Increasing Dominance With No Efficiency Effect

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Abstract

I uncover a new force towards increasing dominance (the property whereby, in dynamic games, the leader tends to increase her lead in expected terms). The new effect results from the strategic choice of covariance in races. I assume that players must choose not the *amount* of resources to spend but *how to allocate* those resources. I show that the laggard has an incentive to chose a different path from the leader. In equilibrium, this results in the laggard choosing a less promising path, in effect trading off lower expected value for lower correlation with respect to the leader. This in turn leads to increasing dominance. In order to make the point as clear as possible and differentiate it from the forces previously characterized, I *assume* that no joint payoff (or efficiency) effect is present.

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1 Introduction

An important question in many dynamic games is whether asymmetries between players tend to increase over time (increasing dominance) or decrease over time (catching up). Gilbert and Newbery (1982) characterize conditions under which monopoly dominance persists over time even though there are opportunities for rival firms to challenge an incumbent monopolist. Budd, Harris and Vickers (1993) identify some of the basic forces leading to increasing dominance in the context of a dynamic model of R&D. Cabral and Riordan (1994) provide sufficient conditions for increasing dominance is the context of dynamic competition with learning-by-doing. More recently, Athey and Schmutzler (1999) derive general conditions for increasing dominance in a reduced-form model of dynamic competition.[1]

One common feature of all of these models is that strategies are defined by some measure of effort in trying to move ahead of competitors. Increasing dominance then results when the leader has a greater incentive to exert effort than the laggard. As Budd, Harris and Vickers (1993) put it, "the question is whether the current leader works harder than the laggard—does the 'gap' between firms tend to increase or decrease?" The answer, they show, is that "the [system] tends to evolve in the direction where joint payoffs are greater."

The increasing dominance results of Gilbert and Newbery (1982), Budd, Harris and Vickers (1993), Cabral and Riordan (1994) and Athey and Schmutzler (1999) are all based on this property of dynamic games, together with some form of the *efficiency effect*, the property whereby total payoffs are greater when asymmetries are greater. In other words: suppose that total payoffs are greater when the leader gets farther ahead in a race. Then, in equilibrium the leader will tend to get farther ahead in the race.

In this paper, I uncover an additional force towards increasing dominance, one that is based on the strategic choice of covariance. Suppose that players must choose not the *amount* of resources to spend but *how to allocate* those resources. Specifically, suppose that each player has a fixed amount of resources to spend and must choose between alternative paths of uncertain success. By choosing the same path, the players' success is perfectly correlated. By choosing different paths, success is independent across players.[2] In this context, I show that the laggard in a race has an incentive to chose a different path from the leader. In equilibrium, this results in the laggard choosing a less promising path, in effect trading off lower expected value for lower correlation with respect to the leader. This in turn leads to increasing dominance.

In order to make the point as clear as possible, I *assume* that no joint payoff (or efficiency) effect is present. In this context, any force towards increasing dominance must originate in something other than the joint payoff and efficiency effects.

The paper is organized as follows. In the next section, I introduce a two-player, infinite

[1] Other results related to increasing dominance include Flaherty (1980), Arthur (1989), Klepper (1996), and Bagwell, Ramey and Spuelber (1997).

[2] Hoernig (1999) considers the choice of R&D effort and type of effort. However, he does not address the issue of correlation across players.

period game where players must choose in each period between two alternative paths. Section 3 presents the main results. Section 4 includes a discussion of the results and some concluding remarks.

2 Model definitions and assumptions

Consider an infinite-period game with two players. In each period, the state of the game is summarized by an integer $z \in \mathbb{Z}$. Short run payoffs are summarized by the functions $p_i(z),\ i = 1, 2$. I assume that payoff functions are monotonic and symmetric, i.e. $p_1(z)$ is increasing in z and $p_2(z) = p_1(-z)$ (and thus $p_2(z)$ is decreasing in z). By an abuse of notation that simplifies the analysis, I denote by $p(n)$ the payoff for a player who is "ahead" is state $z = n \geq 0$ (the "leader"); the payoff for the rival player (the "laggard") is therefore $p(-n)$.

One useful way of thinking about the model is that two firms attempt to move up a quality ladder (or down a cost ladder) by exerting R&D effort. In each period, payoffs are determined by the difference in quality levels, $n = q_i - q_j$. Motion across states is therefore determined by the firms' success in moving up the ladder.

A crucial feature of the model is that players must choose between two alternative paths, a and b; and, once a path is chosen, a fixed amount of effort is spent in following that path. If we interpret the model as one of R&D competition, then this amounts to assuming that the R&D budget is fixed and that the only choice is between different research paths. Each path allows players to move up the ladder one step with a positive probability, α and β, respectively, where both α and β are strictly between 0 and 1. If players were to choose paths based on expected value only, then the choice would be trivial—a if $\alpha > \beta$ and b if $\beta > \alpha$. However, selecting a particular path also implies a particular correlation with respect to the rival player's motion. Specifically, I assume that, if both players choose the same path, then either both players move up one step or neither one does. If players choose different paths, however, then the probability of success is independent across players. Finally, if players choose each of the paths with strictly positive probability, then the players' motion is positively but imperfectly correlated. Formally:

Assumption 1 *Success is perfectly correlated for a given period and path, independent across periods and paths.*

A Markov strategy for player i is a map $x_i(n)$, giving the probability of choosing path a in state n. A pair of strategies $x_i(n)$, together with the (common) discount factor δ, induce value functions $v_i(n)$. I treat value functions in terms of average period payoff, so $v_i(n) = (1 - \delta)p_i(n) + \delta v_i^+$, whare v_i^+ is player i's expected continuation value. Moreover, I restrict to symmetric equilibria. For simplicity, if with some abuse of notation, I denote strategies and value functions by $x(n)$ and $v(n)$, respectively.

The main result of the paper is that increasing dominance results even when there is no joint payoff effect. For this purpose, I assume that total payoffs are constant, that is, independent of the state. Specifically, I assume that

Assumption 2 $0 \leq p(n) \leq 1; \;\; p(n) + p(-n) = 1.$

Assumption 2 implies that, if increasing dominance occurs, it does not result from the joint payoff effect. I also assume that a leader achieves her maximum payoff for a finite lead length:

Assumption 3 *There exists an $\bar{n}$ such that, for $n > \bar{n}$, $p(n) = 1$.*

3 Main results

I now present the main results of the paper. Lemma 1 and Corollary 1 characterize the equilibrium when the two paths are equally promising ($\alpha = \beta$). I show that the race then has the nature of a "matching pennies" game, the equilibrium being for players to choose each path with probability $\frac{1}{2}$. If $\alpha > \beta$, however, then the leading player chooses the most promising path with greater probability, which in turn implies increasing dominance—Proposition 1.

Lemma 1 *Suppose that $\alpha = \beta$. Then, in equilibrium and for $n > \bar{n}$ it must be that*

$$x(n) = \begin{cases} 1 & \text{if } x(-n) > 1/2 \\ 0 & \text{if } x(-n) < 1/2 \\ [0,1] & \text{if } x(-n) = 1/2 \end{cases} \qquad x(-n) = \begin{cases} 0 & \text{if } x(n) > 1/2 \\ 1 & \text{if } x(n) < 1/2 \\ [0,1] & \text{if } x(n) = 1/2 \end{cases}$$

Proof: Expected payoff in state n is given by

$$v(n) = (1-\delta)p(n) + \delta\phi v(n-1) + \delta\phi v(n+1) + \delta(1-2\phi)v(n),$$

where

$$\phi \equiv \alpha(1-\alpha)\Big(x(n)\big(1-x(-n)\big) + \big(1-x(n)\big)x(-n)\Big).$$

First notice that it must be $0 < v(n) < 1$ for all n. In fact, suppose the opposite is true and that $v(-n') = 0$, $v(-n'+1) > 0$. This is only possible if, in state n', $\phi = 0$, which corresponds to the case when $x(n')$ and $x(-n')$ are equal to each other and equal to 0 or 1. But clearly this is not an equilibrium, for in state n' the player receiving a payoff of zero would increase his or her value by choosing a different $x(-n')$. This argument also implies that, if $n > \bar{n}$, $\phi > 0$.

Suppose that $\phi \neq 0$. It follows that

$$v(n) - \frac{1}{2}v(n-1) - \frac{1}{2}v(n+1) = \frac{1-\delta}{2\phi\delta}\Big(p(n) - v(n)\Big).$$

If $n > \bar{n}$, $1 = p(n) > v(n)$. Consequently, $v(n)$ is locally concave and $v(-n) = 1 - v(n)$ is locally convex. This implies that the leader's optimal $x(n)$ is that which minimizes ϕ, whereas the laggard's optimal $x(-n)$ is that which maximizes ϕ. The best responses in the lemma then follow. ■

In words, Lemma 1 states that the leader's best response is to "imitate" the laggard, whereas the laggard's best response is to "differentiate" from the leader. In fact, if the leader wants to maximize the probability of selecting the same path as the laggard, then she should take path a with probability 1 if the laggard selects a with probability greater than $1/2$, as indicated by the Lemma. Likewise, if the laggard wants to minimize the probability of selecting the same path as the leader, then he should take path b if the leader selects a with probability greater than $1/2$, as indicated by the Lemma.

The intuition for this result is that a leader's current payoff is greater than her discounted value, that is, "things can only get worse." This implies that her value function is concave: she has less to gain from extending her lead than she has to lose from being caught up by the laggard. She thus prefers to minimize the variance of motion across states, which she does by maximizing the correlation with respect to the laggard. Conversely, the laggard's current payoff is lower than his discounted value, that is, "things can only get better." This implies that his value function is convex: he has less to lose from letting his lag extend than he has to gain from catching up with the leader. He thus prefers to maximize the variance of motion across states, which he does by minimizing the correlation with respect to the leader.

An immediate implication of Lemma 1 is that the game has the nature of a "matching pennies" game, the equilibrium of which is for players to equally mix between the two possible paths:

Corollary 1 *Suppose that $\alpha = \beta$. Then, in equilibrium and for $n > \bar{n}$, $x(n) = x(-n) = 1/2$.*

Lemma 1 and Corollary 1 characterize the equilibrium when the two paths are equally promising ($\alpha = \beta$). I now consider the case when one of the paths is more promising ($\alpha > \beta$) and show that increasing dominance results in equilibrium:

Proposition 1 (increasing dominance) *There exists an $\bar{\alpha}$ such that, if $\beta < \alpha < \bar{\alpha}$, then in equilibrium and at state $n > \bar{n}$ the system moves away from zero in expected value.*

Proof: From the proof of Lemma 1, we conclude that, in equilibrium, players mix between the two paths. In fact, value functions are strictly concave (leader) and convex (laggard), so that, even if α is changed by a small amount, the nature of the best response functions remains as before: the leader preferring to "imitate" the laggard, the latter preferring to differentiate from the former.

Expected payoff at state n is given by

$$v(n) = (1 - \delta)p(n) + \delta\phi^- v(n - 1) + \delta\phi^\circ v(n) + \delta\phi^+ v(n + 1),$$

where

$$\begin{aligned}
\phi^- &\equiv x(n)\left(1-x(-n)\right)(1-\alpha)\beta + \left(1-x(n)\right)x(-n)(1-\beta)\alpha \\
\phi^+ &\equiv x(n)\left(1-x(-n)\right)\alpha(1-\beta) + \left(1-x(n)\right)x(-n)\beta(1-\alpha). \\
\phi^\circ &\equiv 1-\phi^- - \phi^+
\end{aligned}$$

The fact that the leader mixes implies that the right-hand side of the value function is invariant with respect to $x(n)$. Substituting 1 and 0 for $x(n)$, equating, and solving for $x(-n)$, we get

$$x(-n) = \frac{(\alpha+\beta-2\alpha\beta)v(n) - \beta(1-\alpha)v(n-1) - \alpha(1-\beta)v(n+1)}{(\alpha+\beta-2\alpha\beta)\left(2v(n)-v(n-1)-v(n+1)\right)}.$$

Differentiating with respect to α at $\alpha=\beta$ yields

$$\left.\frac{\partial\, x(-n)}{\partial\, \alpha}\right|_{\alpha=\beta} = -\frac{v(n+1)-v(n-1)}{4\alpha(1-\alpha)\left(2v(n)-v(n-1)-v(n+1)\right)}.$$

Since $p(n) \leq 1,\ n < \bar{n}$ (strict inequality for $n < -\bar{n}$) and $p(n) = 1,\ n > \bar{n}$, $v(n)$ is increasing in n for $n > \bar{n}$. Moreover, by the same argument as in the proof of Lemma 1, $v(n)$ is concave for $n > \bar{n}$. It follows that the above derivative is negative, which in turn implies that $x(-n) < 1/2$ (recall that, for $\alpha = \beta$, $x(n) = x(-n) = 1/2$). An analogous argument implies that $x(n) > 1/2$. Since path a is better than b (in expected value), the result follows. ■

The above results are based on several assumptions regarding the value of α and n. The results are tight in the sense that one can find counterexamples when those assumptions fail. Specifically, if α is much greater than β, it is no longer the case that players mix between the two paths. In fact, for α sufficiently greater than β, both players choose path a. Moreover, if n is less than $\bar{n}$, one can find examples whereby the system moves toward zero in expected value: it suffices to assume that δ is close to zero and $p(n)$ convex.

■ **Alternative formulations of main result.** The previous results are limited in that they only apply for the case when the leader is "far" ahead of the laggard. However, imposing additional restrictions on the value of δ, I can prove similar versions of the increasing dominance result which apply at every state. The following results dispense with Assumption 3.

Proposition 2 *Suppose that $p(n)$ is strictly concave for $n > 0$. There exist $\bar{\alpha}$ and $\bar{\delta}$ such that, if $\beta < \alpha < \bar{\alpha}$, and $\delta < \bar{\delta}$, then the system moves away from zero in expected value.*

Proof: If δ is close to zero, then the payoff function provides a first-order approximation to the value function. Concavity of $p(n)$ therefore implies concavity of $v(n)$. The rest of the proof proceeds as as in Lemma 1. ■

Proposition 3 *Suppose that $p(n) > \frac{1}{2}$ iff $n > 0$. There exist $\bar{\alpha}$ and $\bar{\delta}$ such that, if $\beta < \alpha < \bar{\alpha}$, and $\delta > \bar{\delta}$, then the system moves away from zero in expected value.*

Proof: Recall that $v(n) = (1-\delta)p(n) + \delta\left(\phi^{-}v(n-1) + \phi^{\circ}v(n) + \phi^{+}v(n+1)\right)$. Together with $v(n) + v(-n) = 1$, this implies that $\lim_{\delta\to 1} v(n) = \frac{1}{2}$. Since $p(n) > \frac{1}{2}$ iff $n > 0$, it follows that for δ large enough, $p(n) > v(n)$. The proof then proceeds as in Lemma 1. ■

4 Discussion

Although my model implies increasing dominance, the reasons for the result are in stark contrast with respect to standard increasing dominance results. In the latter, total payoffs are increasing when increasing dominance takes place. Typically, this results from a convex payoff function, that is, a function with the properties that the leader has more to gain from extending her lead than the laggard has to lose from falling farther behind. By contrast, my model features constant total payoffs, so that the above effect is absent. Instead, the crucial feature of the equilibrium is that the leader has less to gain from moving farther ahead than she has to lose from being caught up by the laggard, whereas the laggard has more to gain from moving closer to the leader than he has to lose from falling farther behind. This implies that the leader prefers low variance of motion in the state space, or, equivalently, high correlation with respect to the laggard; whereas the laggard prefers the opposite, that is, low correlation with respect to the leader.

In the standard increasing dominance results, convexity of the payoff function translates into equilibrium strategies whereby the leader makes a greater effort than the laggard. By contrast, my model features constant total effort, so that the previous effect is absent. Instead, convexity of the laggard's value function translates into an equilibrium strategy whereby the laggard trades off a lower expected value for a lower correlation with respect to the leader.

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