Expectations and Information in Second Generation Currency Crises Models

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This version: January 2000

We are very grateful to Matteo Bugamelli, Robert Flood, Olivier Jeanne, Francesco Lippi, Alessandro Prati, Nicolas Sahuguet and Fabiano Schivardi for useful comments and helpful discussions. We also thank seminar participants at the Bank of Italy and at Georgetown University (1999 Southeast International Economics Conference) for many insightful suggestions. This paper does not necessarily reflect the views of the Bank of Italy.

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Abstract

We explore the role of expectations in second generation currency crises models, proving that sudden shifts in speculators' behavior can trigger currency devaluations, even without any sizable worsening of the fundamentals. Our model shows that "small" (mean-preserving) changes of speculators' beliefs may drive agents to a unique equilibrium with a self-fulfilling attack. Following a recent line of research, we also compare the results of private and public information models, finding the following paradox: releasing public information seems to be more convenient when fundamentals are bad.

JEL classification: F31, D82, D84.
Keywords: Speculative Attacks, Exchange Rate Crises, Global Games.

1 Introduction

The financial turmoil that invested East Asian countries in the summer of 1997 has revealed the limits of theoretical models in explaining actual currency crises episodes. According to many accounts, the event supposed to be the most likely cause of a crisis (a definite worsening of the fundamentals, possibly implied by an unsustainable stance of the economic policy) did not occur, at least in some of the Asian economies struck by speculative attacks.[1] Thus, many economists believe that other factors might have a crucial role in determining the dynamics of a crisis.

In first generation currency crises models (FGMs), originally developed by Krugman (1979) and Flood and Garber (1984), financial crises follow a deterioration of the fundamentals, typically due to inconsistent economic policies. By contrast, second generation models (SGMs), first developed by Obstfeld (1986), turned the attention to the costs and benefits of the fixed exchange rate policy, stressing the importance of the trade-off faced by the government between defending a fixed currency peg and other policy targets.[2] In these models, a devaluation is the government's optimal response to the actions of speculators and can take place as a result of self-fulfilling beliefs, without a previous worsening of the fundamentals. Since speculative attacks raise the cost of defending a fixed exchange rate, SGMs may exhibit self-fulfilling multiple equilibria.[3] In SGMs the space of fundamentals is usually divided in three parts: when fundamentals are "good", there is a unique equilibrium in which the exchange rate is maintained; when fundamentals are "bad", the currency depreciates; when fundamentals fall in an "intermediate" range (the "ripe for attack" zone), both outcomes are feasible.

In a recent paper, Morris and Shin (1998) started a promising strand of analysis, by developing a SGM with incomplete information. They consider speculators having a uniform prior probability distribution over the state of fundamentals that is updated according to the observation of a private

[1] Corsetti et al. (1998) and Radelet and Sachs (1998) express different views of the causes of the Asian crisis.

[2] Targets of the economic policy that can conflict with the defense of a fixed currency peg include: achieving a low level of unemployment, stimulating economic growth, reducing the fiscal burden, supporting a sound banking system. For an overview, see Obstfeld (1996).

[3] For a discussion about the self-fulfilling feature of currency crises, see Obstfeld (1994), Krugman (1996) and the commentaries therein.

signal. Their model, as well as the earliest complete information models, does not allow to examine the role of the distribution of agents' beliefs about the fundamentals. This issue has been neglected in the literature, presumably because one could think that, in an incomplete information framework, only the mean of speculators probability assessment over the fundamentals matters. Hence, if we figure unbiased agents' expectations, the incomplete information framework would not enrich the benchmark analysis and would not modify the structure of the equilibria. Nevertheless, agents' beliefs have often been put forward to explain actual currency crises. For instance, after the Russian crisis, many commentators pointed to an increase in agents' uncertainty as a possible explanation for the transmission of the speculative pressures to other countries (especially in Latin America) that had very limited trade linkages with Russia [see also IMF (1999)]. Yet, typical SGMs do not explain why uncertainty should influence speculative attacks.

In this paper we present two different variants of the incomplete information model studied by Morris and Shin (1998). Our first SGM allows to study the role played by agents' beliefs about the fundamentals in a standard "currency crisis" game. By explicitly introducing these beliefs, we find that the mean of the distribution over the fundamentals is not the sole relevant parameter. In particular, we show that mean-preserving changes of speculators' expectations can result in a shift from a model with multiple equilibria to a model with a unique "attack" equilibrium. Hence, currency crises can be triggered by "small" changes in the distribution of agents' beliefs, even without any underlying deterioration of the fundamentals. Our model differs from Morris and Shin's because it takes into account a generic prior probability distribution and studies how equilibria change together with it. To focus on the effects of uncertainty of agents' beliefs, we simplify the model by neglecting private information.

Our study supports the thesis that *uncertainty matters* and offers also some interesting insights about the multiple equilibria zone of the complete information game. When fundamentals are in this region and agents have unbiased expectations, we show that if uncertainty is sufficiently large, the incomplete information model has a unique equilibrium that entails a speculative attack. In other words, the "good" equilibrium in the "ripe for attack" zone *is not robust* to an increase in uncertainty.

In the second SGM presented in this paper, we analyze how public information affect the structure of the equilibria. Along the lines of the global games study of Carlsson and van Damme (1993), Morris and Shin (1998) proved that the removal of the hypothesis of common knowledge of agents' behavior, implicit in their private information framework, determines a unique equilibrium. Here, we develop a model in which agents observe the same public signal (so that the common knowledge of agents' actions is restored), and we find that multiple equilibria no longer disappear.

Interestingly, by comparing the results of private and public information models, the following paradox emerges: providing public information seems to be more convenient when fundamentals are bad! An inspection on the reasons behind this paradox provides some useful insights on the likelihood of the events that determine the equilibria. When the private information model of Morris and Shin (1998) predicts an equilibrium with a currency attack, the equilibrium with no attack in our public information model envisages an implausible (but feasible) situation in which speculators waste a big payoff that could otherwise be obtained with a small coordination effort. Analogously, when the private information model predicts an equilibrium with no attack, the equilibrium with a currency attack in the public information model foresees speculators getting a small payoff and requires very large coordination. Hence, this comparison highlights that the equilibria ruled out by the private information model that re-emerge in the public information framework seem to be hardly plausible. Therefore, the paradox warns that policy conclusions drawn from models with multiple equilibria can be misleading, especially when considerations on the likelihood of the outcomes are neglected.

The paper is organized as follows. The next section briefly reviews the benchmark model with complete information. In section 3, we analyze a basic incomplete information framework where speculators' expectations over the fundamentals are distributed according to a generic prior probability distribution and we study the consequences of changing its variance. In section 4, we present the model with public information, and compare our results with those of Morris and Shin. Section 5 concludes.

2 The complete information model

In this section, we present the simple game-theoretic formulation of a SGM of complete information proposed by Morris and Shin (1998). This model provides the standard framework on which we further develop the analysis of currency crises in the following sections.

Let us suppose that the economy is characterized by a state of fundamentals θ that can take values over the set $[0,1]$, with $\theta = 1$ corresponding to a situation of "strong fundamentals". In the absence of government interventions, the "natural" (or shadow) exchange rate is given by $f(\theta)$, where f is a continuous and strictly increasing function.[4] The actual exchange rate is pegged by the government at a level e^*, with $e^* \geq f(\theta)$ for all θ. A speculator can either attack the currency (e.g., by short-selling one unit of it over the exchange rate market), or refrain from doing so. If she attacks the currency and the government abandons the defense of the peg, the speculator gets the difference between the previous peg and the "natural" exchange rate, free of the transaction cost: $e^* - f(\theta) - t$; on the other hand, if the government successfully defends the peg, the speculator ends up with a negative payoff (the transaction cost t). If the speculator refrains from attacking the currency, she gets 0. We assume also that $e^* - f(1) < t$ holds; namely, in the best state of fundamentals the "natural" exchange rate is sufficiently close to the pegged level e^*, so that the profit coming from a depreciation is outweighed by the transaction cost t.

The government derives a value $v > 0$ from defending the peg, but he also faces management costs. In particular, the cost of defending the peg is a function of the state of fundamentals and of the proportion of speculators attacking the currency. We denote this cost function by c. Hence, if the government defends the peg when a proportion α of speculators attacks the currency and the fundamentals are at the level θ, his payoff is $v - c(\alpha, \theta)$; if he abandons the peg, we assume that his payoff is zero. The cost function is supposed to be continuous, differentiable, increasing in α and decreasing in θ. Moreover, two hypotheses are introduced to make the problem economically interesting: $c(0,0) > v$ and $c(1,1) > v$. The former inequality states that in the worst state of fundamentals, the cost of defending the peg exceeds the benefit coming from maintaining it, even when no speculator attacks; the latter establishes that when all speculators attack the currency, the cost of defending the peg exceeds the value v, even in the best state of fundamentals.

Let us denote with $\underline{\theta}$ the value of the fundamentals that solves $c(0, \underline{\theta}) = v$; i.e., $\underline{\theta}$ is the value of θ at which, in the absence of any speculative selling, the government is indifferent between defending the peg and leaving it. Hence, when $\theta < \underline{\theta}$, the government finds it profitable to abandon the peg even if no speculator attacks the currency. Similarly, we denote by $\bar{\theta}$ the value that

[4] The exchange rate is defined in terms of units of foreign currency per unit of national currency.

solves $e^* - f(\bar{\theta}) - t = 0$. Whenever $\theta > \bar{\theta}$, although speculators could force the government to abandon the peg, they would get a negative payoff from successfully attacking the currency. Assuming that $\underline{\theta} \leq \bar{\theta}$,[5] we can therefore classify the soundness of the fixed exchange rate by referring to the states of fundamentals:

• In the interval $[0, \underline{\theta})$ fundamentals are such that the government's cost of defending the peg exceeds the value v, even if no speculator attacks. Hence, in this interval the peg is "unstable" since the currency certainly depreciates.

• In the interval $[\underline{\theta}, \bar{\theta}]$, if all speculators attack the currency, the government's cost of defending the peg exceeds the value v and the currency depreciates; on the other hand, if no speculator attacks the currency, the government finds it profitable to hold the peg, and no devaluation occurs. This interval is usually referred to as the "ripe for attack" zone, to underline the possibility of a positive gain from short-selling.

• In the interval $(\bar{\theta}, 1]$ the "natural" exchange rate $f(\theta)$ is sufficiently close to the pegged level e^*, so that any profit coming from the depreciation of the currency is offset by the transaction cost. Then, in this interval the peg is "stable" and no devaluation occurs.

The above tripartition of the state of fundamentals shows that there are two zones in which there exists a unique equilibrium (attack/devalue in the first interval, refrain from attacking/defend the peg in the third one), and one in which the game between speculators and the government entails multiple equilibria. These multiple equilibria have the characteristic of making speculators' expectations self-fulfilling. In fact, consider the interval $[\underline{\theta}, \bar{\theta}]$, and suppose that the government decides whether to defend the peg after having observed the choices of his opponents. If each speculator believes that the currency will depreciate, her best action is to attack the peg. In this case, although the government might have successfully defended the peg with only a "limited" attack, speculators' coordinated choice of attacking forces him to devalue, thereby vindicating their beliefs. On the other hand, if speculators "feel" that the peg is going to be maintained, their best action is to refrain from the attack. This, in turn, allows the government's successful defense of the peg, confirming again his opponents' beliefs.

The most important drawback of the existence of multiple equilibria is that they do not allow precise predictions about the outcome of the game. The sole advise is that there exists an interval of the fundamentals for which

[5] This assumption holds if v is "large" and t is "small".

an attack is possible, but one cannot say whether or when the attack will happen. Besides, the two equilibria in the "ripe for attack" zone (except for the sole value $\bar{\theta}$) are asymmetric: speculators can gain a positive payoff by attacking the currency, while they get just a null payoff by refraining from the attack.

Next two sections provide some extensions to this standard model by analyzing contexts in which speculators are uncertain about the true θ.

3 Incomplete information: the role of agents' expectations

In this section we assume that speculators do not know the true state of fundamentals, but only have expectations about it, in the form of a probability distribution over [0,1]. We also assume that this distribution is common knowledge, that is absolutely continuous with "full support" over [0,1], and we denote by η its probability density function. Such a modification of the original framework allows a better understanding of the role of expectations in second generation currency crises models.

Since θ is not known when agents choose their actions, there cannot be multiple equilibria in the sense of a complete information model (i.e. for a given value of θ). Hence, we can ask whether there are multiple equilibria for a given p.d. η over [0,1]. It is a simple task to verify that multiple equilibria are still possible;[6] nevertheless, by inquiring the conditions that must be fulfilled by η in order to have a model with multiple equilibria, we can get some important insights on the effects of speculators' expectations.

The government, who knows the state of fundamentals θ, takes his decision about the defense of the peg after speculators' have made their choices. Hence, he will use a decision rule $\psi\left(\alpha,\theta\right)$ such that:[7]

$$\psi(\alpha,\theta)=\begin{cases} leave, \text{ if } v-c(\alpha,\theta)\leq 0 \\ defend, \text{ otherwise}\end{cases}.$$

[6] For instance, if η has support only over the subset $(\underline{\theta},\overline{\theta})$, agents know for sure that θ is in the interval with multiple equilibria and can coordinate either on the 'good' (refrain from attack) or on the 'bad' (attack) equilibrium.

[7] We assume that the government chooses to abandon the peg when he is indifferent.

In particular, $\psi(1,\theta) = leave$ for any θ in [0,1] and $\psi(0,\theta) = defend$ for any θ in $(\underline{\theta}, 1]$.

Let us determine the expected payoff of a generic speculator. It is enough to verify that:

• if a speculator i refrains from attacking, she gets 0 whatever the others do;

• if a speculator i attacks while all other agents attack, her expected payoff is given by:

$$\int_0^1 (e^* - f(\theta) - t)\eta(\theta)d\theta = e^* - E\left[f\left(\widetilde{\Theta}\right)\right] - t, \tag{1}$$

since, for any level of θ, an attack by all speculators induces the government to leave the defense of the peg;

• if a speculator i attacks while all other agents refrain from attacking, her expected payoff is:

$$\int_0^{\underline{\theta}} (e^* - f(\theta) - t)\eta(\theta)d\theta - \int_{\underline{\theta}}^1 t\eta(\theta)d\theta, \tag{2}$$

since in the interval $[0, \underline{\theta}]$ the government leaves the defense of the peg, while on $(\underline{\theta}, 1]$ the peg is maintained.

Let us denote the integral (1) with $u(a_i, a_{-i})$, and the expression (2) with $u(a_i, n_{-i})$. The strategy profile in which all agents attack the currency is an equilibrium *iff* $u(a_i, a_{-i}) \geq 0$; the strategy profile in which all agents refrain from attacking is an equilibrium *iff* $u(a_i, n_{-i}) \leq 0$. Let also p be the probability that the state of fundamentals is not larger than $\underline{\theta}$. We can refer to p as the probability of an "unforced" currency depreciation, because the government devalues even if $\alpha = 0$. Thus, we have:

$$u(a_i, a_{-i}) = e^* - E\left[f\left(\widetilde{\Theta}\right)\right] - t, \text{ and}$$

$$u(a_i, n_{-i}) = e^*p - E\left[f\left(\widetilde{\Theta}\right) \mid \widetilde{\Theta} \leq \underline{\theta}\right] p - t.$$

It is easy to show that, being $u(a_i, a_{-i}) \geq u(a_i, n_{-i})$,[8] speculators' choices of attacking the currency are strategic complements. Hence, noting that both $u(a_i, a_{-i}) \geq 0$ and $u(a_i, n_{-i}) \leq 0$ can hold for the same η, one can distinguish three situations:

• if $u(a_i, a_{-i}) > 0$ and $u(a_i, n_{-i}) > 0$ there is only one equilibrium: all speculators attack the currency and the government abandons the peg;

• if $u(a_i, a_{-i}) < 0$ and $u(a_i, n_{-i}) < 0$ there is only one equilibrium: all speculators refrain from attacking the currency, while the government either abandons or maintains the peg, depending on $\theta \leq \underline{\theta}$ or $\theta > \underline{\theta}$;

• if $u(a_i, a_{-i}) \geq 0$ and $u(a_i, n_{-i}) \leq 0$ there are multiple equilibria: agents can either attack the currency (and force a devaluation) or refrain from doing so (so that the peg is maintained, provided that $\theta > \underline{\theta}$).

We want to focus on the situation with multiple equilibria. After some simple algebra we can get that there are multiple equilibria *iff*:

$$e^* \in \left[E\left[f\left(\widetilde{\Theta}\right)\right] + t, E\left[f\left(\widetilde{\Theta}\right) \mid \widetilde{\Theta} \leq \underline{\theta}\right] + t/p\right]. \qquad (3)$$

Let us denote the above interval with $E \equiv [e_1, e_2]$. We can verify that the condition $e^* \in E$ is a reasonable requirement for multiple equilibria. When the level of the peg e^* is "large" (i.e. $e^* > e_2$), speculators expect a large gain from a successful attack; hence, the strategy profile where agents refrain from the attack is ruled out and the unique equilibrium of the model is the one that yields a massive speculative attack. On the other hand, if e^* is "small" ($e^* < e_1$), speculators expect a negative payoff from a successful attack and the unique equilibrium of the model predicts that speculators do not attack the currency. For "intermediate" levels of e^*, both outcomes are equilibria of the game.

We can get a simple necessary condition for multiple equilibria, by requiring that E is not empty. One can find that $E \neq \emptyset$, *iff*:

$$p \leq \frac{t}{t + E\left[f\left(\widetilde{\Theta}\right)\right] - E\left[f\left(\widetilde{\Theta}\right) \mid \widetilde{\Theta} \leq \underline{\theta}\right]} = s.$$

where, clearly, $s \in (0, 1)$.

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[8] Recall that, by hypothesis, $e^* \geq f(\theta)$ for all θ; then, the condition $e^* \geq E\left[f\left(\widetilde{\Theta}\right) \mid \widetilde{\Theta} > \underline{\theta}\right]$ is always satisfied.

This condition can be used to show that "small" modifications of agents' expectations can have breaking consequences on the exchange rate and make the equilibrium with a self-fulfilling speculative attack the unique feasible outcome. Let us assume that η is such that $p \leq s$ and $e^* \in E$, so that this specification of the parameters entails multiple equilibria. Consider an economy characterized by those parameter levels and where speculators coordinate on a "good" equilibrium, so that the government can maintain the peg. Suppose that speculators' expectations suddenly change from η to a new p.d. η' on $[0,1]$, such that p increases at a level $p' > s$, while $E\left[f\left(\widetilde{\Theta}\right)\right]$, $E\left[f\left(\widetilde{\Theta}\right) \mid \widetilde{\Theta} \leq \underline{\theta}\right]$ and t remain constant.[9] With the new probability distribution η' over $[0,1]$, there cannot be multiple equilibria and the unique equilibrium of the game is the one where all speculators attack the currency, forcing a devaluation.[10] This should not be surprising since an increase in p means that the speculators' expectation of an "unforced" devaluation of the currency increases. However, the change in speculators' beliefs does not necessarily imply a "generalized" worsening of the expectations over the fundamentals, because an increase in p can occur with η' preserving the same mean as η. This point can be clarified by considering the following simple discrete example.

Assume that fundamentals can take only three values: $\theta \in \{\theta_1, \theta_2, \theta_3\}$ with $\theta_1 < \theta_2 < \theta_3$. We suppose also that the parameters are such that: when $\theta = \theta_1$ the government devalues the currency even if no speculator attacks; when $\theta = \theta_2$ there are multiple equilibria; when $\theta = \theta_3$ no speculator attacks the currency and the peg can be maintained. Specifically, we need to assume:

$$\begin{gathered} e^* - f(\theta_3) - t < 0, \\ e^* - f(\theta_2) - t > 0, \\ v - c(0, \theta_1) < 0, \\ v - c(0, \theta_2) > 0. \end{gathered}$$

[9] For instance, one can simply suppose that the new p.d. η' is such that: $\eta'(\theta) = \lambda\eta(\theta)$ for $\theta \in [0, \underline{\theta}]$ (so that $E\left[f\left(\widetilde{\Theta}\right) \mid \widetilde{\Theta} \leq \underline{\theta}\right]$ does not change), and $\lambda > 1$ is big enough to make $p' > s$. Then, one can define η on $(\underline{\theta}, 1]$ to keep also $E\left[f\left(\widetilde{\Theta}\right)\right]$ unchanged. Of course, infinite other choices are also feasible.

[10] We can infer that the "attack" equilibrium is the one that survives, by checking that $u(a_i, a_{-i}) \geq 0$ still holds. Therefore, this modification of the parameters (in particular, the increase in p) makes $u(a_i, n_{-i}) > 0$.

Let us keep the symbol η to denote the probability distribution over the state of fundamentals and denote (coherently with the previous notation) $p = Prob\left(\widetilde{\Theta} = \theta_1\right)$ and $q = Prob\left(\widetilde{\Theta} = \theta_3\right)$, with $p, q > 0$ and $p + q < 1$. The necessary and sufficient conditions to achieve an equilibrium with a speculative attack and an equilibrium without a speculative attack are, respectively:

$$e^* - E\left[f\left(\widetilde{\Theta}\right)\right] - t \geq 0; \tag{4}$$

$$e^* p - f(\theta_1) p - t \leq 0. \tag{5}$$

Hence, the necessary condition for multiple equilibria becomes:

$$p \leq \frac{t}{t + E\left[f\left(\widetilde{\Theta}\right)\right] - f(\theta_1)} = s. \tag{6}$$

Consider an economy characterized by parameter levels such that the inequalities (4), (5) and (6) hold and where agents coordinate their choices so that they refrain from attacking ("good" equilibrium). Small modifications of the parameters can induce a unique equilibrium with a currency devaluation. In fact, if speculators update their expectations to a p.d. η' with a higher probability of an "unforced" devaluation of the currency (so that (5) and (6) no longer hold), they trigger the attack that forces the government to abandon the peg. Notably, the updated p.d. η' can preserve the same mean as η. Indeed, there are infinite p.d. that maintain the same mean as η and have a bigger p: all of them are characterized by an increase in the variance of the speculators' expectation over the fundamentals. In fact, in order to keep the mean constant, p and q must both increase. Hence, in this simple example, the currency attack is triggered by the growth of the uncertainty on the state of fundamentals.[11]

Finally, let us come back to the general case and suppose that η is an unbiased p.d. (i.e. the agents' expected level of θ coincides with the true state

[11] The assertion concerning the uncertainty about the fundamentals cannot be easily generalized to more complex examples. In the general case, the main mean-preserving modification of agents' beliefs that can induce a unique speculative attack is the increase in p.

of fundamentals). It is easy to verify that, when f is linear, the condition $\theta \leq \overline{\theta}$ is necessary and sufficient to have an equilibrium with a speculative attack. Hence, suppose that θ is in the "ripe for attack zone" (so that $\theta \leq \overline{\theta}$), if p is sufficiently high, the strategy profile with no attack can be ruled out, and the only feasible outcome turns out to be the equilibrium that entails a devaluation of the currency. Therefore, even when fundamentals are in a zone that, with complete information, would generate multiple equilibria (and, thus, where agents could refrain from attacking the currency, allowing the peg to be maintained), in the incomplete information case there might be a unique equilibrium with a speculative attack, depending on how big is the speculators' probability assessment of an "unforced" devaluation.[12]

This result highlights the weakness of an economy whose fundamentals are such that the currency is "ripe for attack". When the fundamentals are in the ripe for attack zone, the economy is considered to be *vulnerable* to a currency attack because speculators can trigger a crisis, but this outcome does not necessarily occurs. The incomplete information model shows, instead, that such an economy can be as *fragile* as it is in the "unstable zone", because for some (unbiased) speculators' expectations there is a unique equilibrium with an attack that forces a devaluation.

4 Public and private information

Morris and Shin (1998) have developed a different incomplete information version of the simple model outlined in section 2. In a framework where agents do not know the true state of fundamentals and only have imperfect private information, they show that there exists a unique equilibrium for each state of fundamentals. Their model highlights the importance of the removal of the *common knowledge* hypothesis for the achievement of the latter result. In fact, along the lines of Carlsson and van Damme (1993), the uniqueness of the equilibrium is not the consequence of the uncertainty on the state of fundamentals *per se*; rather, it follows from each agent's uncertainty on the

[12] Perhaps, the following remark is even more worrying: if f is concave, a necessary and sufficient condition for an equilibrium with a speculative attack is $\theta \leq \widehat{\theta}$, for some $\widehat{\theta} > \overline{\theta}$. Hence, with incomplete information, virtually any state of fundamentals could give rise to a speculative attack. Moreover, this result does not depend on the presumed risk-propensity of speculators (since f is not an utility function), but only on the characteristics of the economy.

other players' actions, due to the impossibility of precisely establishing the information received by them. In other words, the uniqueness of the equilibria is not produced by the removal of the hypothesis of common knowledge of the fundamentals, but it follows from the removal of the hypothesis of common knowledge of each player's action, that is implicit in a private information framework.

In this section we compare Morris and Shin's results with those that we achieve in a model with public information; i.e. in a model in which there is common knowledge of the information available to each player, since information is given by an imperfect public signal. Hence, in this model, we restore the hypothesis of common knowledge of agents' actions (because speculators' strategies are given in any equilibrium), and we maintain the lack of common knowledge of fundamentals. Coherently with Morris and Shin, we find that the model has multiple equilibria, although this happens in a zone that might "slightly" differ from the "ripe for attack" zone of the complete information model. Moreover, comparing the two different incomplete information frameworks, we find the following interesting paradox: the diffusion of public information can be more convenient when fundamentals are "bad"!

Let us briefly recall the private information model. With respect to the framework presented in the previous section, Morris and Shin consider a specific prior probability distribution η, given by the uniform p.d.f. over $[0,1]$. They also assume that each agent i observes a signal x_i uniformly and independently (conditional to θ) drawn from the interval $[\theta-\epsilon,\theta+\epsilon]$, where ϵ is a small positive number.[13]

Thus, the game is structured as follows: a) Nature chooses the state of fundamentals according to a uniform p.d.f. over $[0,1]$; b) speculators observe their signal x_i and decide simultaneously whether to attack the currency or to refrain from the attack; c) the government, who knows the true value of θ, observes the share of speculators attacking the currency, and then decides whether to defend the peg or to leave the defense of the exchange rate, allowing a devaluation of the currency to its "natural" level $f(\theta)$.

Within this framework, it is possible to prove the following theorem:

Theorem 1 (Morris and Shin, 1998): There is a unique level θ^* of the fundamentals such that, in any equilibrium of the game

[13] Heinemann and Illing (1999) provide a generalization of Morris and Shin's framework. Their model shows that the uniqueness of the equilibrium holds also with more general probability distributions.

with imperfect private information, the government abandons the currency peg *iff* $\theta \leq \theta^*$.

The reason why multiple equilibria are ruled out can be explained by considering different *orders of knowledge*. For the sake of simplicity, let us suppose that each agent refrains from attacking the currency *iff* she is sure that θ is not in $[0, \underline{\theta}]$ and she is sure that all the other agents also refrain from the attack. Of course, this behavior is not optimal since in the original problem agents can do better by trying to precisely evaluate "where" is θ and to estimate in detail "how many" players attack the currency. A player is sure that θ is not in $[0, \underline{\theta}]$, only if she receives a message $x_i \geq \underline{\theta} + \epsilon$. Hence, if agents do not consider the other players' behavior, they will choose according to a rule R_1 that tells them to attack only if $x_i < \underline{\theta} + \epsilon$. But if an agent assumes that the others are deciding according to R_1, she will be sure that all the others are refraining from the attack only when she receives a message $x_i < \underline{\theta} + 3\epsilon$, because her message may differ by no more than 2ϵ from other agents' messages. Hence, if she wants to refrain from the attack only when she is sure that also all the other players are not attacking, she will use a decision rule R_2 that tells her to attack only if $x_i < \underline{\theta} + 3\epsilon$. However, again, if an agent thinks that the others are deciding according to R_2, she will have to use a decision rule R_3 that tells her to attack only if $x_i < \underline{\theta} + 5\epsilon$. By iterating this argument, one realizes that it is never common knowledge that θ is in a zone where agents surely refrain from the attack.

To get a different intuition for the result, consider that, for small values of ϵ and for some θ in the "ripe for attack' zone, each player can be sure that θ is effectively in the "ripe for attack" zone and can also be sure that her opponents know that θ is in that zone. However, it is not common knowledge that θ is in the "ripe for attack" zone yet. As Carlsson and van Damme (1993) put it, " there is a sharp separation between *knowledge* and *common knowledge*" and this prevents the strict equilibria of the complete information game to be both equilibria also in the game with private information.[14] In fact, for any feasible message the lack of common knowledge forces speculators to compare the potential gains and losses that follow the choice of

[14] The existence of a sharp difference between common knowledge and knowledge (or, more precisely, between common knowledge and "almost common knowledge") was noticed also by Rubinstein (1989), in his famous "electronic mail game".

attacking. Hence, speculators will have to solve their decision problem for the optimal switching point, thereby determining a unique equilibrium.

We now consider a model where any agent receives the same public message x, that we assume to be uniformly drawn from the interval $[\theta - \delta, \theta + \delta]$. This hypothesis maintains the uncertainty over θ and restores the common knowledge of speculators' actions. We can prove the following theorem:

> **Theorem 2**: Let be $\delta \leq \left(\bar{\theta} - \underline{\theta}\right)/4$, there exists a subset $M \equiv (m_1, m_2] \subset [0, 1]$, with $m_1 \in (\underline{\theta}, \underline{\theta} + 2\delta)$ and $m_2 \in (\overline{\theta} - 2\delta, \overline{\theta} + 2\delta)$, such that, if $\theta \in M$, the game with imperfect public information has multiple equilibria.

The proof of the theorem shows that the existence of multiple equilibria is entirely based on the assumption of common knowledge of the players' actions. We provide here a sketch of the proof, and we defer the most technical part to the Appendix.

First of all, we determine the optimal strategy of the government at the last stage of the game. Then, we build two different equilibria and verify that there exist values of the fundamentals for which they both hold.

Let $\beta(\theta)$ be the smallest share of speculators that, by attacking, induce the government to leave the defense of the peg. Of course, $\beta(\theta) = 0$ for $\theta \in [0, \underline{\theta}]$ and β is such that $c(\beta, \theta) - v = 0$ for $\theta \in (\underline{\theta}, 1]$. One can easily find that: i) $\beta \geq 0$; ii) β is a continuous function of θ; iii) $\beta' > 0$.

Then, the government chooses according to the following optimal rule:

$$\psi(\alpha, \theta) = \begin{cases} \textit{leave}, \text{ if } \alpha \geq \beta(\theta) \\ \textit{defend}, \text{ if } \alpha < \beta(\theta) \end{cases}, \text{ for any } \theta \in [0, 1] \, .$$

Given the optimal strategy of the government, we can solve the reduced-form game played by the speculators. Recall that a strategy for a speculator is a function $\varphi : [0, 1] \rightarrow \{\textit{attack}, \textit{do not attack}\}$. Depending on the public message observed, we can single out three cases:

- if $x < \underline{\theta} - \delta$ all the agents know that $\theta \in [0, \underline{\theta})$;
- if $x \in \left(\underline{\theta} + \delta, \overline{\theta} - \delta\right)$ all the agents know that $\theta \in (\underline{\theta}, \overline{\theta})$;
- if $x > \overline{\theta} + \delta$ all the agents know that $\theta \in \left(\overline{\theta}, 1\right]$.

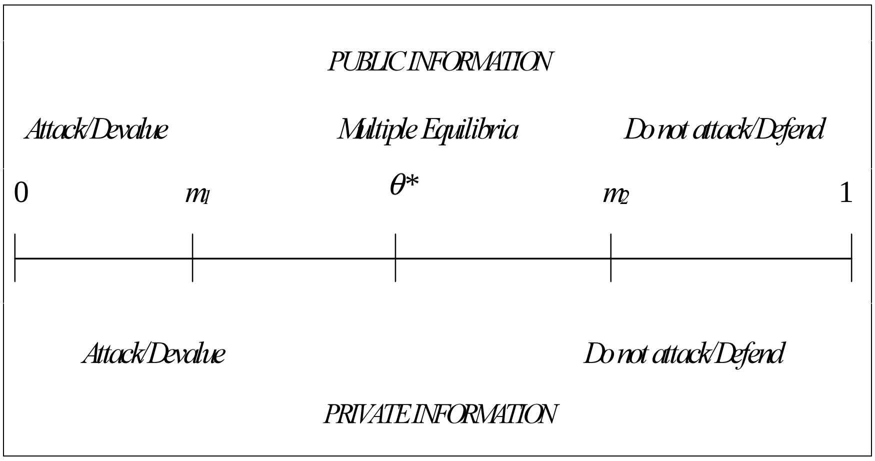
We are going to build two different equilibria. In the first equilibrium $(E1)$ all agents use a strategy φ_{E1} such that $\varphi_{E1}(x) = \textit{attack}$ if $x \leq \underline{\theta} - \delta$,

$\varphi_{E1}(x) =$ *do not attack* if $x \geq \underline{\theta} + \delta$, and $\varphi_{E1}(x)$ has to be determined for $x \in (\underline{\theta} - \delta, \underline{\theta} + \delta)$. In the second equilibrium $(E2)$ all agents use a strategy φ_{E2} such that $\varphi_{E2}(x) =$ *attack* if $x \leq \overline{\theta} - \delta$, $\varphi_{E2}(x) =$ *do not attack* if $x \geq \overline{\theta} + \delta$ and $\varphi_{E2}(x)$ has to be determined for $x \in \left(\overline{\theta} - \delta, \overline{\theta} + \delta\right)$.

It is easy to verify that no speculator has incentive to deviate from φ_{E1} when all the others are following the same strategy, for the values of φ_{E1} that we have specified. The same applies to φ_{E2}. The determination of $\varphi_{E1}(x)$ when $x \in (\underline{\theta} - \delta, \underline{\theta} + \delta)$ and of $\varphi_{E2}(x)$ when $x \in \left(\overline{\theta} - \delta, \overline{\theta} + \delta\right)$ is rather technical and the proof is presented separately in the Appendix. However, the intuition is straightforward. In the first equilibrium the utility from attacking when $x = \underline{\theta} - \delta$ is strictly positive, while it is strictly negative when $x = \underline{\theta} + \delta$. In the Appendix we show that there is a unique message x_1 that, when received, makes the utility from attacking equals to zero. Analogously, in the second equilibrium, the utility from attacking is strictly positive when $x = \overline{\theta} - \delta$, it is strictly negative when $x = \overline{\theta} + \delta$, and we prove in the Appendix that there is a unique message x_2 such that the utility from attacking equals to zero. The two signals x_1 and x_2 are the two switching point of the agents' optimal cut-off strategies. From x_1 we get the unique level of the fundamentals, that we call m_1, such that in the equilibrium $E1$ the currency depreciates *iff* $\theta \leq m_1$. Similarly, from x_2 we get the unique level of the fundamentals, that we call m_2, such that in the equilibrium $E2$ the currency depreciates *iff* $\theta \leq m_2$. Hence, we find an interval $M \in (m_1, m_2]$ where the currency is maintained or not, depending on agents coordinating on $E1$ or $E2$.

The results achieved with the public information model clearly show that the existence of a multiple equilibria zone is not caused by the hypothesis of common knowledge of the fundamentals, but it is due to the hypothesis of common knowledge of the players' actions. In fact, from the one hand we have proved that the relaxation of the former hypothesis alone is not sufficient to get rid of the zone in which there exist more than one equilibrium. On the other hand, the reintroduction of the common knowledge of the agents' actions, still maintaining the uncertainty about the true state of the fundamental, is the cause of the re-emerging of a multiple equilibria framework.

We can finally compare the results of the private and the public information model. By an inspection of the graphical argument proposed by Morris and Shin, one realizes that, in general, it is $\theta^* \in (\underline{\theta}, \overline{\theta} + 2\epsilon)$. However, it is easy to fix parameter values for which $\theta^* \in interior(M)$. The previous models have shown that if the true state of fundamentals is in (m_1, θ^*) and agents only have private information, they will trigger a speculative attack that forces a currency devaluation. Instead, for the same levels of θ, if agents only have public information, there is a "hope" that the currency will not depreciate because agents could coordinate on the "good" equilibrium with no attack.[15] Now suppose that θ is in $(\theta^*, m_2]$; if agents only have private information there will be no speculative attack, while if agents only have public information there could be a speculative attack (because agents can coordinate on the bad equilibrium). This generates the following paradox: providing public information seems to be more convenient when fundamentals are "rather bad" (i.e. in $(m_1, \theta^*]$) than when fundamentals are "rather good" (i.e. in $(\theta^*, m_2]$)![16]



The paradox can be explained by a precise comparison of the private and the public information model, considering the "amount of coordination" required to achieve the higher payoff equilibrium and the "size" of this payoff. This comparison sheds light on the likelihood of the outcomes in the case of

[15] This holds also if agents have both private and public information, as long as δ is "much smaller" then ε. E.g., one can think of releasing public information with $\delta = 0$.

[16] In general, we could also say that providing public information seems to be more convenient when fundamentals are 'bad' (i.e. when $\theta < \theta^*$) than when fundamentals are 'good' (i.e. when $\theta > \theta^*$). In fact, if $\theta^* \leq m_1$ ($\theta^* > m_2$) providing public information does not make any difference since the peg is abandoned (maintained) anyway. Note also that the paradox exists as long as $\theta^* \in interior(M)$. In fact, if $\theta^* = m_2$ ($\theta^* = m_1$), providing public information is always (never) convenient.

multiple equilibria. In the multiple equilibria region, the equilibrium with a coordinated currency attack always yields the highest payoff for speculators. Hence, in order to compare different equilibria for the same state of fundamentals, we can consider the "amount of coordination" required to achieve the highest payoff and the "size" of this payoff.

When fundamentals are "good", speculators holding only private information do not attack the currency because their expected gain from a successful attack is "low", the "amount of coordination" required to get that payoff is "high" (the share of attackers that forces the government to abandon the peg is high) and this makes their expected payoff smaller than zero. The availability of public information, eliminating the uncertainty on the others, offers speculators the opportunity to coordinate on the bad equilibrium and to get also that small positive payoff. However, it does not seem very likely that speculators succeed in achieving a high "amount of coordination" to get just a low payoff.

On the other hand, when fundamentals are "rather bad", speculators' payoff from a successful attack grows and the "coordination effort" required to achieve that payoff decreases. Hence, speculators holding only private information attack the currency because, given that a small "coordination effort" is sufficient to get a high payoff, they think it is likely that also the others attack. With public information, instead, they could refrain from the attack. However, this event does not seem to be very likely, because it foresees speculators wasting a big payoff that could be easily achieved.[17]

This result can be also compared to the comparative statics exercise in Heinemann and Illing (1999). Building on Morris and Shin's private information model, Heinemann and Illing show that a decrease in ε makes θ^* to decrease. According to the authors, the government could reduce ε by committing to a more transparent economic policy. Hence, their result states that an increased transparency of government's policy reduces the likelihood of a currency attack. With respect to our model, this finding implies that when government's economic policy is transparent, the region (m_1, θ^*), where it is convenient for the policy-maker to release public information, becomes smaller. Thus, committing to a more transparent economic policy seems to reduce the benefit from a strategic use of public information.

An interesting extension of the present study would be a more detailed

[17]Note that a similar comparison can be made also between the results of complete and private information models.

examination of the strategic use of public information. This analysis should take into account the possibility that the government finds it profitable to release distort information, and it is potentially complicated by the presence of multiple equilibria (that re-emerge as ε goes to zero or as one explicitly considers precise public information as in our model). The exploration of this issue is beyond the scope of the present paper. However, it could be an important development of the SGMs that could also provide the correct theoretical tool for the evaluation of the benefits from the adherence to programs like the Special Data Dissemination Standard of the International Monetary Fund [see IMF(1999)].

5 Conclusion

The game-theoretic approach of SGMs has proven to be an important line of research for the investigation of the role of speculators' expectations and information in the onset of a currency crisis. A promising strand of analysis is offered by the study of global games, initiated by Carlsson and van Damme (1993), applied to speculative attacks by Fukao (1994), and to second generation currency crises models by Morris and Shin (1998). On the theoretical ground, global games show the importance of the hypothesis of common knowledge of agents' actions for the result of multiple equilibria and give also some insights about the likelihood of the equilibria of complete and public information games.

The results achieved in the paper are consistent with this theory. In fact, with public information, the reintroduction of the common knowledge of agents' actions (still maintaining some uncertainty over the states of fundamentals), leads us to determine the existence of a multiple equilibria zone. Moreover, the comparison between the results of the public and the private information models highlights an interesting paradox: the government has more convenience in providing public information when fundamentals are "bad" than when fundamentals are "good"! However, one realizes that this occurs because the equilibria of the public information model that are eliminated in the private information game rely on the occurrence of implausible events: e.g., getting a small payoff with a large amount of coordination or giving up a big payoff that could have been achieved with a small amount of coordination.

Our example shows that it is easy to draw deceptive conclusions from models with multiple equilibria, especially when any consideration about the likelihood of the outcomes is neglected. Hence, the paradox calls for some equilibrium selection procedure since with multiple equilibria not only game theory can be a "weak and uninformative theory" [Harsany and Selten (1988)] but it can also can lead to wrong policies. Global games, that lead to a unique equilibrium in a wide class of models, can be a very powerful tool in this perspective.

By focusing on speculators' expectations, we also prove that mean-preserving changes of speculators' probability assessments over the state of fundamentals may be sufficient to drive agents to a unique "bad" equilibrium with a self-fulfilling currency attack. Hence, the model suggests an explanation for sudden shifts in speculators' behavior that trigger currency devaluations and that do not seem to be justified by the fundamentals of the economy. In fact, modifications of agents' beliefs that induce speculative attacks can occur even without a worsening of the expected state of fundamentals. Moreover, the model highlights that a crisis can be triggered by an increase of the *uncertainty* over the state of the fundamentals (measured, say, by the variance of the distribution) or, in general, by an increase of the subjective probability of an "unforced" currency devaluation.

Finally, by assuming unbiased players' beliefs, we prove that a mean-preserving change of expectations that induces a shift to a unique equilibrium with a currency crisis may occur when the true state of fundamentals is in the "ripe for attack" zone. Therefore, the model shows that an economy whose fundamentals are in that zone *is not robust* to changes in agents' beliefs due, say, to an increase in the uncertainty. Note that such an economy would be considered *vulnerable* to a currency attack in a complete information model because speculators can trigger a crisis, but this outcome might not occur. The suggestion coming from our incomplete information model is, instead, that such an economy should be regarded as *fragile* as it is in the "unstable" zone because for some unbiased agents' expectations there can be a unique "bad" equilibrium with a speculative attack and a devaluation of the currency.

A Appendix

The proof of Theorem 2 can be completed by using "locally" – i.e. over the intervals $(\underline{\theta} - \delta, \underline{\theta} + \delta)$ and $\left(\overline{\theta} - \delta, \overline{\theta} + \delta\right)$ – the method applied by Morris and

Shin in their private information model. However, our proof can be simplified since we are not interested in proving the uniqueness of the different equilibria for each state of fundamentals. Moreover, the public information framework allows to eliminate any ambiguity in agents' beliefs: when agents use the same decision rule, since they receive exactly the same public message x, they choose also the same actions. Hence, their beliefs about the share of attackers is always either 0 or 1 and, in equilibrium, it coincides with agents' actual behavior.[18] The proof is in five steps and makes use of a lemma that we prove separately in the fourth step.

1. Beliefs

For a given strategy profile of speculators, let $\pi(x)$ be their belief about the share of attackers when the public message is x. This belief is to be determined in equilibrium and must be consistent with speculators' equilibrium strategies. Given θ, the actual share of attackers depends on π (because of the consistency condition) and on the stochastic realization x. Hence, let $\alpha(\theta, \pi)$ be the expected share of attackers given θ and π. Since messages are uniformly distributed on $[\theta - \delta, \theta + \delta]$, we have:

$$\alpha(\theta, \pi) = \frac{1}{2\delta} \int_{\theta-\delta}^{\theta+\delta} \pi(x)dx.$$

2. Expected payoff

Let be $A(\pi) = \{\theta : \alpha(\theta, \pi) \geq \beta(\theta)\}$; when $\theta \in A(\pi)$, agents expect that the currency will be depreciated. Hence, the payoff of an agent that attacks when the state of fundamentals is θ and her belief is π, can be written as:

$$h_a(\theta, \pi) = \begin{cases} e^* - f(\theta) - t, & \text{if } \theta \in A(\pi) \\ -t, & \text{if } \theta \notin A(\pi) \end{cases}.$$

[18] In a private information model, agents' actual choices can be diverse, because they can receive different messages. However, Morris and Shin assume that agents' beliefs about the "aggregate selling strategy" are always 0 or 1, even if, in equilibrium, the actual share can be inside that interval.

However, when agents decide, they do not know θ and they only observe the public message x. Hence, the expected payoff from attacking the currency when they receive x and their belief is π, is given by:

$$u_a(x,\pi) = \frac{1}{2\delta}\int\limits_{x-\delta}^{x+\delta} h_a(\theta,\pi)d\theta = \frac{1}{2\delta}\int\limits_{C_x}(e* - f(\theta))d\theta - t,$$

$$\text{where } C_x = A(\pi) \cap [x-\delta, x+\delta]$$

Speculators' expected payoff from not attacking the currency, instead, is always given by $u_n(x,\pi) = 0$.

3. Strategies

Let us consider a symmetric equilibrium (all speculators use the same strategy). Since speculators receive the same public message, in equilibrium we have $\pi(x) = 0 \Leftrightarrow u_a(x,\pi) \leq 0$ and $\pi(x) = 1 \Leftrightarrow u_a(x,\pi) > 0$.[19] We also consider equilibria where speculators' strategies are given by a cut-off rule of the following kind:[20]

$$\varphi(x) = \begin{cases} \textit{attack}, & \text{if } x < k \\ \textit{do not attack}, & \text{if } x \geq k \end{cases}.$$

These strategies imply that the belief function is an indicator function:

$$\pi(x) = I_k(x) = \begin{cases} 1, & \text{if } x < k \\ 0, & \text{if } x \geq k \end{cases}.$$

We now make use of a lemma demonstrated by Morris and Shin (1998). For ease of the reader, in the following step we report the lemma and its proof (corrected for a minor imperfection), with a notation consistent with the previous steps

[19] We are assuming that speculators choose to refrain from attacking whenever they are indifferent.

[20] In the general framework of global games, Carlsson and van Damme (1993) prove that a cut-off rule is the unique optimal strategy.

4. Lemma: The function $u_a(k, I_k)$ is strictly decreasing and continuous in k.

Consider the function $\alpha(\theta, I_k)$, that represents the expected share of attackers when fundamentals are at level θ and speculators' belief is I_k. Since x is drawn from a uniform p.d.f., we have:

$$\alpha(\theta, I_k) = \begin{cases} 1, & \text{if } \theta \leq k - \delta \\ \frac{1}{2} - \frac{1}{2\delta}(\theta - k), & \text{if } k - \delta < \theta \leq k + \delta \\ 0, & \text{if } \theta < k + \delta \end{cases}.$$

Given k, let us define $\underline{\lambda}(k)$ as the minimum value of λ such that the following relation holds:

$$\alpha(k + \lambda, I_k) = \beta(k + \lambda). \tag{A1}$$

Recall that when $\alpha \geq \beta$ the currency depreciates. Observe that $\underline{\lambda}(k) = \delta$ if $k \leq \underline{\theta} - \delta$, and - $\delta < \underline{\lambda}(k) < \delta$ if $k > \underline{\theta} - \delta$.[21] In particular, in the latter case $\underline{\lambda}(k)$ solves:

$$\frac{1}{2} - \frac{\underline{\lambda}(k)}{2\delta} = \beta(k + \underline{\lambda}(k)). \tag{A2}$$

Since the government abandons the peg only if θ lies in the interval $[0, k + \underline{\lambda}(k)]$, the payoff function from attacking becomes:[22]

$$u_a(k, I_k) = \frac{1}{2\delta} \int_{k-\delta}^{k+\underline{\lambda}(k)} (e^* - f(\theta))d\theta - t. \tag{A3}$$

[21] Note that if $k \geq \underline{\theta} - \delta$ there is a unique value of λ such that (A1) holds; this is not true if $k < \underline{\theta} - \delta$. Thus, here we depart from Morris and Shin's definitions and make the proof more accurate. In particular, we define $\underline{\lambda}(k)$ as " the *minimum* value of λ such that (A1) holds", so that we can deal with a function and not with a correspondence.

[22] It is indifferent to take the minimum or, say, the maximum of λ such that (A1) holds, since between those values we have $f(\theta) = e^*$. Therefore, a different definition for $\underline{\lambda}$ would not modify the value of the integral (A3).

By hypothesis, $e^* - f(\theta)$ is strictly decreasing in θ. Therefore, to prove that $u_a(k, I_k)$ is strictly decreasing in k it is enough to show that $\underline{\lambda}(k)$ is non increasing.

Differentiating (A2) with respect to k yields:

$$\underline{\lambda}'(k) = -\frac{\beta'(\theta)}{\beta'(\theta) + 1/(2\delta)}.$$

Hence, it is $\underline{\lambda}'(k) \leq 0$, which is sufficient to prove the strict monotonicity of u_a. Finally, the continuity follows from the fact that u_a is an integral in which the limits of integration are themselves continuous in k.

5. Equilibrium

We can finally turn to the two equilibria $E1$ and $E2$ defined in section 4. Consider the equilibrium $E1$ where agents attack if $\theta \leq \underline{\theta} - \delta$ and do not attack if $\theta \geq \underline{\theta} + \delta$. Of course, we have: $u_a(\underline{\theta} - \delta, I_{\underline{\theta}-\delta}) > 0$ and $u_a(\underline{\theta} + \delta, I_{\underline{\theta}+\delta}) < 0$. Hence, from the lemma, it follows that there exists a unique value $x_1 \in (\underline{\theta} - \delta, \underline{\theta} + \delta)$ such that $u_a(x_1, I_{x_1}) = 0$. Therefore, agents' optimal rule will be:

$$\varphi_{E1}(x) = \begin{cases} \textit{attack}, & \text{if } x < x_1 \\ \textit{do not attack}, & \text{if } x \geq x_1 \end{cases}.$$

With a simple graphic argument it is easy to show that there is a unique value of the fundamentals $m_1 \in (\underline{\theta}, \underline{\theta} + 2\delta)$ such that, if $\theta \leq m_1$ the currency depreciates, and if $\theta > m_1$ the peg is maintained.

Analogously, in the equilibrium $E2$ where agents attack if $\theta \leq \overline{\theta} - \delta$ and do not attack if $\theta \geq \overline{\theta} + \delta$, since $u_a(\overline{\theta} - \delta, I_{\overline{\theta}-\delta}) > 0$ and $u_a(\overline{\theta} + \delta, I_{\overline{\theta}+\delta}) < 0$, from the lemma follows that there exists a unique value $x_2 \in \left(\overline{\theta} - \delta, \overline{\theta} + \delta\right)$such that $u_a(x_2, I_{x_2}) = 0$. Therefore, agents optimal rule is:

$$\varphi_{E2}(x) = \begin{cases} \textit{attack}, & \text{if } x < x_2 \\ \textit{do not attack}, & \text{if } x \geq x_2 \end{cases}.$$

Hence, we infer that there is a unique value of the fundamentals $m_2 \in (\underline{\theta} - 2\delta, \underline{\theta} + 2\delta)$ such that, if $\theta \leq m_2$ the currency depreciates, and if $\theta > m_2$ the peg is maintained.

Thus, we have found an interval $M \equiv (m_1, m_2]$ where the currency is maintained or not depending on agents coordinating on $E1$ or $E2$. The proof is completed by checking that, if $\delta \leq \left(\overline{\theta} - \underline{\theta}\right)/4$, M is not empty.

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