolist. The profit (net of investment costs) of a firm with quality $\overline{u}(\widetilde{n})$ is of the form $\Pi^*(\widetilde{n}) = \phi(\widetilde{n}; \beta, \sigma, S, F_0) \cdot \gamma(\widetilde{n}; \beta, \sigma)$, where $\phi(\widetilde{n}; \beta, \sigma, S, F_0) > 0$, and

$$\gamma(\widetilde{n}; \beta, \sigma) \equiv \beta(1-\sigma)\left[2(1-\sigma) + (2\widetilde{n} - 1)\sigma\right] - 2\left[2 + 3(\widetilde{n} - 2)\sigma + (\widetilde{n}^2 - 5\widetilde{n} + 5)\sigma^2\right].$$

It is easy to verify that $\gamma(1; \beta, \sigma) > 0$ and $\lim_{\widetilde{n} \to \infty} \gamma(\widetilde{n}; \beta, \sigma) = -\infty$. Since $\gamma(\widetilde{n}; \beta, \sigma)$ is a quadratic function of $\widetilde{n}$, it has a unique root $\widetilde{n}^*$ such that $\partial\gamma(\widetilde{n}^*; \beta, \sigma)/\partial\widetilde{n} < 0$, which is given by

$$\widetilde{n}^* = \frac{1}{2\sigma}\left[(\beta - 3) - (\beta - 5)\sigma + \sqrt{(\beta - 1)^2 - 2\left(\beta^2 - 3\beta + 3\right)\sigma + \left(\beta^2 - 4\beta + 5\right)\sigma^2}\right].$$

We claim that the (maximum) equilibrium number of firms that invest in quality is given by $\min\left\{\left[\widetilde{n}^*\right], n\right\}$. To see this, notice first that qualities of rival products (with positive sales) are strategic substitutes and that $\overline{u}(\widetilde{n})$ is decreasing in $\widetilde{n}$. Hence, if a firm finds it unprofitable to invest in quality if $[\widetilde{n}^*]$ firms have quality level $\overline{u}\left([\widetilde{n}^*] + 1\right)$, then no firm will find it profitable if the same number of firms offer quality $\overline{u}\left([\widetilde{n}^*]\right)$. That is, firms offering zero quality in equilibrium have no incentive to deviate. Firms offering positive quality levels in the candidate equilibrium can not profitably deviate either since their quality levels are given by the first-order conditions, which satisfy the second-order conditions. Notice, however, that more concentrated equilibria may exist.[16]

We now turn to the analysis of the first stage at which firms simultaneously decide whether or not to enter the market. Since $\phi(\widetilde{n}; \beta, \sigma, S, F_0) \to \infty$ as $S \to \infty$, it follows from the analysis of the investment stage that the maximum equilibrium number of entering firms, n^*, is given by $[\widetilde{n}^*]$ in the limit as market size S tends to infinity, provided that $\widetilde{n}^*$ is not an integer. If $\widetilde{n}^*$ is an integer, then the limit number of entrants is $\widetilde{n}^* - 1$.

In endogenous sunk cost industries, the equilibrium number of entering firms is bounded, no matter how large the market. Even in the absence of mergers, concentration in endogenous sunk cost industries can not get arbitrarily small by increasing the size of the market. This implies that proposition 4 can not hold in endogenous sunk cost industries. Our result is nothing else but an instance of a fundamental result in the analysis of industrial market structure: the "nonconvergence property", due to Shaked and Sutton (1987), according to which fragmented outcomes can not be supported as equilibrium outcomes in endogenous sunk cost industries. The question of interest, not addressed by Shaked and Sutton, is whether it is possible to sustain *arbitrarily* concentrated outcomes.

The limit number of entering firms is decreasing in σ for two reasons. First, the higher is σ, the less variety is offered by the market, and hence the less consumers spend on the goods on offer in this industry, holding prices fixed. Second, the larger the degree of substitutability between goods, the tougher is price competition, and thus the lower

[16]A pathological multiplicity arises if $\widetilde{n}^*$ is an integer. In this case, $\min\left\{[\widetilde{n}^*] - 1, n\right\}$ can also be sustained in equilibrium.

are profit margins. The number of entrants is increasing in β since the more convex the investment cost function is, the less firms will outspend each other in equilibrium. An instructive way of expressing the comparative statics results is the following. The limit number of entrants, as market size tends to infinity, is one if and only if $\sigma \in (\sigma_1(\beta), 1)$; it is equal to n, $n \geq 2$, if and only if $\sigma \in (\sigma_n(\beta), \sigma_{n-1}(\beta)]$, where

$$\sigma_n(\beta) \equiv \frac{(2n-3)\beta - 6(n-1) + \sqrt{(2n+1)^2\beta^2 - 4(2n^2+5n+1)\beta + 4(n^2+6n+1)}}{2[(2n-1)\beta + 2(n^2-3n+1)]}.$$

We can now analyse two limit cases. First, if $\beta \to \infty$, then $\sigma_n(\beta) \to 1$, and for no value of σ is the limit number of entrants finite. Indeed, this limit can be interpreted as the "exogenous sunk cost case", where every active firm chooses quality level 1 in equilibrium, and the number of firms grows without bound as market size increases.[17] Second, if $\beta \to 2$, then $\sigma_n(\beta) \to 0$, and for all values of σ the limit number of entering firms is one. This limit may be dubbed the "natural monopoly case".

Let us summarise our results in the following proposition.

Proposition 5 *If firms are not allowed to merge, the equilibrium number of firms remains finite, no matter how large the market. In particular, if goods are sufficiently good substitutes (σ close to 1) or investment is sufficiently effective (β close to 2), only one firm will enter the market, even as market size (relative to setup costs) tends to infinity.*

If firms are allowed to merge, then more concentrated outcomes will emerge in equilibrium, and more firms will enter the market, than in the absence of mergers. We have shown that the most fragmented market structure in endogenous sunk cost industries may involve arbitrarily concentrated outcomes in large markets. The empirical prediction for endogenous sunk cost industries can thus be summarised as follows.

Corollary 2 *In endogenous sunk cost industries, arbitrarily high one-firm sales concentration ratios may be supported in equilibrium, even in the limit as market size tends to infinity. That is, the upper bound to concentration does not decrease with market size.*

It is important to point out that this prediction is *not* a mere consequence of the non-convergence result, according to which it is impossible to sustain arbitrarily fragmented market structures in large endogenous sunk cost industries. Our corollary states that it is possible to support *arbitrarily* concentrated outcomes; this holds independently of the size of the market and the level of setup cost.

[17] More precisely, we consider the number of entrants as both S and β tend to infinity in such a way that $S/\beta \to \infty$ and $(S/\beta)^{2/(\beta-2)} \to 1$.

4 Ex-Post Entry and the Limits to Concentration

The aim of this section is to investigate the extent to which the possibility of "ex-post entry" constrains industry structure, and in particular the emergence of concentrated outcomes. Recall that post-merger entry was the second force mentioned by Stigler (1950) which prevents firms from successfully monopolising markets through horizontal mergers. In fact, we show that the possibility of "ex-post entry" prevents the emergence of concentrated outcomes in large exogenous sunk cost industries. Moreover, allowing for ex-post industry, we can be very general along several dimensions, and relax previous assumptions on coalition formation and product selection.

Indeed, the main predictions of the last section, regarding the relationship between market size and the upper bound to concentration, have been derived under quite special assumptions. First, we have modelled coalition formation, i.e. mergers, as an open membership game. Second, we have assumed that multiproduct firms can only emerge through mergers; firms are not allowed to choose the number of products they would like to offer. Due to the first assumption, our previous analysis leaves open the question whether the "instability" of concentrated outcomes is a consequence of coordination failures, which may or may not occur under different assumptions on coalition formation. After all, for a given number of firms in the industry, joint profits are maximised under monopoly. The second assumption implies that the only way to sustain a concentrated outcome is through mergers. This is clearly both unrealistic and restrictive, especially in the presence of antitrust laws.

To show that our predictions do not depend on such assumptions, we do neither specify explicitly the extensive form of the game nor the strategy space of players. Instead, we apply a recent equilibrium concept, due to Sutton (1997), which is defined not in the space of strategies, but in the space of outcomes. This equilibrium concept involves two rather weak assumptions, "viability" (no firm makes losses) and "stability" (there is one smart agent who would fill a profitable opportunity in the market), both of which are implied by subgame perfection.

4.1 The Model

There are n_0 firms that can take actions at certain specified stages. Each firm's action space is denoted by A. Actions may include entry decisions, the choice of the number of products, merger decisions, the choice of product quality (in the case of endogenous sunk cost industries), takeover bids, and so on. Firm i's actions in the entire game are summarised by the vector $\mathbf{a}_i$, $\mathbf{a}_i \subset A$. Each firm may decide not to enter the market, i.e. to choose the "null action", denoted by $\mathbf{a}_i = \emptyset$. The outcome of the game can then be described by the n_0-tuple $(\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_{n_0})$. Suppose n, $n \in \{1, ..., n_0\}$, firms decide to enter the market, i.e. to choose a non-null action Then, deleting all inactive firms and relabelling the remaining active firms, yields the n-tuple

$$\mathbf{a} \equiv (\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_n),$$

which is referred to as a *configuration*.

The total payoff (profit) of firm i from the set of actions $\mathbf{a}_i$, when rivals' actions are given by $\mathbf{a}_{-i} \equiv (\mathbf{a}_1, ..., \mathbf{a}_{i-1}, \mathbf{a}_{i+1}, ..., \mathbf{a}_n)$, is written as

$$\overline{\Pi}(\mathbf{a}_i; \mathbf{a}_{-i}).$$

If firm i decides not to enter the market ($\mathbf{a}_i = \emptyset$), then its payoff is zero:

$$\overline{\Pi}(\emptyset; \mathbf{a}_{-i}) = 0.$$

The function $\overline{\Pi}(\mathbf{a}_i; \mathbf{a}_{-i})$ summarises not only the final-stage profits but also possible costs from taking the set of actions $\mathbf{a}_i$ (e.g. costs from entering a new product) as well as payments between firms (resulting from merger or takeover decisions).

To exclude nonviable markets, we assume that there is some action $\mathbf{a}_0$, $\mathbf{a}_0 \neq \emptyset$, such that

$$\overline{\Pi}(\mathbf{a}_0; \emptyset) > 0. \tag{11}$$

Furthermore, it is assumed that the number of potential entrants, n_0, is sufficiently large such that if all firms choose to enter the market, then there is at least one that makes a negative profit. The idea is that entering the market requires a minimum setup cost of ϵ, and the sum of final-stage payoffs is bounded above by $(n_0 - 1)\epsilon$.

All we have to specify about the extensive form of the game is the following. There are T stages at which firms can enter the market and take actions; associated with these actions are certain costs and payments between firms. Additionally, at stage $T+1$, firms engage in some kind of price (or quantity) competition. All payoffs are summarised by the reduced-form payoff function $\overline{\Pi}(\cdot;\cdot)$. Firm i is free to take actions at any date t, $t \in \{t_i, t_i + 1, ..., T\}$, where $t_i \in \{1, ..., T\}$ is firm i's "date of arrival". This allows for "first-mover advantages". The important feature of the extensive form is that there is some penultimate stage, T, at which firms take actions simultaneously, and new firms can enter the market, before firms engage in price competition. We do not allow for actions that are effectively conditioning on the outcome of this penultimate stage.

A configuration $\mathbf{a}^*$ is called an *equilibrium configuration* if the following two conditions are satisfied:

(i) (viability) For all firms i,

$$\overline{\Pi}(\mathbf{a}_i^*; \mathbf{a}_{-i}^*) \geq 0.$$

(ii) (stability) There is no set of actions $\mathbf{a}_{n+1}$ such that entry is profitable. That is,

$$\overline{\Pi}(\mathbf{a}_{n+1}; \mathbf{a}^*) \leq 0.$$

Condition (i) requires that no firm makes a loss in equilibrium, while condition (ii) says that if there is a profitable opportunity in the market, then there is some smart agent who will fill it. It may be worth pointing out that both conditions are consistent with boundedly rational agents who do not fully maximise their payoffs. Moreover, both conditions are implied by subgame perfection. To see this, notice that if the viability condition was not satisfied in a candidate SPE, then a firm could profitably deviate by choosing the null action ("do not enter"), and make zero profit. Similarly, if the stability condition was not satisfied, then an inactive firm could profitably deviate by entering the market at stage T. We thus have the following "inclusion" property.

Proposition 6 (Sutton 1997) *Any outcome that can be supported in an SPE in pure strategies is an equilibrium configuration.*

The concept of an equilibrium configuration has bite for empirical applications if the conditions of viability and stability can be expressed in the space of observable outcomes. This is indeed the case if firms' actions merely consist in the choice of the number of products (exogenous sunk cost industries) or product quality (endogenous sunk cost industries). If we want to allow for side payments between firms, however, then we have to re-formulate these conditions in the space of observables. Let us now apply the equilibrium concept to study the limits of concentration in exogenous and endogenous sunk cost industries.

Exogenous Sunk Cost Industries. In the case of exogenous sunk cost industries, a profile of firms' actions, i.e. a configuration, $\mathbf{a}$, induces a profile of products (a coalition structure)

$$\mathbf{m} \equiv (m_1, m_2, ..., m_l),$$

$l \in \{1, ..., n\}$, where m_i gives the number of products in firm (or coalition) i's portfolio. The demand structure is as in section 3.1, and firms (coalitions) are assumed to compete in prices at the ultimate stage. Suppose the total number of products offered in the industry is given by $\overline{m} \equiv \sum_{j=1}^{l} m_i$. Then, provided it has chosen to enter the market, firm i's profit from the final price competition stage is given by $S\Pi(m_i; \mathbf{m}_{-i})$, which can be derived from equation (3) in section 3.1. The setup cost per product is denoted by ϵ.

Suppose now that $\mathbf{a}^*$ forms an equilibrium configuration, which induces the profile of products, $\mathbf{m}^*$. We may express the viability and stability conditions for exogenous sunk cost industries in the space of observables (i.e. in the space of profiles of product numbers) as follows:

(i') For all $i \in \{1, ..., l\}$,

$$S\Pi\left(m_i^*; \mathbf{m}_{-i}^*\right) - m_i^* \epsilon \geq 0.$$

(ii') There does not exist an m_{n+1}, $m_{n+1} \in \{1, 2, ...\}$, such that

$$S\Pi(m_{n+1}; \mathbf{m}^*) - m_{n+1}\epsilon > 0.$$

Condition (ii') is slightly weaker than (ii) in that we restrict the actions of an additional entrant to the choice of the number of its products. To understand condition (i'), notice that $S\Pi\left(m_i^*; \mathbf{m}_{-i}^*\right) - m_i^* \epsilon$ is an upper bound on firm i's total payoff under the assumption that the implicit price (in case of coalition formation: the profit share), or the explicit price (in case of takeovers: the takeover bid), of acquiring a product from another firm is at least the setup cost per product, ϵ. (The rationale for this assumption is that, otherwise, there would be some firm which sells a product to a rival below its cost. This assumption, however, is somewhat stronger than the viability condition, which only requires that a firm makes no loss on its combined activities.) Conditions (i') and (ii') coincide with (i) and (ii) if firms' actions merely consist in selecting the number of their products. In the equilibrium analysis below we show that these two rather weak requirements are powerful enough to obtain strong empirical predictions.

Endogenous Sunk Cost Industries. The application of the equilibrium concept to the case of endogenous sunk cost industries proceeds similarly to the case of exogenous sunk cost industries. A configuration $\mathbf{a}$ induces a profile of qualities

$$\mathbf{u} \equiv (\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_l),$$

$l \in \{1, ..., n\}$, where $\mathbf{u}_i$ is the vector of qualities offered by firm (coalition) i. Firm i's final stage profit is denoted by $S\Pi(\mathbf{u}_i; \mathbf{u}_{-i})$. We use the demand system of section 3.2 and assume that firms compete in prices, so that $S\Pi(\mathbf{u}_i; \mathbf{u}_{-i})$ is the Nash equilibrium profit as described in section 3.2. The cost of investment is given by equation (7); the entry cost per product is again ϵ.

Suppose now that $\mathbf{a}^*$ forms an equilibrium configuration, which induces the profile of qualities $\mathbf{u}^*$. Denote by M_i the set of products in firm i's portfolio at the end of the game. The viability and stability conditions can then be expressed in the space of observables as follows:

(i") For all i, $i \in \{1, ..., l\}$,

$$S\Pi\left(\mathbf{u}_i^*; \mathbf{u}_{-i}^*\right) - \sum_{k \in M_i} \left(F_o u_k^\beta + \epsilon\right) \geq 0.$$

(ii") There does not exist a vector of qualities, $\mathbf{u}_{n+1}$, $\mathbf{u}_{n+1} \neq \mathbf{0}$, such that

$$S\Pi\left(\mathbf{u}_{n+1}; \mathbf{u}^*\right) - \sum_{k \in M_{n+1}} \left(F_o u_k^\beta + \epsilon\right) > 0.$$

Again, if firms' actions merely consist in the choice of product qualities (and the number of products), then these two conditions coincide with (i) and (ii).

4.2 Equilibrium Configurations

In the following, we study equilibrium configurations in exogenous and endogenous sunk cost industries, respectively. We investigate, in particular, whether concentrated outcomes can be sustained as equilibrium configurations in large markets.

4.2.1 Exogenous Sunk Cost Industries

Before turning to the equilibrium analysis of the exogenous sunk cost case, let us introduce some further notation. We denote by $S\pi(m_i;\mathbf{m}_{-i})$ firm i's final-stage profit *per product*, i.e. $\pi(m_i;\mathbf{m}_{-i}) \equiv \Pi(m_i;\mathbf{m}_{-i})/m_i$. From equation (3),

$$\pi\left(m_i;\mathbf{m}_{-i}\right) = \frac{(1-\sigma)\left[1-\sigma+(\overline{m}-m_i)\sigma\right]}{2\left[1-\sigma+\overline{m}\sigma\right]\left[2(1-\sigma)+(2\overline{m}-m_i)\sigma\right]^2\left[1-\sigma\sum_{j=1}^{l}\frac{m_j}{2(1-\sigma)+(2\overline{m}-m_j)\sigma}\right]^2}.$$

For a given number of products, industry profits are clearly maximised under monopoly. Does this imply that monopoly will endogenously emerge in equilibrium? Not necessarily, as we have seen in the section on endogenous horizontal mergers. The reason is that a firm, by staying out of a coalition, may be better off than by joining. In fact, we have shown (abusing notation slightly) that

$$\pi(1;\overline{m}-1) > \pi(\overline{m};0) \text{ for } \overline{m} \text{ sufficiently large.} \tag{12}$$

In an open membership game, firms will, therefore, not endogenously merge to monopoly if the number of firms in the industry is sufficiently large.

This "inefficient" outcome may be viewed as being due to some coordination failure. One may, therefore, think that if firms were allowed to renegotiate on coalition formation, and make side payments, monopoly could be achieved. However, any renegotiation should be modelled explicitly, and it is a priori not clear whether such renegotiation would lead to an efficient outcome. Firms, anticipating renegotiation, would have even less incentives to merge prior to renegotiation. More importantly, renegotiation has bite only if it takes place after all entry has occurred. The present model formalises this idea: there is a penultimate stage at which firms may renegotiate earlier agreements and, simultaneously, new entry may occur.

In the first part of the paper, we have shown that (12) holds. Below, we prove a stronger result:

$$\pi(1;\overline{m}) > \pi(\overline{m};0) \text{ for } \overline{m} \text{ sufficiently large.} \tag{13}$$

Ex-post entry in conjunction with this claim are sufficient to imply that monopoly will not occur in large markets. To see this, suppose that $(\overline{m},0)$ is sustainable as an equilibrium configuration. Viability and stability require

$$\pi(\overline{m};0) \geq \epsilon/S$$

and

$$\pi(1;\overline{m}) \leq \epsilon/S, \tag{14}$$

which imply

$$\pi(\overline{m};0) \geq \pi(1;\overline{m}). \tag{15}$$

Let $\widehat{m}(S/\epsilon)$ denote the maximum integer such that $\pi(1;\widehat{m}(S/\epsilon)) \geq \epsilon/S$. Since $\pi(1;\overline{m})$ is strictly decreasing in $\overline{m}$, and $\lim_{\overline{m}\to\infty}\pi(1;\overline{m}) = 0$, we have $\widehat{m}(S/\epsilon) \to \infty$ as $S/\epsilon \to \infty$. Configuration $(\overline{m},0)$ satisfies the stability condition (14) if $\overline{m} \geq \widehat{m}(S/\epsilon)$. From equation (13), it then follows that $\pi(1;\overline{m}) > \pi(\overline{m};0)$ in sufficiently large markets. But this is in contradiction to (15).

Let us now show that equation (13) does indeed hold. It is straightforward to compute that

$$\pi(\overline{m};0) = \frac{1}{8\left[1-\sigma+\overline{m}\sigma\right]}$$

and

$$\pi(1;\overline{m}) = \frac{(1-\sigma)\left[1+(\overline{m}-1)\sigma\right]\left[2+\overline{m}\sigma\right]^2}{2[1+\overline{m}\sigma]\left[4+4(\overline{m}-1)\sigma-\overline{m}\sigma^2\right]^2}.$$

Taking the limit as $\overline{m}$ tends to infinity, we obtain

$$\lim_{\overline{m}\to\infty}\left\{\pi(\overline{m};0)-\pi(1;\overline{m})\right\} < 0,$$

which proves the claim.

We have thus shown that it is possible to exclude monopoly outcomes in large exogenous sunk cost industries. In fact, we are able to obtain a much stronger result: the upper bound to concentration goes to zero as market size tends to infinity.

Proposition 7 *For any $(\sigma,\gamma) \in (0,1)^2$, there exists a threshold level $\left(\overline{S/\epsilon}\right)(\sigma;\gamma)$ such that for all $S/\epsilon \geq \left(\overline{S/\epsilon}\right)(\sigma;\gamma)$, the market share of the largest firm is bounded above by γ in any equilibrium configuration.*

The proof of proposition 7 is similar to that of proposition 2, and can be found in the appendix. The proposition shows that our previously derived predictions regarding exogenous sunk cost industries are robust; they do not depend on the details of the extensive form of the game, provided we allow for ex-post entry.

Proposition 7 characterises equilibrium configurations. Existence of an equilibrium configuration in the exogenous sunk cost case can be shown as follows. Let $\mathbf{1}_n$ denote the n-tuple $(1,1,...,1)$. Then, define $n(\epsilon/S)$ as the maximum integer n such that $\pi\left(1;\mathbf{1}_{n-1}\right) \geq \epsilon/S$. From assumption (11), $n(\epsilon/S) \geq 1$. We claim that the maximally fragmented market structure $\mathbf{1}_{n(\epsilon/S)}$ can be sustained as an equilibrium configuration. To see this, notice that $\mathbf{1}_{n(\epsilon/S)}$ satisfies the viability condition by definition. Moreover, for any $m \in \{1,2,...\}$, $\pi\left(m;\mathbf{1}_{n(\epsilon/S)}\right) \leq \pi\left(1;\mathbf{1}_{n(\epsilon/S)}\right) < \epsilon/S$, so that the stability condition holds as well. This proves the claim.

4.2.2 Endogenous Sunk Cost Industries

In the rather specific endogenous horizontal merger model of section 3.2, we have shown that, in endogenous sunk cost industries, it is possible to sustain very concentrated outcomes, even monopoly, no matter how large the market. We are now in the position to show that this prediction carries over to the current setting.

Proposition 8 *If products are sufficiently good substitutes (σ close to 1), or investment in quality enhancement sufficiently effective (β close to 2), monopoly can be sustained in an equilibrium configuration. This holds independently of the level of market size relative to setup costs, provided the market is not too small so as to support at least one firm.*

Proof. Using the notation of section 3.2, suppose the candidate monopolist offers one product, which is of quality $\overline{u}(1)$. In section 3.2, we have shown that if $\sigma \in (\sigma_1(\beta), 1))$, then, even in the limit as market size relative to setup costs tends to infinity, is entry unprofitable for a firm, which is restricted to offer only one product. Moreover, the monopolist makes positive profits, provided market size (relative to setup costs) is not too small. (Notice that the level of σ has no impact on the profit of a single-product monopolist.) The proof of the extension of the result to the case of a multiproduct entrant proceeds as follows. The first step consists in showing that a multiproduct entrant optimally chooses the same quality for all of its products. The second and final step consists in observing that the final stage profit *per product* is decreasing in the number of own products, holding quality fixed. ■

Although the possibility of ex-post entry works against the emergence of concentrated outcomes, there are several reasons why, in the present model, monopoly may be sustained in equilibrium for a larger set of parameter values than in the model of section 3.2. First, in the present model, entry deterrence through quality investment (and product proliferation) is consistent with the concept of an equilibrium configuration. In contrast, in the model of section 3.2, entry deterrence is not consistent with subgame perfection since all firms invest simultaneously in quality. Second, the present equilibrium concept does not require the candidate monopolist to maximise profits. In particular, the monopolist may overinvest in quality relative to the profit-maximising level. Third, in the earlier model, a multiproduct firm can only emerge through mergers, and mergers are, potentially, subject to coordination failures. In the present setup, such coordination failures are muted; for instance, the candidate monopolist may simply select the number of products so as to deter entry.

Let us show the existence of an equilibrium configuration for *any* set of parameter values satisfying (11). Using the notation of section 3.2, suppose n firms offer an product of quality $\overline{u}(n)$ each. Hence, each firm's profit (gross of entry cost ϵ) is given by $\Pi^*(n)$. Let $\overline{n}(\epsilon)$ denote the largest integer n such that $\Pi^*(n) \geq \epsilon$; from (11), $\overline{n}(\epsilon) \geq 1$. Then, we claim that $(\overline{u}(\overline{n}(\epsilon)), ..., \overline{u}(\overline{n}(\epsilon)))$ can be sustained as an equilibrium configuration. Indeed, the viability condition is satisfied by construction. Moreover, as we have already shown in section 3.2, additional entry by a single-product firm can not be profitable. Following the argument in the proof of proposition 8, entry by a multiproduct firm must be unprofitable as well. Hence, the stability condition holds as well.

5 Conclusion

The aim of this paper has been to sharpen the predictions of the game-theoretic literature on industrial market structure. In his book on the relationship between market size and concentration, Sutton (1991) showed that *fragmented outcomes* can in general be sustained in large exogenous sunk cost industries, but not in large endogenous sunk cost industries. The question addressed in this paper has been whether it is possible to make predictions as to in what kind of industries it is possible to sustain *concentrated outcomes*, and in what kind of industries it is not. Using an endogenous horizontal merger model with free but costly entry, we have shown, in the first part of the paper, that it is impossible to sustain concentrated outcomes in large exogenous sunk cost industries. More precisely, the upper bound to the one-firm concentration ratio goes to zero as market size (relative to setup costs) tends to infinity. In contrast, in endogenous sunk cost industries, where firms can invest in some fixed R&D or advertising outlays to increase the (perceived) quality of their products, arbitrarily concentrated outcomes can be sustained even in the absence of mergers, no matter how large the market. In the second part of the paper, we have shown that the same results obtain independently of the details of the extensive form of the game, and allowing for side payments between firms and endogenous product choice, provided one allows for ex-post entry.

We believe that the predictions of this paper are robust. For instance, it is possible to show that the conclusions of the paper do not hinge on the assumption of price competition. In fact, under quantity competition, the incentive to take a free ride on rivals' effort to restrict output is larger than under price competition, so that it is more difficult to obtain concentrated outcomes in exogenous sunk cost industries. The possible emergence of very concentrated outcomes in large endogenous sunk cost industries is not affected. More research on the robustness of our results is needed.

6 Appendix

Proof of lemma 1. Since $U(\mathbf{x}; Y - \sum_k p_k x_k)$ is strictly concave in $\mathbf{x}$, there exists a unique utility-maximising consumption bundle, given any price vector $\mathbf{p}$. That is, each consumer has a well-defined demand function for good k, $d_k(\mathbf{p})$; market demand is $D_k(\mathbf{p}) = S d_k(\mathbf{p})$. Recall that, by assumption, $Y > \sum_k p_k d_k(\mathbf{p})$ in equilibrium. Hence, if $d_k(\mathbf{p}) > 0$, demand for good k is implicitly defined by the first-order condition

$$1 - 2(1-\sigma) d_k(\mathbf{p}) - 2\sigma \sum_{l=1}^{n} d_l(\mathbf{p}) - p_k = 0.$$

Relabel products in increasing order of price, i.e. $p_k \leq p_{k+1}$ for all $k \in \{1, ..., n-1\}$. Define the integer $\overline{n}(\mathbf{p})$ as follows. If $p_1 > 1$, then $\overline{n}(\mathbf{p}) = 0$; otherwise, let $\overline{n}(\mathbf{p})$ be the largest integer v, $v \leq n$, such that $(1-\sigma)(1-p_v) - v\sigma\left(p_v - (1/v)\sum_{l=1}^{v} p_l\right) \geq 0$. Demand (per consumer) for good k can now be written as

$$d_k(\mathbf{p}) = \frac{(1-\sigma)(1-p_k) - \overline{n}(\mathbf{p})\sigma\left(p_k - \frac{1}{\overline{n}(\mathbf{p})}\sum_{l=1}^{\overline{n}(\mathbf{p})} p_l\right)}{2(1-\sigma)\left[1-\sigma+\overline{n}(\mathbf{p})\sigma\right]} \tag{16}$$

if $k \leq \overline{n}(\mathbf{p})$, and $d_k(\mathbf{p}) = 0$ otherwise. Notice that, although $\overline{n}(\mathbf{p})$ takes only integer values, $d_k(\mathbf{p})$ is continuous in $\mathbf{p}$.

Let $\mathbf{p}_{M_i}$ denote the vector of coalition M_i's prices, and $\mathbf{p}_{-M_i}$ the price vector of its rivals. Coalition M_i sets the prices of its products so as to maximise its profit:

$$\max_{\{p_k\}_{k\in M_i}} \sum_{k\in M_i} p_k D_k(\mathbf{p}_{M_i}, \mathbf{p}_{-M_i}).$$

Since the demand function possesses (a finite number of) kinks, a firm's best-reply function is not continuous everywhere. Denote by $\mathbf{p}_{-k} \equiv (p_1, ..., p_{k-1}, p_{k+1}, ..., p_n)$ the vector of prices of all goods other than k. Let us now make three observations. First, there exists a $\overline{p}(\mathbf{p}_{-k})$ such that $d_k(p_k, \mathbf{p}_{-k}) > 0$ if and only if $p_k < \overline{p}(\mathbf{p}_{-k})$. It is easy to see that $\overline{p}(\mathbf{p}_{-k}) > 0$ for all nonnegative $\mathbf{p}_{-k}$, which implies that each firm make positive profit in equilibrium. Second, each product makes positive sales in equilibrium. To see this, suppose that good k, $k \in M_i$, makes zero sales. But then, M_i could raise its profit by setting p_k slightly below $\overline{p}(\mathbf{p}_{-k})$, holding all other prices fixed. Third, coalition M_i's profit is continuous in $\mathbf{p}$; it is strictly concave in $\mathbf{p}_{M_i}$ for any $\mathbf{p}_{-M_i}$, provided that prices are such that $p_k < \overline{p}(\mathbf{p}_{-k})$ for all $k \in M_i$. These observations together imply that the set of first-order conditions is necessary and sufficient for $\mathbf{p}^*$ to form a Nash equilibrium.[18] Hence, equilibrium price p_k^*, $k \in M_i$, is implicitly defined by

$$1 + \frac{2\sigma}{1-\sigma}\sum_{l\in M_i} p_l^* + \frac{\sigma}{1-\sigma}\sum_{j\notin M_i} p_j^* = 2\left(\frac{1-\sigma+n\sigma}{1-\sigma}\right)p_k^*. \tag{17}$$

Since the left-hand side of (17) is independent of k, it follows that $p_k^* = p_{M_i}^* < \overline{p}(\mathbf{p}_{-k}^*)$ for all $k \in M_i$. That is, a merged entity sets the same price for each of its products.

We can now rewrite the first-order condition as follows

$$p_{M_i}^* = \frac{1-\sigma+\sigma\sum_{j\in Z} m_j p_{M_j}^*}{2(1-\sigma)+(2n-m_i)\sigma}. \tag{18}$$

Multiplying both sides with m_i, and summing over all coalitions, gives

$$\sum_{j\in Z} m_j p_{M_j}^* = \frac{(1-\sigma)\sum_{j\in Z}\frac{m_j}{2(1-\sigma)+(2n-m_j)\sigma}}{1-\sigma\sum_{j\in Z}\frac{m_j}{2(1-\sigma)+(2n-m_j)\sigma}}. \tag{19}$$

[18] In the following, we suppress the dependence of strategies on $\mathbf{z}$ for notational simplicity.

Inserting (19) into (18) yields the (unique) equilibrium price of coalition M_i's products, as given by equation (2).

Using (16) and (2), we can now calculate the market demand per product of coalition M_i as

$$D_{M_i}(\mathbf{p}^*) = S\frac{1-\sigma+(n-m_i)\sigma}{2(1-\sigma)\left[1-\sigma+n\sigma\right]} \cdot p^*_{M_i}. \tag{20}$$

It is straightforward to verify that $\sum_k p^*_k d_k(\mathbf{p}^*) < 1/8\sigma$, so that the assumption on income indeed ensures that income is higher than the consumer's equilibrium expenditure on the n substitute goods.[19] ■

Proof of proposition 1. Let $S\pi\,(m; n-m)$ denote the profit per product of a coalition with m members, facing a single nonempty rival coalition with $n-m$ members. Merger to monopoly can be sustained in equilibrium if and only if

$$\pi(n; 0) \geq \pi(1; n-1).$$

Using (3), this condition can be rewritten as

$$\begin{aligned} &\left[4(1-\sigma)^2 + 4n\sigma(1-\sigma) + 3(n-1)\sigma^2\right]^2 \\ &-4(1-\sigma)\left[1-\sigma+(n-1)\sigma\right]\left[2(1-\sigma)+(n+1)\sigma\right]^2 \;\; \geq \;\; 0, \end{aligned}$$

which simplifies to

$$\phi(n,\sigma) \equiv (n-1)^2\left[4n-7\right]\sigma^2 + 4(n-1)\left[-n^2+6n-7\right]\sigma + 4\left[-n^2+4n-3\right] \geq 0.$$

It is easily checked that $\phi(2,\sigma) = \sigma^2 + 4\sigma + 4$ and $\phi(3,\sigma) = 20\sigma^2 + 16\sigma$. Hence, if $n \in \{2,3\}$, then merger to monopoly is sustainable for all $\sigma \in (0,1)$. If $n \geq 4$, then $\phi(n,0) < 0$, and $\phi(n,\sigma)$ has a unique positive root, $\widehat{\sigma}(n)$, given by

$$\widehat{\sigma}(n) = \frac{2\left(n^2-6n+7\right) + 2\sqrt{n^4-8n^3+27n^2-44n+28}}{(n-1)(4n-7)}.$$

Note that $\widehat{\sigma}(n) \in (0,1)$ for all $n \geq 4$. That is, if $n \geq 4$, then merger to monopoly can be supported for all $\sigma \in [\widehat{\sigma}(n), 1)$. ■

Proof of proposition 2. Suppose the assertion is false. Then, there exist an increasing sequence $\left\{n^k\right\}_{k=1}^{\infty}$ of numbers of active firms and a sequence of coalition M_i's number of products, $\left\{m_i^k\right\}_{k=1}^{\infty}$, such that M_i's market share, γ_i^k, as measured by the relative number of its products, m_i^k/n^k, is bounded below by γ, i.e. $\gamma_i^k \geq \gamma$ for all k, and such that $\lim_{k\to\infty}\gamma_i^k = \gamma_i^\infty$. (Notice that it is always possible to find a convergent

[19] Note that expenditure is maximised under merger to monopoly, in which case $\sum_k p^*_k d_k(\mathbf{p}^*) = n/\left[8(1-\sigma+n\sigma)\right]$.

subsequence since $\gamma_i^k \in [\gamma, 1]$.) For this to be an equilibrium, a member of M_i should have no incentive to deviate and form a coalition on its own. Formally,

$$\frac{\left[1-\sigma+n^k(1-\gamma_i^k)\sigma\right]}{\left[2(1-\sigma)+n^k(2-\gamma_i^k)\sigma\right]^2 [\Psi^k]^2} \geq \frac{\left[1-\sigma+n^k(1-1/n^k)\sigma\right]}{[2(1-\sigma)+n^k(2-1/n^k)\sigma]^2 \left[\Psi^k+\Phi_i^k\right]^2},$$

where

$$\Psi^k \equiv 1-\sigma \sum_{j\in Z} \frac{n^k\gamma_j^k}{2(1-\sigma)+n^k(2-\gamma_j^k)\sigma},$$

and

$$\begin{aligned}\Phi_i^k \equiv & \frac{\sigma n^k \gamma_i^k}{2(1-\sigma)+n^k(2-\gamma_i^k)\sigma} - \frac{\sigma n^k\left[\gamma_i^k - 1/n^k\right]}{2(1-\sigma)+n^k(2-\gamma_i^k+1/n^k)\sigma} \\ & -\frac{\sigma}{2(1-\sigma)+n^k(2-1/n^k)\sigma}.\end{aligned}$$

This condition can be rewritten as

$$\left(\frac{1-\sigma+n^k(1-\gamma_i^k)\sigma}{1-\sigma+n^k(1-1/n^k)\sigma}\right)\left(\frac{2(1-\sigma)+n^k(2-1/n^k)\sigma}{2(1-\sigma)+n^k(2-\gamma_i^k)\sigma}\right)^2 \geq \left(\frac{\Psi^k}{\Psi^k+\Phi_i^k}\right)^2. \tag{21}$$

Observe that $\lim_{k\to\infty}\Phi_i^k = 0$ since $n^k \to \infty$, and $\gamma_i^k \to \gamma_i^\infty$, as $k \to \infty$. Hence, if Ψ^k is bounded away from zero, the right-hand side of equation (21) converges to 1 as $k \to \infty$, whereas the left-hand side converges to $4(1-\gamma_i^\infty)/(2-\gamma_i^\infty)^2 < 1$. That is, if Ψ^k is bounded away from zero, then for n^k sufficiently large, the above inequality can not hold – a contradiction.

If $\lim_{k\to\infty}\Psi^k = 0$, however, the right-hand side of (21) may not converge to 1. Notice that this case occurs if and only if there exists a firm j such that $\gamma_j^\infty = 1$, and hence $\gamma_k^\infty = 0$ for all $k \neq j$. Now, if $i \neq j$, we are done. The interesting case is when $i = j$, i.e. $\gamma_i^\infty = 1$, so that the left-hand side of (21) converges to zero. In fact, the right-hand side of the equation converges to zero as well, provided that firm 1 is a monopolist, $\gamma_i^k = 1$, for k sufficiently large; but we already know from corollary 1 that monopoly can not be sustained in equilibrium for n sufficiently large. It, therefore, remains to show that the right-hand side of equation (21) is bounded away from zero if $\gamma_i^k \leq (n^k-1)/n^k$, and hence if $\Psi^k > 0$, for sufficiently large k. To show this, remark first that $\phi(\gamma) \equiv n\gamma/\left[2(1-\sigma)+n(2-\gamma)\sigma\right]$ is increasing and convex in γ, which implies, first, that Ψ^k is decreasing in the industry level of concentration (where a rise in the level of concentration is defined as a transfer of a certain number of products from some firm to a weakly larger one) and, second, that Φ_i^k is increasing in γ_i^k. Let us define

$$\underline{\Psi}^k \equiv 1-\frac{\left(n^k-1\right)\sigma}{2(1-\sigma)+(n^k+1)\sigma}-\frac{\sigma}{2(1-\sigma)+(2n^k-1)\sigma},$$

and

$$\overline{\Phi}_i^k \equiv \frac{\left(n^k-1\right)\sigma}{2(1-\sigma)+(n^k+1)\sigma} - \frac{\left(n^k-2\right)\sigma}{2(1-\sigma)+(n^k+2)\sigma} - \frac{\sigma}{2(1-\sigma)+(2n^k-1)\sigma}.$$

If firm i is the largest firm, and $\gamma_i^k \leq (n^k-1)/n^k$, we thus have $\Psi^k \geq \underline{\Psi}^k > 0$, $0 < \Phi_i^k \leq \overline{\Phi}_i^k$, and hence

$$\frac{\Psi^k}{\Psi^k+\Phi_i^k} \geq \frac{\underline{\Psi}^k}{\underline{\Psi}^k+\overline{\Phi}_i^k}.$$

It is straightforward to check that the right-hand side of this inequality is bounded away from zero. Hence, for k sufficiently large, equation (21) does not hold. This completes the proof. ■

Proof of lemma 2. Suppose that income is sufficiently large so that $Y > \sum_k p_k x_k$ in equilibrium. Then, if $d_k(\mathbf{p};\mathbf{u}) > 0$, demand per consumer for good k is given by the first-order condition

$$u_k - 2(1-\sigma)d_k(\mathbf{p};\mathbf{u})/u_k - 2\sigma\sum_{l=1}^{n} d_l(\mathbf{p};\mathbf{u})/u_l - p_k u_k = 0.$$

Relabel firms such that $u_k(1-p_k) \geq u_{k+1}(1-p_{k+1})$ for all $k \in \{1,...,n-1\}$. Define the integer $\overline{n}(\mathbf{p};\mathbf{u})$ in the following way. If $p_1 > 0$, then $\overline{n}(\mathbf{p};\mathbf{u}) = 0$; otherwise, let $\overline{n}(\mathbf{p};\mathbf{u})$ be the largest integer v, $v \leq n$, such that

$$(1-\sigma)u_v(1-p_v) + v\sigma\left[u_v(1-p_v) - \frac{1}{v}\sum_{l=1}^{v} u_l(1-p_l)\right] \geq 0.$$

Demand (per consumer) for k can then be written as

$$d_k(\mathbf{p};\mathbf{u}) = \frac{(1-\sigma)u_k(1-p_k) + \overline{n}(\mathbf{p};\mathbf{u})\sigma\left[u_k(1-p_k) - \frac{1}{\overline{n}(\mathbf{p};\mathbf{u})}\sum_{l=1}^{\overline{n}(\mathbf{p};\mathbf{u})} u_l(1-p_l)\right]}{2(1-\sigma)\left[1-\sigma+\overline{n}(\mathbf{p};\mathbf{u})\sigma\right]} \cdot u_k$$

if $k \leq \overline{n}(\mathbf{p};\mathbf{u})$, and $d_k(\mathbf{p};\mathbf{u}) = 0$ otherwise. To simplify the algebraic expressions, let us define $y_k(\mathbf{p};\mathbf{u}) \equiv d_k(\mathbf{p};\mathbf{u})/u_k$, $q_k \equiv p_k u_k$, $\overline{u}_N(\mathbf{q};\mathbf{u}) \equiv (1/\overline{n}(\mathbf{q};\mathbf{u}))\sum_{l=1}^{\overline{n}(\mathbf{q};\mathbf{u})} u_l$, and $\overline{q}_N(\mathbf{q};\mathbf{u}) \equiv (1/\overline{n}(\mathbf{q};\mathbf{u}))\sum_{l=1}^{\overline{n}(\mathbf{q};\mathbf{u})} q_l$. We thus get the following (normalised) demand function for good k, $k \leq \overline{n}(\mathbf{q};\mathbf{u})$,

$$y_k(\mathbf{q};\mathbf{u}) = \frac{(1-\sigma)u_k + \overline{n}(\mathbf{q};\mathbf{u})\sigma\left[u_k - \overline{u}_N(\mathbf{q};\mathbf{u})\right] - (1-\sigma)q_k - \overline{n}(\mathbf{q};\mathbf{u})\sigma\left[q_k - \overline{q}_N(\mathbf{q};\mathbf{u})\right]}{2(1-\sigma)\left[1-\sigma+\overline{n}(\mathbf{q};\mathbf{u})\sigma\right]},$$

which is continuous in $\mathbf{q}$ and $\mathbf{u}$.

Coalition M_i's best reply to $\mathbf{q}_{-M_i}$ is given by the solution of the following optimisation programme:

$$\max_{\{q_k\}_{k\in M_i}} \sum_{k\in M_i} y_k(\mathbf{q};\mathbf{u})q_k.$$

In the following, we attempt to characterise equilibrium. Suppose $\mathbf{q}^*$ forms a Nash equilibrium.[20] If $y_k(0, \mathbf{q}_{-k}) > 0$, define the threshold price $\overline{q}_k(\mathbf{q}_{-k})$ such that $y_k(0, \mathbf{q}_{-k}) > 0$ if and only if $q_k < \overline{q}_k(\mathbf{q}_{-k})$; otherwise, let $\overline{q}_k(\mathbf{q}_{-k}) = 0$.[21] Observe that M_i's profit is strictly concave in q_k on $(0, \overline{q}_k(\mathbf{q}_{-k}))$, holding all other prices fixed. Hence, if $q_k^* \in (0, \overline{q}_k(\mathbf{q}_{-k}^*))$, then q_k^* is implicitly defined by the first-order condition

$$(1-\sigma)u_k + \overline{n}\sigma(u_k - \overline{u}_N) - (1-\sigma)q_k^* - \overline{n}\sigma(q_k^* - \overline{q}_N^*) + \overline{m}_i\sigma\overline{q}_{M_i}^* - (1-\sigma+\overline{n}\sigma)q_k^* = 0, \quad (22)$$

where $\overline{m}_i$ is the number of M_i's products with positive sales, and $\overline{q}_{M_i}^*$ the average normalised price of these products. Clearly, $\overline{n} = \sum_{j \in Z} \overline{m}_j$. Taking averages over M_i's products with positive sales, yields

$$\overline{q}_{M_i}^* = \frac{(1-\sigma)\overline{u}_{M_i} + \sigma \sum_j \overline{m}_j(\overline{u}_{M_i} - \overline{u}_{M_j}) + \sigma \sum_j \overline{m}_j \overline{q}_{M_j}^*}{2(1-\sigma) + (2\overline{n} - \overline{m}_i)\sigma}. \quad (23)$$

Multiplying both sides with $\sigma\overline{m}_i$, and summing over all coalitions, gives

$$\sigma \sum_j \overline{m}_j \overline{q}_{M_j}^* = \frac{\sum_j \frac{\sigma\overline{m}_j\left[(1-\sigma+\overline{n}\sigma)\overline{u}_{M_j} - \sum_i \sigma\overline{m}_i\overline{u}_{M_i}\right]}{2(1-\sigma)+(2\overline{n}-\overline{m}_j)\sigma}}{1 - \sum_j \frac{\sigma\overline{m}_j}{2(1-\sigma)+(2\overline{n}-\overline{m}_j)\sigma}}. \quad (24)$$

We obtain M_i's average equilibrium price, (8), by inserting (24) into (23). Using (22), we finally get the (normalised) equilibrium price of good k, $k \in M_i$, provided it makes positive sales:

$$q_k^* = \overline{q}_{M_i}^* + \frac{u_k - \overline{u}_{M_i}}{2}.$$

■

Proof of proposition 7. For stability condition (ii') to hold, we must have $\overline{m} \to \infty$ as $S/\epsilon \to \infty$. Hence, it suffices to show that, for large $\overline{m}$, the market share of any firm is bounded above by γ. The proof proceeds along the lines of that of proposition 2.

Suppose the assertion is false. Then, there exist an increasing sequence $\left\{\overline{m}^k\right\}_{k=1}^{\infty}$ of number of products and a sequence of the number of firm i's products, $\left\{m_i^k\right\}_{k=1}^{\infty}$, such that firm i's market share γ_i^k, as measured by the relative number of its products, $m_i^k/\overline{m}^k$, is bounded below by γ, i.e. $\gamma_i^k \geq \gamma$, and such that $\lim_{k\to\infty} \gamma_i^k \equiv \gamma_i^\infty$. For $\left(\overline{m}^k\boldsymbol{\gamma}^k\right)$ to be sustainable in an equilibrium configuration, we must have

$$\pi\left(\overline{m}^k\gamma_i^k; \overline{m}^k\boldsymbol{\gamma}_{-i}^k\right) \geq \pi\left(1; \overline{m}^k\boldsymbol{\gamma}^k\right),$$

[20] In the remainder, we suppress dependence of strategies on $(\mathbf{z}, \mathbf{u})$ for notational simplicity. Moreover, we abstract from the problem that products with zero sales may nevertheless constrain equilibrium.

[21] The function $\overline{q}_k(\cdot)$ varies across k for two reasons. First, different products are produced by different coalitions. Second, goods produced by the same coalition may differ in quality.

where $\boldsymbol{\gamma}_{-i}^k$ is the vector of market shares of firm i's rivals. Let us reformulate this inequality as

$$\frac{\pi\left(\overline{m}^k\gamma_i^k;\overline{m}^k\boldsymbol{\gamma}_{-i}^k\right)}{\pi\left(1;\overline{m}^k\boldsymbol{\gamma}^k\right)}=\left(\frac{\Psi^k+\Omega^k}{\Psi^k}\right)^2\Theta_i^k\geq 1, \tag{25}$$

where

$$\begin{aligned}\Psi^k &\equiv 1-\sigma\sum_j\frac{\overline{m}^k\gamma_j^k}{2(1-\sigma)+\overline{m}^k(2-\gamma_j^k)\sigma},\\ \Omega^k &\equiv -\frac{\sigma}{2(1-\sigma)+(2\overline{m}^k+1)\sigma}\\ &\quad +\sigma\sum_j\left(\frac{\overline{m}^k\gamma_j^k}{2(1-\sigma)+\overline{m}^k(2-\gamma_j^k)\sigma}-\frac{\overline{m}^k\gamma_j^k}{2(1-\sigma)+\overline{m}^k\left(2\left(1+\frac{1}{\overline{m}^k}\right)-\gamma_j^k\right)\sigma}\right),\end{aligned}$$

and

$$\Theta_i^k\equiv\frac{\left[1-\sigma+\overline{m}^k(1-\gamma_i^k)\sigma\right]\left[1-\sigma+(\overline{m}^k+1)\sigma\right]\left[2(1-\sigma)+(2\overline{m}^k+1)\sigma\right]^2}{\left[1-\sigma+\overline{m}^k\sigma\right]^2\left[2(1-\sigma)+\overline{m}^k(2-\gamma_i^k)\sigma\right]^2}.$$

It is straightforward to check that

$$\lim_{k\to\infty}\Theta_i^k=\frac{4(1-\gamma_i^\infty)}{(2-\gamma_i^\infty)^2}<1.$$

Notice that $\Theta_i^k,\Psi^k\geq 0$, with the inequalities being strict under all market structures other than monopoly. Let $\lambda\left(\overline{m};m_j\right)\equiv -m_j/\left[2(1-\sigma)+(2\overline{m}-m_j)\sigma\right]$. Since $\eta(m_j)\equiv\partial\lambda\left(\overline{m};m_j\right)/\partial\overline{m}$ is convex in m_j, the candidate equilibrium market structure that maximises Ω^k for a given $\overline{m}^k$, is monopoly; that is,

$$\Omega^k\leq-\frac{\sigma}{2(1-\sigma)+(2\overline{m}^k+1)\sigma}+\left(\frac{\sigma\overline{m}^k}{2(1-\sigma)+\overline{m}^k\sigma}-\frac{\sigma\overline{m}^k}{2(1-\sigma)+\left(\overline{m}^k+2\right)\sigma}\right).$$

It is easy to see that the right-hand side of this inequality converges to zero as $\overline{m}^k\to\infty$; hence, $\lim_{k\to\infty}\Omega^k=0$. If Ψ^k does not converge to zero, i.e. if there is no firm j with $\gamma_j^\infty=1$, one obtains

$$\lim_{k\to\infty}\left(\frac{\Psi^k+\Omega^k}{\Psi^k}\right)\Theta_i^k=\frac{4(1-\gamma_i^\infty)}{(2-\gamma_i^\infty)^2}<1,$$

which is in contradiction to equation (25).

If $\lim_{k\to\infty}\Psi^k=0$, however, $\left(\Psi^k+\Omega^k\right)/\Psi^k$ may not converge to 1. This case occurs if and only if there is a firm j such that $\gamma_j^\infty=1$, and $\gamma_l^\infty=0$ for all $l\neq j$. Accordingly, suppose $\gamma_i^\infty=1$. Since we have already shown in the text that monopoly can not be sustained as an equilibrium configuration in large markets, let us assume, moreover, that $\gamma_i^k \leq \left(\overline{m}^k - 1\right)/\overline{m}^k$. Under this assumption,

$$\Psi^k \geq \underline{\Psi}^k \equiv 1 - \frac{\left(\overline{m}^k - 1\right)\sigma}{2(1-\sigma) + (\overline{m}^k + 1)\sigma} - \frac{\sigma}{2(1-\sigma) + (2\overline{m}^k - 1)\sigma},$$

and

$$\begin{aligned}\Omega^k \leq \overline{\Omega}^k \quad \equiv \quad & -\frac{\sigma}{2(1-\sigma) + (2\overline{m}^k + 1)\sigma} \\ & + \sigma\left(\frac{1}{2(1-\sigma) + (2\overline{m}^k - 1)\sigma} - \frac{1}{2(1-\sigma) + (2\overline{m}^k + 1)\sigma}\right) \\ & + \sigma\left(\frac{\overline{m}^k - 1}{2(1-\sigma) + (\overline{m}^k + 1)\sigma} - \frac{\overline{m}^k - 1}{2(1-\sigma) + (\overline{m}^k + 3)\sigma}\right),\end{aligned}$$

since $\lambda(\overline{m}; m_j)$ is concave in m_j, and $\partial\lambda(\overline{m}; m_j)/\partial\overline{m}$ convex in m_j, respectively. Accordingly,

$$\frac{\Psi^k + \Omega^k}{\Psi^k} \leq \max\left\{\frac{\underline{\Psi}^k + \overline{\Omega}^k}{\underline{\Psi}^k}, 1\right\}.$$

It is easily verified that the right-hand is bounded from above. Hence, if $\gamma_i^\infty = 1$, we obtain

$$\lim_{k\to\infty}\left(\frac{\Psi^k + \Omega^k}{\Psi^k}\right)\Theta_i^k = 0.$$

Again, this is in contradiction to equation (25). ■

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