Markets with Simultaneous Signaling and Screening

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Abstract

We model markets with adverse selection as matching markets. In a given match the informed or the uniformed party is chosen to make a take-it-or-leave-it proposal. This allows to account for the simultaneous presence of signaling and screening. Moreover, the possibility to dissolve matches unsuccessfully allows to endogenize the distribution of types in the market. It will be shown that this approach overcomes the well-known trade-off between ensuring existence (in signaling games) and obtaining clear-cut results (in screening games).

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1. Introduction

The analysis of markets with adverse selection has attracted much attention during the last three decades. Given the prevalence of private information in many real world settings such as insurance, labor markets, or investor-bank relationships, this seems justified. On the other side, it is fair to say that there exists no commonly accepted notion of competitive equilibrium in markets with adverse selection. This holds even if we restrict attention to contributions in the literature which, in the words of Gale (1996), emphasize the non-cooperative nature of equilibrium.[1]

To fix ideas, consider a labor market where workers have private information about their productivity type. A labor contract signed between a single firm and a single worker specifies a wage and additionally a sorting variable such as training or working hours. (The worker's payoff satisfies a standard sorting (or single crossing) condition in this variable.) Moreover, assume that firms compete for workers as they constitute the long side of the market. There exist two canonical approaches to analyze this setting. The most prominent approach is to consider a two-stage screening game. At the first stage, the *uninformed* agents, i.e. firms, simultaneously offer a menu of contracts. At the second stage, workers pick an individual firm and sign a contract.[2] It turns out that equilibrium contracts are unique and separating. However, as found by Rothschild and Stiglitz (1976), an equilibrium in pure strategies may fail to exist.[3] A different approach is to consider a game of signaling where the *informed* party proposes a contract. Though contractual games of signaling have been predominantly analyzed for bilateral monopolies,[4] the analysis of the two-stage game in a frictionless market is identical.[5] Though existence is no longer an issue, signaling games are plagued with a multiplicity of equilibria.

From this brief overview of existing non-cooperative models, the following drawbacks emerge. First, the outcome is very sensitive to the choice of the game, i.e. to the sequence of moves.

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[1] As we restrict ourselves to the non-cooperative strand of the literature, we refer the reader to Gale (1996) for a list of alternative approaches.

[2] The standard approach is to assume that competing firms have unlimited capacities (or vacancies). Only recently, Inderst and Wambach (1999a, 1999b) have analyzed the case where capacities are constrained. We should note that this makes no difference if workers can visit firms without costs. In contrast, if it is costly to be rationed as visiting another firm entails search costs, the non-existence problem of Rothschild and Stiglitz (1976) disappears.

[3] In response to the non-existence problem, Wilson (1977) and Riley (1979) have proposed alternative solution concepts. These concepts can be given a game-theoretic foundation by extending the original two-stage game. Hellwig (1987) summarizes several attempts in this direction.

[4] For an overview see Kreps and Sobel (1994).

[5] To our knowledge, the only attempt to embed signaling games in a market enviroment with frictions is Inderst (1999).

Hellwig (1987) notes that this discrepancy between the predictions in models of screening and signaling presents a fundamental dilemma for applied economists. While the order in which people move is crucial for the predictions, it may not be observable and may not even be fixed in a given market. Second, each approach has its own serious problems. While the screening approach suffers from the problem of non-existence, the signaling approach fails to make clear predictions as there exist multiple equilibria with highly different outcomes. Thirdly, the picture of a *frictionless* market which is assumed in either setting is surely an abstraction from reality. In fact, in settings with complete information, the issue how to model decentralized markets has been addressed in the literature on matching and search markets (see the overview in McMillan and Rothschild (1994)).

The model presented in this paper intends to overcome all three drawbacks of the existing approaches. Hence, we will allow for the simultaneous presence of both signaling and screening. Moreover, we explicitly introduce frictions by embedding the respective contractual games in a matching market environment. The approach will allow to derive clear-cut results regarding the equilibrium allocation (of contracts), while preserving existence. Additionally, it allows to address issues which could not be analyzed previously such as the endogenization of the distribution of types in the market.

To fix ideas, we stick to the picture of the labor market. We assume that each firm has a single vacancy and that each job seeker can work for at most one firm. Moreover, we consider a stationary environment where each moment (in discrete time) a fixed measure of potential employers and employees appear on the market fringe and may decide to enter the market. To simplify the analysis, we restrict attention to the case where potential employers outnumber potential employees. This is formally equivalent to a zero-profit condition for firms and is surely reasonable in the labor market context. In the market individuals are pairwise matched. In a given match a party is chosen randomly to make a take-it-or-leave-it offer. Hence, if the firm makes the proposal, we encounter a game of screening. Otherwise, the parties play a game of signaling. If the offer is rejected, the match is dissolved and the two parties reenter the market. Waiting to be matched anew is costly. We are mainly interested in the case where frictions become arbitrarily small.

If we require that the market size remains bounded as frictions vanish, we can derive the following main results of the paper. First, we show that equilibrium contracts converge to a uniquely defined set of least-cost-separating contracts as frictions vanish. Hence, the uniqueness result of the standard screening approach is preserved in the matching market. Secondly, focusing again on low frictions, we can establish existence. The key to the existence result is that the matching market environment allows to endogenize the distribution of types in the market. To our knowledge, the possibility that variations in the *circulation time* of different types are used to endogenize the distribution is new to the literature on markets with adverse selection.[6]

The rest of this paper is organized as follows. In Section 2 we introduce the model. Section 3 derives the convergence result, while Section 4 addresses existence. In Section 5 we discuss equilibria where the stock of agents in the market grows beyond any boundary as frictions disappear. In these equilibria the efficiency gains from a decrease in frictions are almost entirely offset by longer waiting times for firms or low-type workers who crowd the market. Section 6 concludes with a discussion of alternative models for the contractual game in a match. Some proofs are relegated to the appendix.

2. The Model

2.1 Players and Payoffs

The market consists of firms which have a single vacancy to fill and of potential workers who may work for at most one firm. A worker has private information about his type denoted by a natural number $i \in I = \{1, ..., \bar{i}\}$ with finite $\bar{i} > 1$. Firms assign probability $\mu(i) > 0$ to type $i \in I$ with $\sum_{i \in I} \mu(i) = 1$. A worker and a firm can conclude a contract specifying two variables t, y, where t is a monetary transfer, while $y \geq 0$ is real valued and may denote, for instance, the number of hours worked. We abbreviate a contract by $c = (t, y)$ with $c \in C = \Re \times \Re_0^+$. Denote the firm's utility under a contract c with type i by $U(i, c)$ and the utility of the worker by $V(i, c)$. Observe that both utilities depend on the worker's type. If a vacancy remains unfilled, the firm's utility is denoted by U^0. Similarly, if the worker is not successful, his reservation value is denoted by V^0, which is assumed to be type-independent.

We make next a series of restrictions on the payoff functions. First, we restrict attention to the case with transferable utility where $V(i, c) = v(i, y) + t$ and $U(i, c) = u(i, y) - t$.[7] The

[6] This seems reminiscent to results derived in the recent literature on decentralized markets with non-transferable utility (see Burdett and Coles (1998)). The speed with which different types leave the market depends on the acceptance behavior of their respective matching partners, which again depends on the aggregate distribution of potential partners in the population. This mutual dependency allows for multiple equilibria. The possibility to adjust the distribution of circulating agents has also been used in the axiomatic setting of Myerson (1995).

[7] The main convergence result of this paper is generalized to the case with non-transferable utility in the working paper version (see Inderst (1999c)).

functions $v(i,\cdot)$ and $u(i,\cdot)$ are twice continuously differentiable. Define the surplus function $s(i,y) = u(i,y) + v(i,y)$. We invoke the following assumptions.

(A.1) $s(i,y)$ is strictly quasiconcave; $lim_{y\to\infty} ds(i,y)/dy < 0$.

(A.2) $dv(j,y)/dy > dv(i,y)/dy$ for all $j > i$

(A.3) $u(j,y) > u(i,y)$ for $j > i$ and $y > 0$.[8]

Observe first that (A.1) admits in particular the case where $s(i,\cdot)$ is linear and strictly decreasing in y. We will frequently use this case as an example. (A.2) is standard in problems of screening of this sort. By $dv(j,y)/dy > dv(i,y)/dy$ for $y > 0$ and $j > i$ the contractual component y is a sorting variable as the worker's payoff satisfies a standard sorting condition with respect to this variable. Finally, by (A.3) firms prefer to conclude a given contract with a higher type.

Before illustrating (A.1)-(A.3) with two examples, consider for any type i the program to maximize $V(i,c)$ subject to $c \in C$ and $U(i,c) \geq U^0$. By (A.1) a unique solution exists, which is denoted by $c^*(i)$.

Examples

- The Spence case: We specify $V(i,c) = t - y/a_i$ and $U(i,c) = a_i - t$ for $i \in I$, where $a_j > a_i > 0$ for $j > i$. Observe that the sorting variable is purely dissipative such that $y^*(i) = 0$ for all $i \in I$. A standard example is education or some non-related training, which better types can manage with less effort and thus less disutility.

- Working hours: Assume that y represents hours of work. A high type is more eager to work, i.e. he incurs less disutility from working additional hours, and he is also more productive both absolutely from (A.3) and on the margin as we assume additionally that $du(j,y)/dy > du(i,y)/dy$ for $j > i$.[9] If we assume that $ds(i,y)/dy > 0$ at $y = 0$, we obtain $y^*(j) > y^*(i) > 0$ for all $j > i$.

Below we will consider a matching market with endogenous entry. To ensure that workers of all types enter if frictions are low, their respective payoff must exceed V^0 regardless of the firms' beliefs. The following assumption, which is particularly reasonable in the considered context of a labor market, proves to be sufficient for this purpose.

[8] In the working paper version (Inderst (1999c)) we also consider the case where $dv(j,y)/dy > dv(i,y)/dy$ in (A.2) holds only almost everywhere, which admits $dv(j,y)/dy = dv(i,y)/dy = 0$ at $y = 0$.

[9] Hence, the ranking of the marginal trade-offs between types is similar for workers and firms. In the terminology of Beaudry and Poitevin (1993) this represents the "S case", which they distinguish from the "RS case" (e.g. insurance contracts).

(A.4) $V(1, c^*(1)) > V^0$ and $V(j, c) \geq V(i, c)$ for all $c \in C$, $j > i$.

2.2 Market

We consider a matching market with endogenous entry.[10] Time runs discretely and the market operates for an infinite number of periods. All agents discount future payoffs by a constant discount factor $0 < \delta < 1$. The primitives of the model are the time invariant measures of agents newly arriving on the market fringe each period. For instance, we may suppose that each period there is a new cohort of job applicants and firms. We denote the respective finite masses by $F^0 > 0$ for firms and by $W^0(i) > 0$ for workers of type $i \in I$. Denote $W^0 = \sum_{i \in I} W^0(i)$ and $\mu^0(i) = W^0(i)/W^0$. Our main assumption on the primitives is that firms constitute the longer side on the market fringe.

(A.5) $F^0 > W^0$.

Technically, this assumption could be replaced by a zero-profit condition for firms as it will imply that firms realize exactly their reservation utility U^0 in the market.

In what follows, we will restrict attention to stationary markets where the measures of stocks as well as that of agents exiting and entering are time invariant. We denote the stock of firms by F and that of workers of type i by $W(i)$. The aggregate stock of workers equals $W = \sum_{i \in I} W(i)$. For $W > 0$ the distribution in the market is given by $\mu(i) = W(i)/W$. Let E^F be the measure of entering firms and $E^W(i)$ that of entering workers of type i. The measures of exits are denoted by X^F and $X^W(i)$.

The matching market operates as follows. We consider an anonymous market with random matching and a proportional matching technology.[11] In this case the market represents an ocean of players who meet randomly irrespective of their type. Hence, if the market opens up, a firm's probability of being matched with a worker of type i is equal to $\mu(i)m$ where $m = W/(F+W)$. Analogously, the probability of a worker to be matched equals $1 - m = F/(F + W)$. If a match is formed, the two agents play a contractual game specified below. If this game leads to the implementation of a contract, both players leave the market. Otherwise, they re-enter the matching market.

[10] In doing so we follow Gale (1987) and Peters (1992). In contrast, Rubinstein and Wolinsky (1985) take the stocks in the market as the primitives and adjust entry flows to ensure stationarity. For more on this distinction see Osborne and Rubinstein (1997, Chapter 7).

[11] Proportional probabilities are assumed, for instance, in Rubinstein and Wolinsky (1985) and Gale (1987). Anonymity and stationarity are standard assumptions. The impact of non-anonymity has been explored in Rubinstein and Wolinsky (1990).

By now it should be obvious that frictions in the market will imply delay, which should always be costly to players. To ensure that this is indeed the case, we must assume that agents are also impatient about realizing their outside options.[12]

(A.6) $U^0 > 0, V^0 > 0.$

2.3 Contractual Games

If a match is formed, the following games are played. With probability $0 < 1 - b < 1$ the worker is chosen to make a one-shot offer. We denote this game by Γ^W. We introduce the following convention: We allow the worker to propose also the null contract $\emptyset$ which leads to the immediate separation of the match. His actions are thus restricted to $C^0 = C \cup \{\emptyset\}$. For notational convenience we further restrict attention to strategies where players randomize at most over a countable number of actions. The mixed strategy of a single worker of type i is thus a distribution over C^0 denoted by $\rho^W(i, c)$. The firm may either accept or reject the offer. Denote the acceptance probability of the firm by $\gamma^W(i, c)$.

With probability b the firm is chosen to make an offer. We specify that the firm can offer a menu of contracts. We restrict the menu to a finite number $N \geq \bar{i}$ of deterministic contracts.[13] It is also convenient to specify that the menu contains the null contract leading to a separation of the match. If $\{c(n)\}_{0 \leq n \leq N}$ denotes a single menu, we specify $c(0) = \emptyset$, while $c(n) \in C$ for $n > 0$. The firm's mixed strategy represents a distribution $\rho^W(\{c(\cdot)\})$ over the set of menus $\{\emptyset\} \cup C^N$. The worker may now choose a particular contract in the menu. Observe that choosing $n = 0$ is equivalent to rejecting the offer. The mixed strategy of a worker of type i when facing a menu $\{c(\cdot)\}$ is thus a distribution $\gamma^F(i, \{c(\cdot)\}, n)$ over $0 \leq n \leq N$.

2.4 Discussion of the Modelling Assumptions

As our model constitutes a comparatively new approach to model markets with adverse selection, we should comment in more detail on the individual building blocks.

Matching market

Three elements are key to the definition of the market, which are discussed in turn.

[12] Technically, (A.6) ensures that the market does not clog up over time. Alternatively, this could be ensured by introducing a positive entry cost or some (additive) search costs.

[13] In what follows, we will restrict ourselves to a characterization for low frictions where only the (adjacent) upwards incentive compatibility constraints become important. As it is well-known, assuming that $d^2v(i, y)/dy^2$ is nonincreasing in i ensures that randomization over contracts in C does not benefit the firm. Moreover, it is straightforward to show that randomization over contracts in C and the possibility to break-up the match (i.e. the null contract $\emptyset$) is not profitable even though reservation values become type-dependent.

Frictions: We assume that agents are impatient and therefore prefer to contract immediately instead of waiting to be matched anew. All results would continue to hold if we specified instead that agents do not discount future payoffs but incur "search" costs $s > 0$ from (re-)entering the matching market.

Matching technology: Though we specify a proportional matching technology, our results rely only on two properties: continuity and monotonicity in the stocks of agents. In particular, we could assume that only the "long" side of the market is rationed, while the "short" side finds a matching partner with probability one.

Flows and stocks: The primitives of our model are the potential entrants arriving at the market fringe each period. An alternative approach would be to take the stocks in the market as primitives and adjust entries and exits to ensure stationarity.[14] We refer the reader to the thorough arguments in Gale (1987) who convincingly argues in favor of the former approach. A completely different and equally attractive setting would, however, be to consider a market with fixed stocks which clears over time. With complete information this approach has been pursued, for instance, by Binmore and Herrero (1988) and Gale (1987). Our arguments and results do not necessarily extend to this setting.

Contractual games

The specification of the contractual games contains two major ingredients.

Both sides are active: By waiting sufficiently long, an agent can be sure that he will be chosen as the proposer in some match. Allowing both sides of the market to become active is a crucial ingredient of any matching model. Otherwise, one encounters the well-known monopoly price paradoxon (see Diamond (1972)) which would cause the market to shut down.

Random choice and one-shot offers: In line with most contributions to the matching market literature we specify that with a fixed probability either side may be chosen to make a one-shot proposal. Given our motivation in the introduction, it seems moreover natural to combine the two standard (one-shot) settings. In Section 6 we comment on alternative specifications such as alternating offers or allowing also workers to propose menus.

2.5 Equilibria

We now derive equilibrium requirements. We first discuss the requirements for strategies in the contractual games and turn next to equilibrium conditions for the matching market.

[14] For more on this approach, see the working paper version (Inderst (1999c)).

Contractual games

We restrict attention to symmetric and stationary strategies.[15] For the signaling game Γ^W, we denote a firm's posterior beliefs if it observes the offer c by $\pi(i,c)$. We require that strategies in Γ^W are sequentially optimal and that firms consistently update their beliefs. By the latter requirement it holds that $\pi(i,c) = \mu(i)\rho^W(i,c)/[\sum_{j\in I}\mu(j)\rho^W(j,c)]$ in case $\rho^W(i,c) > 0$ for some $i \in I$. (Recall that $\mu(\cdot)$ denotes the distribution of types in the market.) Similarly, we require for the screening game Γ^F that strategies are sequentially optimal. We summarize strategies and beliefs for Γ^W by $\sigma^W = (\rho^W, \gamma^W, \pi)$ and strategies for Γ^F by $\sigma^F = (\rho^F, \gamma^F)$.

Given strategies for both games and both agents, we can define allocations as follows. For Γ^W define for all $c \in C^0$ the probability $\alpha^W(i,c) = \rho^W(i,c)\gamma^W(i,c)$. We denote the support by $B^W(i)$. Similarly, we define for Γ^F the allocations

$$\alpha^F(i,c) = \sum_{\{\{c(\cdot)\}\in\{\emptyset\}\cup C^N | c(n)=c\}} \rho^W(\{c(\cdot)\})\gamma^F(i,\{c(\cdot)\},n)$$

and denote the support by $B^F(i)$. We define the aggregate support by $B(i) = B^W(i) \cup B^F(i)$. If $B(i) \cap C$ is non-empty, we may also define for type i the distribution of contracts $c \in C$ which he implements in the market. By stationarity, the distribution function is given by

$$\beta(i,c) = \frac{b\alpha^F(i,c) + (1-b)\alpha^W(i,c)}{1 - b\alpha^F(i,\emptyset) - (1-b)\alpha^W(i,\emptyset)}. \tag{1}$$

Matching market

We already noted that we restrict attention to stationary market environments implying that we can neglect the time subscripts for stocks, entries, and exits for all agents. As is well known, there always exists the trivial case where the market fails to open up as agents of either side will not enter. In what follows, we will neglect this possibility. We next require that the decisions to enter (or not) are optimal. To evaluate this choice, we must calculate reservation values realized in the market. As entering the market requires one unit of time, reservation values are equivalent to an agent's expected utility after dissolving a match unsuccessfully. They are denoted by U^R for firms and by $V^R(i)$ for workers of type i. We introduce the convention that $U(i,\emptyset) = U^R$ and $V(i,\emptyset) = V^R(i)$. Given strategies played in Γ^F and Γ^W, we define the expected utilities

$$V^F(i) = \sum_{c\in B^F(i)} \alpha^F(i,c)V(i,c),$$

[15] As players are allowed to randomize and as we consider a continuum of agents on either side, the symmetry restriction is only for convenience. Stationarity of strategies implies in particular that reservation values (in the market) become stationary. If contracts specified only a transfer, stationarity of reservation values would follow from the assumed stationarity of the market (see Gale (1987)). To see why this is not sufficient in our case, observe that firms may be indifferent between several menus under which the information rent left to some types varies.

$$
\begin{aligned}
V^W(i) &= \sum_{c \in B^W(i)} \alpha^W(i,c) V(i,c), \\
U^F &= \sum_{i \in I} \mu(i) \sum_{c \in B^F(i)} \alpha^F(i,c) U(i,c), \\
U^W &= \sum_{i \in I} \mu(i) \sum_{c \in B^W(i)} \alpha^W(i,c) U(i,c).
\end{aligned}
$$

These can be substituted to obtain the reservation values by

$$
\begin{aligned}
U^R &= \delta \left[(1-m) U^R + m(bU^F + (1-b)U^W) \right], \qquad (2) \\
V^R(i) &= \delta \left[m V^R(i) + (1-m)(bV^F(i) + (1-b)V^W(i)) \right].
\end{aligned}
$$

For instance, the agent of type i realizes in a given match the expected utility $bV^F(i) + (1-b)V^W(i)$ as the game Γ^F (Γ^W) is played with probability b $(1-b)$. With probability m the agent has to wait for the next round.

The optimality requirement for firms' entry decisions is:

$$
E^F = \begin{cases} 0 & \text{if } U^R < U^0 \\ \in [0, F^0] & \text{if } U^R = U^0 \\ F^0 & \text{if } U^R > U^0 \end{cases} .
$$

The respective condition for workers is analogous.

Summary of equilibrium conditions

A market equilibrium is described by a profile $\psi = (\sigma^W, \sigma^F, F, \{W(i)\}, E^F, \{E^W(i)\})$ satisfying the following requirements:

1. Strategies in Γ^F are symmetric, stationary, and sequentially optimal.

2. Strategies in Γ^W are symmetric, stationary, and sequentially optimal, while beliefs are consistently updated.

3. Entry decisions are optimal.

4. The market is stationary, i.e. $E^W(i) = X^W(i)$ and $E^F = X^F$.[16]

3. Convergence of Contracts

In this section we show that the distribution of contracts implemented in the matching markets converges as frictions vanish if we impose additionally an intuitive restriction on the set of equilibria. We start by deriving a family of contracts which becomes essential in characterizing the limit outcome.

[16] Note that exit flows are fully determined by stocks and strategies in the contractual games.

3.1 The Rothschild-Stiglitz Contracts

Define a family of contracts $\left\{c^{RS}(i)\right\}_{i\in I}$ as follows:

i) For $i = 1$ the contract $c^{RS}(1)$ maximizes $V(1, c)$ subject to $c \in C$ and $U(1, c) \geq U^0$.

ii) For $i > 1$ the contract $c^{RS}(i)$ maximizes $V(i, c)$ subject to $c \in C$, $U(i, c) \geq U^0$, and $V(i-1, c) \leq V(i-1, c^{RS}(i-1))$.

Define the realized utilities by $V^{RS}(i) = V(i, c^{RS}(i))$. The following result is standard given (A.1)-(A.3).

Lemma 0. *The family* $\left\{c^{RS}(i)\right\}_{i\in I}$ *is uniquely determined and satisfies* $U(i, c^{RS}(i)) = U^0$, $y^{RS}(i) \geq y^{RS}(j)$ *for all* $j > i$, *and global incentive compatibility (i.e.* $V^{RS}(i) \geq V^{RS}(i, c^{RS}(j))$ *for all* $i, j \in I$).

We call $\left\{c^{RS}(i)\right\}_{i\in I}$ the Rothschild-Stiglitz (RS) contracts as these contracts would emerge in the two-stage screening game described in the introduction.[17] For an illustration, consider again the examples introduced in Section 2.

Examples

- The Spence case: Recall that we specify in this case $V(i, c) = t - y/a_i$ and $U(i, c) = a_i - t$ for $i \in I$, where $a_j > a_i$ for $j > i$. For $\bar{i} = 2$ it is easily checked that the RS contracts specify $y^{RS}(1) = 0$ and $t^{RS}(1) = a_1 - U^0$, $y^{RS}(2) = a_1(a_2 - a_1)$ and $t^{RS}(2) = a_2 - U^0$. The respective utilities are equal to $V^{RS}(1) = a_1 - U^0$ and $V^{RS}(2) = a_2 - a_1(a_2 - a_1)/a_2 - U^0$.

- Working hours: Recall that in this case the sorting variable y, which represents working hours, is not purely dissipative. Indeed, we ensured for the first-best choices that $y^*(j) > y^*(i)$ for $j > i$. Hence, in contrast to the Spence case, first-best choices of y are type-dependent. This may even ensure that the family of first-best contracts $\{c^*(i)\}_{i\in I}$ is incentive compatible such that $c^{RS}(i) = c^*(i)$ for all types.

3.2 Convergence Result

Observe that in our model the stock of agents in the market is determined endogenously as the primitives are the constant flows of potential entrants arriving at the market fringe each period. As a consequence, the model does not impose any inherent restrictions on the size of stocks

[17] This family of contracts features also prominently in the signaling literature. In standard (monotonic) signaling games it would be selected by the Divinity criterion of Cho and Sobel (1990), while Kreps and Sobel (1994) show that it is selected by the Intuitive Criterion if utilities are transferable and the game exhibits a "take-it-or-leave-it setup".

other than the requirement that entries must be equal to exits. We regard it as (economically) reasonable to impose such a restriction. If the market size remains bounded (as frictions vanish), it can be shown that the distribution of equilibrium contracts converges. Moreover, we show existence of a sequence of equilibria with this property.

The important implications of imposing an upper bound on the stocks of agents are that neither the measure of firms nor the measure of low-type agents explodes as frictions vanish. Indeed, in Section 5 we construct sequences of equilibria where one of these requirements fails and the convergence result does not hold. To be more precise, convergence fails if search costs incurred by firms do not vanish for $\delta \to 1$. Formally, this is the case if

$$f = \frac{\delta mb}{1 - \delta(1 - mb)}$$

does not converge to one if $\delta \to 1$. Observe that for this to hold $m \to 0$ is necessary but not sufficient. Hence, in what follows we will impose the requirement that $f \to 1$, which is weaker than requiring that the measure of firms remains bounded. The second requirement is now that the distribution of types in the market remains bounded away from the boundaries of the simplex Δ_I. As it can be shown that all workers enter for low frictions and that workers of the highest type will always conclude a contract in the screening game Γ^F, this is equivalent to requiring that the measure of workers remains bounded for $\delta \to 1$.

For the rest of this section we thus impose the following restriction on equilibria, where $M > 0$ is some upper bound.

Equilibrium selection: Define for $\delta < 1$ the set $\Psi_\delta(M)$ satisfying:

1) For all $\psi \in \Psi_\delta(M)$ the aggregate stock of workers in the market must satisfy $W < M$.

2) For any $\varepsilon > 0$ there exists $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$ and equilibria $\psi \in \Psi_\delta(M)$ it holds that $f > 1 - \varepsilon$.

In Section 4 we will prove that there exists a finite M such that the set of equilibria $\Psi_\delta(M)$ satisfying 1) and 2) is in fact non-empty for all sufficiently large values of δ.

The derivation of our convergence result proceeds now in two steps. We first prove that reservation values must converge as frictions vanish. This result will then be used to derive the convergence of the distribution of implemented contracts.

Proposition 1. *For any $\varepsilon > 0$ there exists $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$, $\psi \in \Psi_\delta(M)$, and $i \in I$ it holds that*

$$V^{RS}(i) - \varepsilon < V^R(i) < V^{RS}(i) + \varepsilon. \tag{3}$$

The proof of Proposition 1 in Appendix A proceeds in several steps. For an intuition we briefly outline the argument. Observe first that, by stationarity, the firms' utility realized in the matching market must be equal to U^0. Otherwise, the market could not be stationary or firms would not find it optimal to enter at all. By (A.4) this implies that all workers will enter as frictions become sufficiently low. As the stock of workers in the market remains bounded, the distribution of types remains bounded away from the boundaries of the simplex Δ_I as $\delta \to 1$. As additionally $f \to 1$ holds by assumption, it becomes virtually costless for firms to wait until they find themselves in a "specific" match, e.g. in a match where they propose a contract to some type i. Suppose now that reservation values do not satisfy (3). By the nature of the RS contracts this would imply that there are some unrealized gains for firms to trade with specific types. These gains would be realized as $\delta \to 1$. Precisely, consider the case of an upper boundary on $V^{RS}(i)$ and assume that the claim holds for all types $j < i$, but not for type i. By construction of the RS family of contracts, any contract c realizing $V(i-1,c) \leq V^{RS}(i)$ and $V(i,c) > V^{RS}(i)$ must yield $U(i,c) < U^0$. Hence, it would only be proposed or accepted by firms if they simultaneously realize more than U^0 with some (higher) types. It is shown that this cannot be the case in equilibrium as firms would be better off by restricting an offer to these (higher) types. The main complication in the proof of Proposition 1 is that we allow both sides to randomize when offering and when responding to a proposal.

Denote next for some contract $c \in C$ the ε-neighborhood by $\Omega(c, \varepsilon)$. The convergence result, which is proved in Appendix B, can then be stated as follows.

Proposition 2. *For any $\varepsilon > 0$ there exists $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$, $\psi \in \Psi_\delta(M)$, $i \in I$, and corresponding distributions of contracts $\beta(i,\cdot)$ with support $B(i)$ it holds that*[18]

$$\sum_{c \in B(i) \cap \Omega(c^{RS}(i), \varepsilon)} \beta(i,c) > 1 - \varepsilon.$$

In words, as frictions become smaller, any type i must implement contracts in a small neighborhood of his respective RS contract with a probability close to one. This result is intuitive given the continuity of payoff functions and the characterization of reservation values in Proposition 2.

Recall now from the introduction that the family of RS contracts is implemented in the unique equilibrium (if firms play pure strategies) of the one-shot screening game. In contrast, multiple equilibria with highly different outcomes are obtained under signaling. Embedding the contract design in a matching market environment allows us to account for the simultaneous presence of

[18] Hence, along a sequence of equilibria where $\delta \to 1$, the distributions for some type i weakly converge to the RS distribution $\beta^{RS}(i)$ which puts mass one on $c^{RS}(i)$. The set of equilibrium distributions converges to $\beta^{RS}(i)$ with respect to the topology derived from the Hausdorff metric.

screening and signaling. The convergence results of Propositions 1-2 are driven by the presence of screening. Intuitively, the same forces as in the standard two-stage model of screening are still active in the matching market framework (under the assumed restrictions). As a consequence, contracts implemented by some worker must truly reflect the worker's type, while ensuring incentive compatibility.

4. Endogenization of the Distribution of Types

In this section we argue that the set of equilibria characterized in Propositions 1-2 is not empty. Moreover, we feel that the way how this is established is itself of economic interest. While typically models of adverse selection specify an exogenous distribution of types, this becomes endogenous in the matching market environment. As the distribution of contracts converges for $\delta \to 1$, the distribution of agents in the market will adjust to ensure that it is optimal for agents to make and accept the respective proposals.

To put this into perspective, we know from Rothschild and Stiglitz (1976) that the family of RS contracts is only interim efficient (in the sense of Holmström and Myerson (1983)) if the probability of low types is sufficiently low.[19] If the family $\left\{c^{RS}(i)\right\}_{i\in I}$ fails to be interim efficient, it cannot arise as an equilibrium in the standard two-stage screening model where firms compete for workers.

We proceed now as follows. We first give a constructive proof of existence for the two-type Spence case, which illustrates the interdependence between the distribution of types in the market and the shape of equilibrium contracts. Finally, we state an existence result for general payoff functions.

Example: The Spence case with two types

Consider for a moment the program of a single firm which faces a single worker. The firm's beliefs are given by μ and the worker's type-dependent reservation values are given by $V^R(i)$ for $i \in I = \{1, 2\}$.[20] Assume additionally that $V^R(2) > V^R(1)$. If the firm must offer both types a feasible contract, we obtain the following results:

i) $\mu(1) > \mu^*(1) = a_1/a_2$: The unique optimal menu specifies $t = V^R(1)$ and $y = 0$ for $i = 1$, $y = (V^R(2) - V^R(1))a_1a_2/(a_2 - a_1)$ and $t = V^R(2) + (V^R(2) - V^R(1))a_1/(a_2 - a_1)$ for $i = 2$.

19 See also Maksin and Tirole (1992) for a more thorough discussion of this issue.

20 Inderst (1999c) considers also the case with three types to show that the proposed method of construction extends to more than two types.

ii) $\mu(1) > \mu^*(1)$: The unique optimal menu specifies $y = 0$ and $t = V^R(2)$ for $i = 1, 2$.

iii) $\mu(1) = \mu^*(1)$: The firm is indifferent between the menus i) and ii).

Return now to the matching market environment and consider a sequence of equilibria $\psi_\delta \in \Psi_\delta(M)$ where $\delta \to 1$. It is intuitive from the above analysis that the distribution of types cannot satisfy $\mu_\delta(1) < \mu^*(1)$ as firms would then strictly prefer to offer a pooling contract.[21] Hence, if the distribution among entrants satisfies $\mu^0(1) < \mu^*(1)$, the resulting distribution in the market must put more weight on low types. This is accomplished by creating different (expected) times of circulation for low and high types. Precisely, we will ensure in this case that the distribution in the market is equal to $\mu^*(1)$. Firms are then indifferent between offering a pooling or a separating menu. If reservation values converge to the RS utilities where $V^{RS}(2) > V^{RS}(1)$, low types strictly prefer to be pooled with high types. This "cross-subsidization" will ensure that the reservation value for $i = 1$ satisfies $V^R(1) = V^{RS}(1)$ even at values $\delta < 1$,[22] which in turn makes firms indifferent between offering the low type an acceptable contract or not. Similarly, in Γ^W the low type becomes indifferent between implementing $c^{RS}(1)$ or dissolving the match unsuccessfully. It then remains to adequately choose the probabilities of breakdown to arrive at $\mu^*(1)$.

We consider now two cases in turn.

Case 1 of the example: $\mu^0(1) \geq \mu^*(1)$

In this case we construct equilibria where the distribution in the market is equal to that among potential entrants. We will index the equilibrium variables by δ. Suppose for Γ^W that type i offers $t = a_i - U^0$, $y = y^{RS}(i)$. In Γ^F firms specify for the low type $y = 0$, $t^F_\delta(1) = V^R_\delta(1)$, and for the high type $y^F_\delta(2) = (V^R_\delta(2) - V^R_\delta(1))a_1 a_2/(a_2 - a_1)$, $t^F_\delta(2) = V^R_\delta(2) + y^F_\delta(2)/a_2$. All matches are successful, and we specify that all workers enter ($E^W_\delta(i) = M^0(i)$), which implies $\mu_\delta(i) = \mu^0(i)$. For firms we specify $E^F_\delta = E^W_\delta$. By substitution we obtain for the reservation values

$$\begin{aligned} V^R_\delta(1) &= \frac{\delta(1-m_\delta)(1-b)}{1-\delta\left[1-(1-m_\delta)(1-b)\right]}\left[a_1 - U^0\right], \qquad (4) \\ V^R_\delta(2) &= \frac{\delta(1-m_\delta)(1-b)}{1-\delta\left[1-(1-m_\delta)(1-b)\right]}\left[a_2 - a_1(a_2-a_1)/a_2 - U^0\right], \\ U^R_\delta &= \frac{\delta m_\delta b}{1-\delta\left[1-m_\delta b\right]}\left[\mu^0(1)(a_1 - V^R_\delta(1)) + \mu^0(2)(a_2 - V^R_\delta(2) - y^F_\delta(2)/a_2)\right]. \end{aligned}$$

[21] The non-existence result is formally derived in Inderst (1999c).

[22] Of course, the probability with which firms choose the pooling contract will go to zero as $\delta \to 1$.

Requiring $U_\delta^R = U^0$ and substituting, we obtain

$$m_\delta = \frac{U^0(1-\delta b)}{\delta b(\mu^0(1)a_1 + \mu^0(2)a_2 - U^0) + \delta(1-b)U^0}. \tag{5}$$

Observe that $\lim_{\delta\to 1} m_\delta = \bar{m}$ for some $0 < \bar{m} < 1$, implying in particular that $f_\delta \to 1$. By specifying the stocks $W^0(i) = W_\delta(i)(1-m_\delta)$ and $W^0 = F_\delta m_\delta$, we ensure that the market is stationary.

Observe next that by (A.6), $\lim_{\delta\to 1} m_\delta = \bar{m}$, and the definition of reservation values, there exists some finite M such that $V_\delta^R(i) > V^0(i)$ holds for $i \in I$ and high δ, while $W_\delta < M$. The first implication ensures that entry is optimal for all workers. As $\mu^0(1) \geq \mu^*(1)$ and $V_\delta^R(i) < V^{RS}(i)$, we know from previous results that firms cannot profitably deviate in Γ^F. Finally, workers' strategies in Γ^W can be supported by pessimistic out-of-equilibrium beliefs. Summing up results, we have found a finite M such that $\Psi_\delta(M)$ is non-empty for sufficiently high δ.

Case 2 of the example: $\mu^0(1) < \mu^*(1)$

In this case we construct equilibria where the fraction of low types in the market strictly exceeds the proportion among potential entrants. Precisely, we will ensure that $\mu_\delta(1) = \mu^*(1)$. This will be established as matches with $i = 1$ will be broken up unsuccessfully with sufficient probability. Denote the probability with which matches with low types are successful by $\bar{\rho}$. Matches with high types are always successful. By stationarity and the assumption that all workers enter, $\mu_\delta(1) = \mu^*(1)$ implies the requirement that

$$\frac{\mu^0(1)}{\bar{\rho}\,[1-\mu^0(1)] + \mu^0(1)} = \mu^*(1).$$

Given some matching probability m_δ, the stock of workers is given by $W^0(2) = (1 - m_\delta)W_\delta(2)$ and $W^0(1) = (1-m_\delta)W_\delta(1)\bar{\rho}$. We turn next to the contractual games. Suppose for Γ^W that $i = 2$ offers $t = a_2 + y^R(2)/a_2$ and $y = y^{RS}(2)$, while $i = 1$ proposes with probability $0 \leq \rho_\delta^W \leq 1$ the contract $t = a_i$, $y = 0$, and with probability $1 - \rho_\delta^W$ the null contract.[23] In Γ^F firms offer with probability $\rho_\delta^{F,P} > 0$ a single (pooling) contract $y = 0$, $t = V_\delta^R(2)$, while offering with probability $\rho_\delta^{F,S}$ the separating menu with $c_\delta^F(1)$ and $c_\delta^F(2)$ described in Case 1. With probability $1 - \rho_\delta^{F,P} - \rho_\delta^{F,S}$ firms offer only $c_\delta^F(2)$ to $i = 2$. Observe that these strategies yield

$$\bar{\rho} = (1-b)\rho_\delta^F + b(\rho_\delta^{F,P} + \rho_\delta^{F,S}). \tag{6}$$

We impose now the requirement that $V_\delta^R(1) = V^{RS}(1)$ holds for all sufficiently high values

[23] Recall that this is equivalent to offering any unacceptable contract.

of δ, which transforms to

$$\frac{\delta(1-m_\delta)b\rho_\delta^{F,P}V_\delta^R(2)}{1-\delta\left[m_\delta+(1-m_\delta)(1-b\rho_\delta^{F,P})\right]}=a_1-U^0. \tag{7}$$

Observe that the reservation value for $i=2$ is still given by (4). To obtain the reservation value of firms, note that by previous results firms are indeed indifferent between all three specified offers as $V_\delta^R(1)=V^{RS}(1)$ and $\mu_\delta(1)=\mu^*(1)$. This yields $U_\delta^R=f_\delta[\mu^*(1)a_1+\mu^*(1)a_2-V_\delta^R(2)]$, which allows to obtain a unique matching probability m_δ from the requirement that $U_\delta^R=U^0$. Indeed, inspection reveals that m_δ is again uniquely determined by (5).[24] It remains to determine the probabilities with which agents randomize over the specified proposals. From (7) we can solve for a unique value $0<\rho_\delta^{F,P}<1$ if δ is sufficiently high.[25] Observe in particular that $\lim_{\delta\to 1}\rho_\delta^{F,P}=0$, which again allows for high values of δ to find a pair $\rho_\delta^{F,S}$,ρ_δ^W satisfying jointly with $\rho_\delta^{F,P}$ the equation (6), while $\rho_\delta^{F,S}+\rho_\delta^{F,P}\leq 1$, $\rho_\delta^W\leq 1$. The way we have constructed the equilibrium candidate, only the sum of $\rho_\delta^{F,S}$ and ρ_δ^W is uniquely determined. The rest of the argument is now analogous to that in Case 1.

We should note that even for Case 1 with $\mu^0(1)\geq\mu^*(1)$ the (refined) set of equilibria $\Psi_\delta(M)$ may be quite large. Though Proposition 2 puts much structure on the distribution of equilibrium contracts for low frictions, this is not the case for the distribution of types in the market.

Observe finally that we have use mixed strategies to construct equilibria in Case 2. Given a continuum of firms, this is equivalent to specifying asymmetric pure strategies. Moreover, the nature of the mixed strategy equilibrium is different to that obtained in a two-stage screening game, where pure strategy equilibria fail to exist if $\mu^0(1)<\mu^*(1)$.[26] In the two-stage model strategies are not ex-post optimal, i.e. individual firms would like to readjust their strategies after observing their opponents choices. Of course, this is not an issue in our model.

General payoff functions

We state next an existence result for general payoff functions satisfying (A.1)-(A.3). Precisely, we want to ensure for sufficiently high δ existence of a sequence of equilibria where for $\delta\to 1$ the stock of workers remains bounded, while $f\to 1$. Recall that these were the requirements imposed for the definition of $\Psi_\delta(M)$ in Section 3.

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[24] To see this, substitute $V_\delta^R(2)$ and observe that $\mu^*(1)a_1+\mu^*(2)a_2=a_2-a_1(a_2-a_1)/a_2$ holds by$\mu^*(1)=a_1/a_2$.

[25] This is possible as $\lim_{\delta\to 1}V_\delta^R(2)=V^{RS}(2)>V^{RS}(1)$ and $\lim_{\delta\to 1}m_\delta=\bar{m}$.

[26] See Dasgupta and Maskin (1986) on existence of the mixed strategy equilibrium in this case.

For the sake of brevity, Appendix C only states the proof for $\bar{i} = 2$. The arguments used in the proof, however, extend to any finite $\bar{i}$.[27].

Proposition 3. *Consider the case with* $\bar{i} = 2$. *Then we can find* $\bar{\delta} < 1$ *and a finite* M *such that there exists for all* $\delta > \bar{\delta}$ *an equilibrium denoted by* ψ_δ *where* $W < M$ *and where, given the specified values of* m_δ, *it holds that* $lim_{\delta \to 1} f_\delta = 1$.

5. Unbounded Markets

In this section we show by example that the convergence results of Propositions 1-2 cease to hold if we do not restrict attention to some selection $\Psi_\delta(M)$. Recall that the restriction consists of two parts. First, search costs for firms must vanish for $\delta \to 1$ as $f \to 1$. Second, the stock of workers must remain bounded. Throughout this section we will use as an example the two-type Spence case.

5.1 Firms Flooding the Market

We derive an example where f does not converge to 1 for $\delta \to 1$. Observe first that $f \to 1$ is surely necessary to obtain the convergence result in Propositions 1-2, as otherwise given the convergence of contracts firms would not be able to realize U^0. What is, however, of more interest is the fact that such a sequence of equilibria exists. To put this into some perspective, suppose first that there is no private information.

The benchmark with complete information

For low frictions it is straightforward to show that there exists a unique equilibrium (of the two-type Spence case) where all workers enter and all matches are successful. Contracts are free of distortions ($y = 0$), while transfers are chosen to make the responding party indifferent between acceptance and rejection. Requiring $U_\delta^R = U^0$ we obtain

$$\begin{aligned} U^0 &= \frac{\delta m_\delta b}{1 - \delta + \delta m_\delta + \delta(1 - m_\delta)(1 - b)}[\mu^0(1)a_1 + \mu^0(2)a_2], \qquad (8) \\ V_\delta^R(i) &= \frac{\delta(1 - m_\delta)(1 - b)}{1 - \delta\left[1 - (1 - m_\delta)(1 - b)\right]}[a_i - U^0], \\ m_\delta &= \frac{U^0(1 - \delta b)}{\delta b(\mu^0(1)a_1 + \mu^0(2)a_2 - U^0) + \delta(1 - b)U^0}. \end{aligned}$$

[27] We should note that the delicate issue is not to establish existence of *an* equilibrium, but of a sequence of equilibria where $\delta \to 1$, $W < M$, and $\lim_{\delta \to 1} f_\delta = 1$

For $\delta \to 1$ the aggregate surplus realized by a newly entering cohort of workers becomes $W^0[\sum_{i\in I}\mu^0(i)s(i,y^*(i)) - U^0 - V^0]$[28].

We construct now an equilibrium with private information where inefficiencies will persist even as frictions vanish. Suppose that $\mu^0(1) \geq \mu^*(1)$. In Γ^W type $i = 1$ proposes $t = a_1 - U^0$ and $y = y^{RS}(1) = 0$, while $i = 2$ proposes $y^W_\delta(2) = y^{RS}(2) + \Delta$ and $t = a_2 - U^0$. The value Δ satisfies $0 < \Delta \leq (a_2 - a_1)^2$. (Recall that $V^{RS}(2) - V^{RS}(1) = (a_2 - a_1)^2/a_2$.) In $\Gamma^F$the firm offers a menu which specifies for the low type $y = 0$ and $t^F_\delta(1) = V^R_\delta(1) - U^0$, and for the high type $y^F_\delta(2) = (V^R_\delta(2) - V^R_\delta(1))a_1a_2/(a_2 - a_1)$ and $t^F_\delta(2) = V^R_\delta(2) + y^F_\delta(2)/a_2$. Moreover, all workers enter, i.e. $E^W_\delta(i) = M^0(i)$. As all matches are successful, this implies $\mu_\delta(i) = \mu^0(i)$. For firms we specify $E^F_\delta = E^W_\delta$. Substituting $V^R_\delta(i)$ into the requirement $U^R_\delta = U^0$ yields a unique matching probability m_δ, satisfying $lim_{\delta\to 1}m_\delta = 0$. As we can specify pessimistic beliefs in Γ^F and as $\mu^0(1) \geq \mu^*(1)$, it is easily checked that strategies in the contractual games are optimal.[29]

As stocks in the market are given by $W_\delta(i) = W^0(i)/(1-m_\delta)$ and $F_\delta = W^0/m_\delta$, F_δ grows beyond any boundary as $\delta \to 1$. More precisely, the stock of firms grows sufficiently fast such that $lim_{\delta\to 1}f_\delta < 1$. In fact, $\bar{f} = lim_{\delta\to 1}f_\delta$ is determined by[30]

$$\bar{f}\left[U^0 + \mu^0(2)\frac{\Delta}{a_2 - a_1}\right] = U^0.$$

To see why this is intuitive, observe that $lim_{\delta\to 1}V^R_\delta(1) = V^{RS}(1)$ and $lim_{\delta\to 1}V^R_\delta(2) = V^{RS}(2) - \Delta/a_2$. As a consequence, firms can realize strictly more than U^0 in Γ^F. However, to keep the market stationary, firms must be kept indifferent between entering or not, implying that their circulation time grows beyond any boundary as $\delta \to 1$.[31] As frictions vanish, the resulting efficiency gains are thus almost entirely offset by increasing the expected waiting time for firms. This type of inefficiency was completely absent in the benchmark case with

[28] We should not that for $\delta < 1$ the market outcome is generally inefficient due to two well known reasons. First, markets where transfers are determined after matches have formed fail to internalize the impact of players' entry decision on the matching probability of other agents (see Hosios (1990). Second, a single matching market cannot adequately adjust to the preferences of heterogenous agents (on one side), i.e. to the different marginal rates of substitution between the speed and the terms of trade (see e.g. Moen (1997)).

[29] Observe in particular that $V^R_\delta(1) < V^{RS}(1)$ ensures that it is not optimal for firms to offer only a contract to $i = 2$.

[30] To see this, observe for Γ^F that in the limit firms realize with high types $U^0 + \Delta/a_2 + y^{RS}(2)/a_2 - y^F_1(2)/a_2$, where we substiuted $a_2 - y^{RS}(2)/a_2 = U^0 + V^{RS}(2)$.

[31] In other words, circulation time and delay of trading become now an essential equilibrating device in a market with adverse selection. This is reminiscent of the Walrasian approach in Gale (1992, 1996) who considers a one-shot setting where probability of trade may vary in various (contractual) submarkets.

complete information.[32][33]

Recall at this point that we constructed an equilibrium in Proposition 3 where high types implement $c^{RS}(2)$ in Γ^F. We showed that the resulting unique matching probability m_δ was identical to that in (8), implying in particular that the low type's utility was unaffected by the presence of private information. In contrast, low types are better off in the "unbounded" equilibrium where firms flood the market.

5.2 Low Types Flooding the Market

We will show that the following strategies constitute an equilibrium for sufficiently high δ. All workers enter and offer in Γ^W the pooling contract with $y = 0$ and $t = a_1 - U^0 + \mu_\delta(2)(a_2 - a_1)$, where the distribution of types in the market will be derived endogenously. Observe that firms are indifferent between accepting and rejecting the proposal, which allows to specify that firms accept with some probability $\rho_\delta^W \in [0, 1]$. Below we will specify that both types do not receive more than their reservation values in Γ^F, which yields

$$V_\delta^R(i) = \frac{\delta(1 - m_\delta)(1 - b)}{1 - \delta[1 - (1 - m_\delta)(1 - b)]} \left[(1 - \rho_\delta^W)V_\delta^R(i) + \rho_\delta^W(a_1 - U^0 + \mu_\delta(2)(a_2 - a_1))\right],$$

implying in particular that $V_\delta^R(1) = V_\delta^R(2)$. In Γ^F firms offer a single contract with $y = 0$ and $t = V_\delta^R(2)$. As this offer makes both types indifferent, we can assume that it is only accepted by the high type. Combining the specification of strategies for the two games and the assumption that all workers enter, The distribution $\mu_\delta(2)$ is equal to $[(1 - b)\rho_\delta^W \mu^0(2)]/[1 + b\mu^0(1)]$. We impose now the requirements that

$$V_\delta^R(i) = V^{RS}(1) = a_1 - U^0 \tag{9}$$

and that firms realize exactly U^0, which by (9) is the case if

$$U^0 = \frac{\delta m_\delta b}{1 - \delta(1 - m_\delta b)}(U^0 + \mu_\delta(2)(a_2 - a_1)). \tag{10}$$

Observe finally that the specified strategies are indeed optimal if equations (9)-(10) have a solution.

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32 We should note that the outcome under complete information depends on the assumption that one side is chosen to make a take-it-or-leave-it offer. Other forms of ex-post agreements tend to generally induce too much entry by traders on the long side of the market (see Peters 1992)) as the bargaining outcomes are relatively insensitive to aggregate demand and supply (see also Bester (1987) and Muthoo (1993) on this issue.)

33 Admittedly, the relevant benchmark under private information is constrained (or interim) efficiency, as defined e.g. by Holmström and Myerson (1983). As the delay of firms does not contribute to separation, the depicted equilibrium will naturally fail an adequately defined notion of interim efficiency. (Circulation time of the informed party can, however, be useful as a separating device in alternative settings which allow for the co-existence of several submarkets. This is analyzed by Inderst and Müller (1999) for a market with durable second-hand goods.)

Existence of a solution is established for high δ in the following Lemma.

Lemma 1. *There exists* $\bar{\delta} < 1$ *such that for* $\delta > \bar{\delta}$ *the system of equations (9)-(10) has a solution* $(m_\delta, \rho_\delta^W)$ *where* $0 < m_\delta < 1$ *and* $0 < \rho_\delta^W < 1$.

Proof. Consider first (10). For given m and $0 < \delta < 1$ there exists a unique value $\mu_\delta^1(m)$ defined by $U^0 = f_\delta[U^0 + \mu_\delta^1(m)(a_2 - a_1)]$. (Observe that it is not guaranteed that $\mu_\delta^1(m) \leq 1$.) Moreover, $\mu_\delta^1(m)$ is continuous and strictly decreasing with $lim_{m\to 0}\mu_\delta^1(m) = \infty$ and $\mu_\delta^1(1) = U^0(1-\delta)/[(a_2 - a_1)\delta b]$. Consider next (9), where we can substitute

$$\rho_\delta^W = \mu_\delta(2)\frac{b\mu^0(1)}{(1-b)(\mu^0(2) - \mu_\delta(2))}, \tag{11}$$

which is well-defined for $\mu_\delta(2) < \mu^0(2)$. (Observe, however, that $\rho_\delta^W \leq 1$ is only satisfied if $\mu_\delta(2)$ is chosen sufficiently low.) For given m and δ, (9) defines a unique value $\mu_\delta^2(m) < \mu^0(2)$ solving $V_\delta^R(1) = V^{RS}(1)$. Observe that $\mu_\delta^2(m)$ is continuous and strictly increasing with $lim_{m\to 1}\mu_\delta^2(m) = \mu^0(2)$ and a finite value $\mu_\delta^2(0)$. It also holds that $lim_{\delta\to 1}\mu_\delta^2(0) = 0$ and $lim_{\delta\to 1}\mu_\delta^1(1) = 0$. By the properties of $\mu_\delta^1(m)$ and $\mu_\delta^2(m)$, we can thus find a threshold $\bar{\delta}_1 < 1$ such that for $\delta > \bar{\delta}_1$ there exists a unique value $0 < m_\delta < 1$ realizing $\mu_\delta(2) = \mu_\delta^1(m_\delta) = \mu_\delta^2(m_\delta) < \mu^0(2)$. It remains to show that $\mu_\delta(2)$ substituted into (11) realizes $\rho_\delta^W \leq 1$ for sufficiently high values of δ. This is implied by the stronger claim that $lim_{\delta\to 1}\rho_\delta^W = 0$. If the latter assertion did not hold, we would obtain for an adequately selected subsequence $lim_{\delta\to 1}\mu_{\delta_n}(2) = \bar{\mu} > 0$. By inspection of (9) this implies $lim_{\delta_n\to 1}m_{\delta_n} = 0$, while by the definition of $\mu_\delta^2(\cdot)$ it must hold that $lim_{\delta_n\to 1}m_{\delta_n} = 1$, which yields a contradiction. By $lim_{\delta\to 1}\rho_\delta^W = 0$ we can thus indeed find some $\bar{\delta}_2 < 1$ such that for $\delta > \bar{\delta}_2$ it holds that $\rho_\delta^W < 1$. Choosing $\bar{\delta} = max\left\{\bar{\delta}_1, \bar{\delta}_2\right\}$ completes the proof. **Q.E.D.**

Inspection of the proof reveals that $lim_{\delta\to 1}\rho_\delta^W = 0$ and therefore $lim_{\delta\to 1}\mu_\delta(2) = 0$. Observe that in the constructed equilibrium low-type workers only implement the contract with $y = 0$ and $t = a_1 - U^0 + \mu_\delta(2)(a_2 - a_1)$, which actually converges to their RS contract. However, as $lim_{\delta\to 1}\rho_\delta^W = 0$, high-type workers will for high δ almost always implement the contract proposed in Γ^F, which specifies $y = 0$ and thus differs from their RS contract. Interestingly, observe that the surplus realized in a successful match specifies the first-best value of the sorting variable $y = 0$. However, as firms realize U^0 and as $lim_{\delta\to 1}V_\delta^R(i) = a_1 - U^0$ for both types $i \in I$, much surplus gets dissipated by search frictions as $\delta \to 1$. (Observe, however, that this time f converges to 1 as $\delta \to 1$.)

In the light of Proposition 2 and the two examples where convergence fails as the market size increases beyond any boundary, it would be interesting to know more about the efficiency properties of different equilibria. The derived results suggest that there is a trade-off between dissipating surplus by excessive search or circulation time and reducing the surplus in individual matches by distorting the sorting variable.

6. Conclusion

This paper explores a new approach to analyze markets with adverse selection. We consider a matching market environment where in a given match either side may have the right of proposal. This allows for the simultaneous presence of signaling and screening. Our approach yields three main insights. First, if the market size remains bounded as frictions vanish, the distribution of implemented contracts converges. Second, the distribution of types in the market must not necessarily reflect the distribution among entrants as different types may have different circulation times depending on how successful their matches are. Third, matching markets with adverse selection may exhibit a new type of inefficiency which is absent in markets with complete information: excessive circulation of either firms or low types (of workers). In essence, this is due to the fact that high types may realize less than their "true" share of the surplus if firms have pessimistic beliefs. The residual surplus is then offset by sufficiently long circulation of either firms or low types. Of course, by the convergence result, this inefficiency vanishes if frictions disappear and the market size remains bounded.

To our knowledge this paper represents the first contribution which models markets with adverse selection in this fashion. To conclude we want to stress one avenue for further research. We conjecture that our convergence result is independent of the particular contractual games, as long as the uninformed side has some right of proposal. However, we would find it worthwhile to explore the following two alternatives. First, we may allow also the informed party to propose a menu of offers from which it can pick any contract after acceptance.[34] This would put additional restrictions on the set of equilibrium outcomes.[35] Second, a natural way of modeling bargaining in a match would be to consider a game of alternating offers. Unfortunately, there is so far almost no literature on bargaining over contracts (with an open time horizon) even in a bilateral monopoly.[36]

Appendix A: Proof of Proposition 1

The proof proceeds in a series of claims. Claims 1-2 derive implications which are intuitive given the primitives of our matching models. In Claim 1 we show that $U^R = U^0$, which is subsequently used to prove that all workers

[34] In a bilateral monopoly this approach has been pioneered by Maskin and Tirole (1992).

[35] Indeed, we conjecture for this specification that we can find for given primitives a threshold $\bar{b} < 1$ such that for $b > \bar{b}$ equilibrium outcomes converge to the RS allocation of contracts as frictions vanish.

[36] To our knowledge, alternating offers with private information and a sorting variable have only been considered in Inderst (1999b). However, this contribution is restricted to two types, private values, and only a subset of parameters (discount factors). Inderst (1998) considers the case where only the uninformed party makes offers but cannot commit to a final proposal.

enter for high δ.

Claim 1. *In all* $\psi \in \Psi$ *it holds that* $U^R = U^0$.

Proof. Recall that we restrict consideration to equilibria where the market opens up, implying $U^R \geq U^0$. As agents exit in pairs, $F^0 > W^0$ implies $U^R \leq U^0$ to ensure stationarity. **Q.E.D.**

Claim 2. *There exists* $\bar{\delta}_1 < 1$ *such that for all* $\delta > \bar{\delta}_1$, $\psi \in \Psi_\delta(M)$, *and* $i \in I$ *it holds that* $E^W(i) = W^0(i)$.

Proof. We first prove by contradiction that $E^W(\bar{i}) = W^0(\bar{i})$. Otherwise, there exists a sequence ψ_δ where $\delta \to 1$, $\psi_\delta \in \Psi_\delta(M)$, and $E^W_\delta(\bar{i}) < W^0(\bar{i})$. (Observe that all variables determined in ψ_δ are indexed by δ.) By (A.4), $E^W_\delta(\bar{i}) < W^0(\bar{i})$ implies $V^R_\delta(i) = V^0$ if $E^W_\delta(i) > 0$ for some $i < \bar{i}$. Observe next that from (A.3)-(A.4) it holds that $U(i, c^{RS}(1)) \geq U^0$ and $V(i, c^{RS}(1)) > V^0$ for all $i \in I$, implying that c with $t = t^{RS}(1) + (V(1, c^{RS}(1)) - V^0)/2$ and $y = y^{RS}(1)$ is strictly acceptable to all types. If a firm rejects all offers and proposes c in Γ^F, the expected utility is bounded from below by $\hat{U} = f_\delta[U^0 + (V(1, c^{RS}(1)) - V^0)/2]$, where

$$f_\delta = \frac{\delta b m_\delta}{1 - \delta\,[1 - b m_\delta]}.$$

If a worker of type $\bar{i}$ behaves similarly, we obtain as a lower boundary $\hat{V} = g_\delta[V^0 + (V(1, c^{RS}(1) - V^0)/2]$, where

$$g_\delta = \frac{\delta(1-b)(1-m_\delta)}{1 - \delta\,[1 - (1-b)(1-m_\delta)]}.$$

As $U^R_\delta = U^0$ by Claim 1, it must hold that $\hat{U} \leq U^0$, which implies $\lim_{\delta\to 1} f_\delta < 1$ and therefore $\lim_{\delta \to 1} g_\delta = 1$. As a consequence, $\hat{V} > V^0$ holds if $\delta > \bar{\delta}^1_1$ for some $\bar{\delta}^1_1 < 1$, which yields a contradiction to $V^R_\delta(\bar{i}) = V^0$.

We next extend the claim to all $i < \bar{i}$. By $E^W_\delta(\bar{i}) = W^0(\bar{i})$ we obtain $m_\delta < m^B_h$ for some $m^B_h < 1$, as otherwise $W_\delta < M$ could not be ensured. If a type i enters the market, his utility is bounded from below by $\hat{V}$ as defined above. As $m_\delta < m^B_h$ implies $\lim_{\delta\to 1} g_\delta = 1$, this exceeds V^0 if $\delta > \bar{\delta}^2_1$ for some $\bar{\delta}^2_1 < 1$. Choosing $\bar{\delta}_1 = \max\left\{\bar{\delta}^1_1, \bar{\delta}^2_1\right\}$ completes the proof. **Q.E.D.**

Claim 2 has the following implications for $\delta > \bar{\delta}_1$ and $\psi \in \Psi_\delta(M)$. We already observed in the proof of Claim 2 that $m_\delta < m^B_h < 1$ must hold to ensure $W_\delta < M$. By the same argument there exists $\alpha^B < 1$ such that $\alpha^W(i, \emptyset) + \alpha^F(i, \emptyset) < \alpha^B$ for all $i \in I$. (Recall that $\alpha^F(i, \emptyset)$ denotes the probability with which a match with type i is broken up in Γ^F.) Moreover, by $W_\delta(i) \geq E^W_\delta(i) = W^0(i)$ and $W_\delta < M$ there exists $\mu^B > 0$ such that $\mu_\delta(i) > \mu^B$ for all $i \in I$. Given the definition of $\Psi_\delta(M)$ and $m_\delta < m^B_h$ it follows for any sequence of equilibria ψ_δ where $\delta \to 1$ and $\psi_\delta \in \Psi_\delta(M)$ that $f_\delta \to 1$ and $g_\delta \to 1$, while by $\mu_\delta(i) > \mu^B$ it also holds for all $i \in I$ that[37]

$$\frac{\delta \mu_\delta(i) b m_\delta}{1 - \delta\,[1 - \mu_\delta(i) b m_\delta]} \to 1. \tag{12}$$

These results will be frequently used in what follows. We proceed by deriving a lower bound on reservation values.

Claim 3. *For any* $\varepsilon > 0$ *there exists* $\bar{\delta}_2 < 1$ *such that for all* $\delta > \bar{\delta}_2$, $\psi \in \Psi_\delta(M)$ *and corresponding reservation values* $V^R(i)$, *it holds that* $V^{RS}(i) - \varepsilon < V^R(i)$.

Proof. The proof is inductive. Consider $i = 1$. We argue to a contradiction and assume that there exists a sequence of equilibria ψ_δ where $\delta \to 1$, $\psi_\delta \in \Psi_\delta(M)$, and $V^R_\delta(1) \leq V^{RS}(i) - \varepsilon$. A firm offering c with

[37] By definition of $\Psi_\delta(M)$ the convergence is uniform.

$t = t^{RS}(1) - \varepsilon/2$ and $y = y^{RS}(1)$ in Γ^F while rejecting all offers in Γ^W receives at least

$$\frac{\delta \mu_\delta(1) bm_\delta}{1 - \delta\left[1 - \mu_\delta(1) bm_\delta\right]}(U^0 + \varepsilon/2), \tag{13}$$

where we use common values and $U(1, c^{RS}(1)) \geq U^0$. By (12), the expression (13) exceeds U^0 if $\delta > \bar{\delta}_2(1)$ for some $\bar{\delta}_2(1) < 1$, which yields a contradiction. Assume now that the assertion holds up to a type $i - 1 < \bar{i}$. We argue again to a contradiction. Recall next that the RS family of contracts is by Lemma 0 globally incentive compatible. By the inductive claim, the contract c with $y = y^{RS}(i)$ and $t^{RS}(i) - t = \varepsilon/2$ is rejected by all types $j < i$ and accepted by i if δ becomes sufficiently large. Hence, by an argument as for $i = 1$, the firms' expected payoff is bounded from below by $[\delta \mu_\delta(i) bm_\delta][U^0 + \varepsilon/2]/[1 - \delta\left[1 - \mu_\delta(i) bm_\delta\right]]$, which again exceeds U^0 if $\delta > \bar{\delta}_2(i)$ for some $\bar{\delta}_2(i) < 1$. Choosing $\bar{\delta}_2 = \bar{\delta}_2(\bar{i})$ proves the claim by the finiteness of I. **Q.E.D.**

We turn next to the upper bound on reservation values. We proceed indirectly by proving a result on the set of implemented contracts, where we discuss first the case of $i = 1$.

Claim 4. *For any $\tilde{\varepsilon} > 0$ there exists $\bar{\delta}_3(\tilde{\varepsilon}) < 1$ such that for all $\delta > \bar{\delta}_3(\tilde{\varepsilon})$ and $\psi \in \Psi_\delta(M)$, it holds that*

$$\sum_{c \in B(1) \cap \{c | V(1,c) \geq V^{RS}(1) + \tilde{\varepsilon}\}} (\alpha^W(1, c) + \alpha^F(1, c)) < \tilde{\varepsilon}$$

Proof. We argue to a contradiction and assume that there exists a sequence $\psi_\delta \in \Psi_\delta(M)$, where $\delta \to 1$, such that there is a non-empty set $C_\delta \subseteq B_\delta(1)$ satisfying $\sum_{c \in C_\delta}(\alpha_\delta^W(1, c) + \alpha_\delta^F(1, c)) \geq \tilde{\varepsilon}$ and $V(1, c) \geq V^{RS}(1) + \tilde{\varepsilon}$ for all $c \in C_\delta$. Define next $C_\delta^F = C_\delta \cap B_\delta^F(1)$ and $C_\delta^W = C_\delta \cap B_\delta^W(1)$. We distinguish between two cases. We can either choose a subsequence δ_n such that $\sum_{c \in C_{\delta_n}^W} \alpha_{\delta_n}^W(1, c) \geq \tilde{\varepsilon}/2$ holds along the sequence, or a subsequence where $\sum_{c \in C_{\delta_n}^F} \alpha_{\delta_n}^F(1, c) \geq \tilde{\varepsilon}/2$ holds.

Case i) Subsequence with C_δ^W

Take the original sequence as the subsequence implying $\sum_{c \in C_\delta^W} \alpha_\delta^W(1, c) \geq \tilde{\varepsilon}/2$. We prove first the following implication.

Assertion 1. *Under the assumption of Case i), there exists (for given $\tilde{\varepsilon}$) a sequence of contracts $\{c_\delta\}$ and types $\{i_\delta\}$, and a value $\bar{\varepsilon} > 0$ such that:*

i) $c_\delta \in C_\delta^W$; $\pi(i_\delta, c_\delta) \geq \bar{\varepsilon}$; and $\gamma_\delta^W(1, c_\delta) > \bar{\varepsilon}$.

ii) $U(i_\delta, c_\delta) \geq U^0 + \bar{\varepsilon}$ with $i_\delta \neq 1$.

Proof. The assumption that $\sum_{c \in C_\delta^W} \alpha_\delta^W(1, c) \geq \tilde{\varepsilon}/2$ implies the existence of a sequence of contracts $\{c_\delta\}$ and of two values $\bar{\varepsilon}_1, \bar{\varepsilon}_2 > 0$ such that the following two claims hold:

- If c_δ is offered in Γ^W, a firm accepts with probability not below $\bar{\varepsilon}_1$.

- Additionally, when observing c_δ, the firm's consistent beliefs put not less than probability $\bar{\varepsilon}_2$ on $i = 1$.[38]

Observe next that, for given $\tilde{\varepsilon} > 0$, there exists some $\bar{\varepsilon}_3 > 0$ such that $U(1, c) \leq U^0 - \bar{\varepsilon}_3$ holds for all $c \in C$ where $V(1, c) \geq V^{RS}(1) + \tilde{\varepsilon}$. (Given transferable utilities and the definition of $c^{RS}(1)$, we can choose $\bar{\varepsilon}_3 = \tilde{\varepsilon}$.) By

[38] More formally, take some equilibrium ψ_δ. Recall that we denote the probability with which some type i proposes $c \in C$ by $\rho_\delta^W(i, c)$ and the acceptance probability of the firm by $\gamma_\delta^W(i, c)$. As $c \in B_\delta^W$, beliefs are defined by Bayes' rule and denoted by $\pi_\delta(i, c) = \mu_\delta(i)\rho_\delta^W(i, c)/\sum_{j \in I} \mu_\delta(j)\rho_\delta^W(j, c)$. Recall next the requirement $\sum_{c \in C_\delta^W} \alpha_\delta^W(1, c) \geq \tilde{\varepsilon}/2$, where $\alpha_\delta^W(i, c) = \rho_\delta^W(i, c)\gamma_\delta^W(i, c)$. This immediately implies existence of two values $\bar{\varepsilon}_1, \bar{\varepsilon}_2' > 0$ and a set $\tilde{C}_\delta^W \subseteq C_\delta^W$ such that $\sum_{c \in \tilde{C}_\delta^W} \rho_\delta^W(1, c) > \bar{\varepsilon}_2'$ and $\gamma_\delta^W(1, c) > \bar{\varepsilon}_1$ for all $c \in \tilde{C}_\delta^W$. Finally, the finiteness of I allows to pick a contract $c_\delta \in \tilde{C}_\delta^W$ and some $\bar{\varepsilon}_2 > 0$ such that indeed $\pi_\delta(1, c_\delta) \geq \bar{\varepsilon}_2$.

optimality, the firm only accepts c_δ if the expected utility is not below U^0. Given $U(1,c_\delta) \leq U^0 - \bar{\varepsilon}_3$, the finiteness of the type set I, and the fact that the firm's posterior beliefs after observing c_δ assign at least probability $\bar{\varepsilon}_2$ to the type $i = 1$, this implies existence of some type $i_\delta > 0$ and a boundary $\bar{\varepsilon}_4 > 0$ such that $U(i_\delta, c_\delta) \geq U^0 + \bar{\varepsilon}_4$. Finally, choose $\bar{\varepsilon} = min\left[\bar{\varepsilon}_1, \bar{\varepsilon}_2, \bar{\varepsilon}_3, \bar{\varepsilon}_3\right]$. **Q.E.D.**

In the remainder of the proof for Case i) we show that firms can profitably deviate as δ becomes sufficiently high. We will construct a contract from c_δ, which will be offered by firms in Γ^F and which will only attract types $i \geq i_\delta$. For this construction we need the following auxiliary result.

Assertion 2. *For any* ε_1 *there exists some* $\hat{\delta}_1(\varepsilon_1) > 0$ *such that* $V_\delta^R(i) > V(i, c_\delta) - \varepsilon_1$ *for all* $i \in I$.

Proof. If type i follows the strategy to reject all offers and to propose c_δ in Γ^W, which by Assertion 1 is accepted with a probability not below $\bar{\varepsilon} > 0$, his expected utility is bounded from below by

$$\hat{V} = \frac{\delta(1-b)(1-m_h^B)\bar{\varepsilon}}{1-\delta\left[1-(1-b)(1-m_h^B)\bar{\varepsilon}\right]} V(i, c_\delta),$$

where we use $m_\delta < m_h^B < 1$. The assertion follows now immediately from the equilibrium requirement that $\hat{V} \leq V_\delta^R(i)$.[39] **Q.E.D.**

Define now by $\bar{C}$ the set of contracts $c \in C$ satisfying $U(i,c) \geq U^0$ and $V(i,c) \geq V^0$ for some $i \in I$. By (A.1), the finiteness of I, and continuity of payoffs the set is compact. By (A.2) this implies existence of two values $\bar{k} > 0$, $\underline{k} > 0$, such that $|ds(i,y)/dy| < \bar{k}$ and $d[v(j,y) - v(i,y)]/dy > \underline{k}$ for all $c \in \bar{C}$. For any small $\varepsilon_2 > 0$ we construct now from c_δ a contract $\tilde{c}_\delta(\varepsilon_2)$, which will be used for a profitable deviation. Define $\tilde{y}_\delta(\varepsilon_2) = y_\delta + \varepsilon_2/\underline{k}$ (where y_δ is the sorting variable in c_δ) and adjust the transfer $\tilde{t}$ to ensure $V(i_\delta, \tilde{c}_\delta(\varepsilon_2)) = V(i_\delta, c_\delta) + \varepsilon_2/2$. Suppose for a moment that contracts on the line combining c_δ with $\tilde{c}_\delta(\varepsilon_2)$ belong to $\bar{C}$. By the derived boundaries on derivatives, this implies $V(j, \tilde{c}_\delta(\varepsilon_2)) < V(j, c_\delta) - \varepsilon_2/2$ for all $j < i_\delta$ and $U(i_\delta, \tilde{c}_\delta(\varepsilon_2)) \geq U(i, c_\delta) - \varepsilon_2(\bar{k}/\underline{k}) - \varepsilon_2/2$. Contracts on the line combining c_δ with $\tilde{c}_\delta(\varepsilon_2)$ belong indeed to $\bar{C}$ for sufficiently small values of ε_2.[40]

Recall next the construction of $\hat{\delta}_1(\varepsilon_1)$ from Assertion 2. Specifying $\varepsilon_1 < \varepsilon_2/4$ thus ensures that all types $j < i_\delta$ reject $\tilde{c}_\delta(\varepsilon_2)$ for $\delta > \hat{\delta}_1(\varepsilon_1)$. By (A.3), the utility realized with types $j > i_\delta$ is not lower than $U(i_\delta, \tilde{c}_\delta(\varepsilon_2))$. Hence, the expected utility realized by a firm which follows the strategy to reject all offers and to propose $\tilde{c}_\delta(\varepsilon_2)$ in Γ^F, is by Assertion 1 bounded from below by

$$\hat{U} = \frac{\delta b m_\delta \mu_\delta(i_\delta)}{1-\delta\left[1-b m_\delta \mu_\delta(i_\delta)\right]}\left[U^0 + \bar{\varepsilon} - \varepsilon_2(\bar{k}/\underline{k}) - \varepsilon_2/2\right].$$

By (12) this strictly exceeds U^0 if ε_2 becomes sufficiently small and δ sufficiently large, which completes the proof for case i).

Case ii) Subsequence with C_δ^F

Take again the original sequence as the subsequence implying $\sum_{c \in C_\delta^F} \alpha_\delta^F(1,c) \geq \tilde{\varepsilon}/2$. In an abuse of notation, we denote for a proposed menu $\{c(\cdot)\}$ the realized utility of type i by $V(i, \{c(\cdot)\}) = max_{n \in N} V(i, c(\cdot))$. (Recall the convention that $c(0) = \emptyset$ such that $V(i, c(0)) = V_\delta^R(i)$. This dependency on the reservation value is suppressed in the notation $V(i, \{c(\cdot)\})$.)

Assertion 3. *Under the assumption of Case ii), there exists (for given* $\tilde{\varepsilon}$*) a sequence of menus* $\{c_\delta(\cdot)\}$ *and a*

[39] To be precise, we need also that $V(i, c_\delta)$ is bounded from above (for any i). If this was not the case, $V_\delta^R(i)$ would grow beyond any bound, which would contradict optimality for firms.

[40] More formally, this follows from Assertion 1 and Claim 3, by which $V(i_\delta, c_\delta) \geq V_\delta^R(i_\delta) > V^0$ holds for low frictions.

value $\bar{\varepsilon} > 0$ *such that:*

i) $\rho_\delta^F(\{c_\delta(\cdot)\}) > 0$; $V(1, \{c_\delta(\cdot)\}) > V^{RS}(1) + \tilde{\varepsilon}$; *and* $\gamma_\delta^F(1, \{c_\delta(\cdot)\}, 0) < 1 - \bar{\varepsilon}$.[41]

ii) $U(1, c) < U^0 - \bar{\varepsilon}$ *for all* $c \in \{c_\delta(\cdot)\} \cap C$ *realizing* $V(1, c) = V(1, \{c_\delta(\cdot)\})$.

iii) For any ε_1 *there exists some* $\hat{\delta}_2(\varepsilon_1) > 0$ *such that* $V_\delta^R(i) > V(i, \{c_\delta(\cdot)\}) - \varepsilon_1$ *for all* $i \in I$ *and* $\delta > \hat{\delta}_2(\varepsilon_1)$.

Proof. By an argument as in Assertion 1, $\sum_{c \in C_\delta^F} \alpha_\delta^F(1, c) \geq \tilde{\varepsilon}/2$ implies existence of a sequence of *menus* denoted by $\tilde{C}_\delta^F$ and a value $\bar{\varepsilon}_1 > 0$ such that $\sum_{\{c(\cdot)\} \in \tilde{C}_\delta^F} \rho_\delta^F(\{c(\cdot)\}) > \bar{\varepsilon}_1$, while $V(1, \{c(\cdot)\}) > V^{RS}(1) + \tilde{\varepsilon}$ and $\gamma_\delta^F(1, \{c(\cdot)\}, 0) < 1 - \bar{\varepsilon}_1$ for all $\{c(\cdot)\} \in \tilde{C}_\delta^F$. In words, the set of menus $\tilde{C}_\delta^F$ is offered with at least probability $\bar{\varepsilon}_1$, and if a menu in this set is proposed, $i = 1$ accepts with at least probability $\bar{\varepsilon}_1$. By the finiteness of I we can choose next some $\bar{\varepsilon}_2 > 0$ and a sequence $\{c_\delta(\cdot)\} \in \tilde{C}_\delta^F$ such that for any $i \in I$ it holds that $\sum_{\{c(\cdot)\} \in \tilde{C}_\delta^F(i)} \rho_\delta^F(\{c(\cdot)\}) > \bar{\varepsilon}_2$, where

$$\tilde{C}_\delta^F(i) = \left\{ \{c(\cdot)\} \in \tilde{C}_\delta^F \mid V(i, \{c(\cdot)\}) \geq V(i, \{c_\delta(\cdot)\}) \right\}.$$

In words, $\{c_\delta(\cdot)\}$ is chosen from $\tilde{C}_\delta^F$ to ensure that for all types $i \in I$ the probability that firms offer menus in $\tilde{C}_\delta^F$ realizing not less than $V(i, \{c_\delta(\cdot)\})$ is not below some threshold $\bar{\varepsilon}_2$. Using the arguments of Assertion 2 this immediately implies the claim iii). Finally, regarding the claim ii), by construction of $c^{RS}(1)$ it holds that $U(1, c) < U^0 - \tilde{\varepsilon}$ in case $V(1, c) > V^{RS}(1) + \tilde{\varepsilon}$ for $c \in C$. We can now choose $\bar{\varepsilon} = min\{\bar{\varepsilon}_1, \bar{\varepsilon}_2, \tilde{\varepsilon}\}$. **Q.E.D.**

We are now in a position to construct a profitable deviation for firms. Assertion 3 allows us to restrict consideration to deviations where firms offer in Γ^F a *single* contract. At this point the argument is, in fact, completely analogous to that in Case i). We are therefore rather brief. Assertion 3 implies that we can again identify a type i_δ with which firms realize strictly more than U^0. Denote first

$$\hat{c}_\delta(i) \in argmax_{c \in \hat{C}_\delta} U(i, c), \text{ where } \hat{C}_\delta = \{c' \in \{c_\delta(\cdot)\} \mid V(i, c') = V(i, \{c_\delta(\cdot)\})\}.$$

By $\mu_\delta(1) > \mu^B$ and i)-ii) in Assertion 3 there exists now some $\bar{\bar{\varepsilon}} > 0$ (depending only on $\tilde{\varepsilon}$) and some type $i_\delta > 0$ such that $V(i_\delta, \hat{c}_\delta(i_\delta)) \geq V_\delta^R(i_\delta)$ and $U(i_\delta, \hat{c}_\delta(i_\delta)) > U^0 + \bar{\bar{\varepsilon}}$. As $V_\delta^R(i) > V(i, \{c_\delta(\cdot)\}) - \varepsilon_1$ holds by iii) in Assertion 3 for all types if $\delta > \hat{\delta}_2(\varepsilon_1)$, we can now construct a deviating contract from $\hat{c}_\delta(i_\delta)$, which is rejected by all types $i < i_\delta$, while it is implemented with probability one by i_δ. The construction is identical to that in Case i).

Having covered all possible cases in i) and ii), the proof of Claim 4 is completed. **Q.E.D.**

We next extend Claim 4 to all higher types.

Claim 5. *For any* $\tilde{\varepsilon} > 0$ *there exists* $\bar{\delta}_4(\tilde{\varepsilon}) < 1$ *such that for all* $\delta > \bar{\delta}_4(\tilde{\varepsilon})$, $\psi \in \Psi_\delta(M)$, *and* $i > 1$ *it holds that*

$$\sum_{c \in B(i) \cap \{c | V(i,c) \geq V^{RS}(i) + \tilde{\varepsilon}\}} (\alpha^W(i, c) + \alpha^F(i, c)) < \tilde{\varepsilon}. \tag{14}$$

Proof. We argue by induction. Suppose the claim holds up to type $i - 1 < \bar{i}$. Precisely, we assume that for any $\tilde{\varepsilon}_1$ there exists some $\hat{\delta}_1(i - 1, \tilde{\varepsilon}_1) < 1$ such that (14) holds for all $\delta > \hat{\delta}_1(i - 1, \tilde{\varepsilon}_1)$ and $j < i$. We first prove an immediate implication of this claim.

[41] Recall that $\rho_\delta^F(\{c_\delta(\cdot)\})$ denotes the probability with which firms offer the menu $\{c_\delta(\cdot)\}$, while $\gamma_\delta^F(1, \{c_\delta(\cdot)\}, n)$ denotes the probability with which type $i = 1$ selects the variant n from the menu $\{c_\delta(n)\}$. In particular, $\gamma_\delta^F(1, \{c_\delta(\cdot)\}, 0)$ denotes the probability of rejection.

Assertion 1. *If (14) holds for all* $j < i$ *and* $\delta > \hat{\delta}_1(i-1, \tilde{\varepsilon}_1)$*, then there also exists* $\hat{\delta}_2(i-1, \tilde{\varepsilon}_1) < 1$ *such that for all* $\delta > \hat{\delta}_2(i-1, \tilde{\varepsilon}_1)$*,* $\psi \in \Psi_\delta(M)$*, and* $j < i$ *it holds that* $V_\delta^R(j) \leq V^{RS}(j) + \tilde{\varepsilon}_1$.

Proof. Recall that $\sum_{c \in B_\delta(j)} (\alpha_\delta^W(j,c) + \alpha_\delta^F(j,c)) > \alpha^B$ for all $j \in I$. The assertion follows then immediately from the definition of reservation values in (2) and from the inductive assumption (i.e. from (14)). **Q.E.D.**

To prove the extension to type i, we argue again to a contradiction and assume that there exists a sequence $\psi_\delta \in \Psi_\delta(M)$, where $\delta \to 1$, and a sequence $C_\delta \subseteq B_\delta(i)$ such that $\sum_{c \in C_\delta} (\alpha_\delta^W(i,c) + \alpha_\delta^F(i,c)) \geq \tilde{\varepsilon}$ and $V(i,c) \geq V^{RS}(i) + \tilde{\varepsilon}$ for all $c \in C_\delta$.

We prove first an auxiliary result which allows us to proceed subsequently as in Claim 4.

Assertion 2. *Given* $\tilde{\varepsilon}$ *there exists* $\hat{\delta}_3 < 1$ *and for* $\delta > \hat{\delta}_3$ *a selection* $\tilde{C}_\delta \subseteq C_\delta$*, as well as some threshold* $\bar{\bar{\varepsilon}} > 0$*, such that the following claims hold:*

i) $\sum_{c \in \tilde{C}_\delta} (\alpha_\delta^W(i,c) + \alpha_\delta^F(i,c)) \geq \bar{\bar{\varepsilon}}$.

ii) $U(i,c) < U^0 - \bar{\bar{\varepsilon}}$ *for all* $c \in \tilde{C}_\delta$.

Proof. Observe first that, given $\tilde{\varepsilon}$, there exists some $\bar{\varepsilon}_1 > 0$ such that $V(i-1,c) \leq V^{RS}(i-1) + \bar{\varepsilon}_1$ and $V(i,c) \geq V^{RS}(i) + \tilde{\varepsilon}$ must imply $U(i,c) < U^0 - \bar{\varepsilon}_1$. (Formally, this follows from the definition of $c^{RS}(i)$ and continuity of payoff functions.) Define the subset $\tilde{C}_\delta \subseteq C_\delta$ where $c \in \tilde{C}_\delta$ if $V(i-1,c) \leq V^{RS}(i-1) + \bar{\varepsilon}_1$. We claim that there exists $\hat{\delta}_3 < 1$ such that $\sum_{c \in \tilde{C}_\delta} (\alpha_\delta^W(i,c) + \alpha_\delta^F(i,c)) \geq \tilde{\varepsilon}/2$ for $\delta > \hat{\delta}_3$. We argue to a contradiction and assume that this is not the case. This allows to select a subsequence δ_n where $\delta_n \to 1$ and $\sum_{c \in C \setminus \tilde{C}_\delta} (\alpha_\delta^W(i,c) + \alpha_\delta^F(i,c)) > \tilde{\varepsilon}/2$. Consider now the following strategy for type $i-1$. By following (partially) the strategy of type i to propose or accept contracts $c \in C \setminus \tilde{C}_\delta$, the expected utility is bounded from below by

$$\frac{\delta(1-m_h^B)\tilde{\varepsilon}/2}{1-\delta[1-(1-m_h^B)\tilde{\varepsilon}/2]} \left[V^{RS}(i-1) + \bar{\varepsilon}_1\right], \tag{15}$$

where we use $m_\delta < m_h^B < 1$. Recall from Assertion 1 that $V_\delta^R(j) \leq V^{RS}(j) + \tilde{\varepsilon}_1$ for $\delta > \hat{\delta}_2(i-1, \tilde{\varepsilon}_1)$ and $j < i$. By choosing $\tilde{\varepsilon}_1$ sufficiently low, the utility in (15) therefore strictly exceeds $V_\delta^R(i-1)$ for sufficiently high δ. This yields a contradiction such that the threshold $\hat{\delta}_3 < 1$ exists. The assertion follows now from the two steps by choosing $\bar{\bar{\varepsilon}} = min\{\tilde{\varepsilon}/2, \bar{\varepsilon}_1\}$. **Q.E.D.**

In what follows we restrict consideration to values $\delta > \hat{\delta}_3$ such that Assertion 2 applies.

We are now in a position to argue as in Claim 4. This time the starting point is the assumed existence of a sequence of sets $\tilde{C}_\delta$ and a threshold $\bar{\bar{\varepsilon}} > 0$ with $\sum_{c \in \tilde{C}_\delta} (\alpha_\delta^W(i,c) + \alpha_\delta^F(i,c)) \geq \bar{\bar{\varepsilon}}$ and $V(i,c) \geq V^{RS}(i) + \tilde{\varepsilon}$ for all $c \in \tilde{C}_\delta$. Denote the sets $\tilde{C}_\delta^F = \tilde{C}_\delta \cap B_\delta^F$ and $\tilde{C}_\delta^W = \tilde{C}_\delta \cap B_\delta^W$. We can again distinguish between two cases where along a subsequence either $\sum_{c \in \tilde{C}_{\delta_n}^W} \alpha_{\delta_n}^W(1,c) \geq \bar{\bar{\varepsilon}}/2$ or $\sum_{c \in \tilde{C}_{\delta_n}^F} \alpha_{\delta_n}^F(1,c) \geq \bar{\bar{\varepsilon}}$.

Case i) (Subsequence $\tilde{C}_\delta^W$*)*

We again take the original sequence as the subsequence such that $\sum_{c \in \tilde{C}_\delta^W} \alpha_\delta^W(1,c) \geq \bar{\bar{\varepsilon}}/2$. As in Assertion 1 of Claim 4 we can next extract a sequence $\{c_\delta\}$ with $c_\delta \in \tilde{C}_\delta^W$ such that for some value $\bar{\varepsilon} > 0$ it holds that $\pi(i, c_\delta) \geq \bar{\varepsilon}$ and $\gamma_\delta^W(i, c_\delta) > \bar{\varepsilon}$. Additionally, it holds that $U(i_\delta, c_\delta) \geq U^0 + \bar{\varepsilon}$ for some $i_\delta \neq 1$. Observe that we can use from Assertion 2 (of this proof) that all $c \in \tilde{C}_\delta^W$ already satisfy $U(i,c) < U^0 - \bar{\bar{\varepsilon}}$ for a fixed threshold $\bar{\bar{\varepsilon}}$. Assertion 2 of Claim 4 carries over immediately, i.e. for any ε_1 there exists some $\hat{\delta}_4(\varepsilon_1) < 1$ such that $V_\delta^R(j) > V(j, c_\delta) - \varepsilon_1$ for all $j \in I$ and $\delta > \hat{\delta}_4(\varepsilon_1)$. Moreover, the construction of a deviating offer $\tilde{c}_\delta(\varepsilon_2)$ from c_δ for all small $\varepsilon_2 > 0$ is again identical. Finally, we can again choose ε_1 and ε_2 sufficiently small and

$\delta > max \left\{ \hat{\delta}_3, \hat{\delta}_4(\varepsilon_1) \right\}$ to complete the argument.

Case ii) (Subsequence $\tilde{C}_\delta^F$)

We again take the original sequence as the subsequence such that $\sum_{c \in \tilde{C}_\delta^W} \alpha_\delta^F(1,c) \geq \overline{\overline{\varepsilon}}/2$. Assertion 3 of Claim 4 carries over with the modification that we have to substitute $\tilde{\varepsilon}$ by the newly derived threshold $\overline{\overline{\varepsilon}}$, which has, however, no qualitative impact on the arguments.

As the set of types is finite, Claim 5 follows from a finite repetition of the argument. **Q.E.D.**

Recall now that $\sum_{c \in B(i)} (\alpha^W(i,c) + \alpha^F(i,c)) > \alpha^B$ holds for all $i \in I$ in all considered equilibria if $\delta > \bar{\delta}_1$. The following result follows then from Claims 4-5 and the definition of reservation values in (2).

Claim 6. *For any $\varepsilon > 0$ there exists $\bar{\delta}_5 < 1$ such that for all $\delta > \bar{\delta}_5$, $\psi \in \Psi_\delta(M)$, and corresponding reservation values $V^R(i)$, it holds that $V^{RS}(i) - \varepsilon > V^R(i)$.*

Proposition 1 follows now from Claims 3 and 6 by choosing $\bar{\delta} = max \left\{ \bar{\delta}_1, \bar{\delta}_2, \bar{\delta}_5 \right\}$. **Q.E.D.**

Appendix B: Proof of Proposition 2

The proof proceeds in a series of steps. Claims 1-2 provide auxiliary results.

Claim 1. *For any $\tilde{\varepsilon} > 0$ there exists $\overline{\varepsilon}_1(\tilde{\varepsilon}) > 0$ such that any contract $c \in C$, which satisfies for some $i \in I$*

$$\begin{aligned} V(i,c) &\geq V^{RS}(i) - \overline{\varepsilon}_1(\tilde{\varepsilon}), \\ V(i-1,c) &\leq V^{RS}(i-1) + \overline{\varepsilon}_1(\tilde{\varepsilon}) \text{ if } i > 1, \\ U(i,c) &\geq U^0 - \overline{\varepsilon}_1(\tilde{\varepsilon}), \end{aligned}$$

must also satisfy $c \in \Omega(c^{RS}(i), \tilde{\varepsilon})$.

Proof. We argue to a contradiction. If the assertion does not hold, there exists a sequence of contracts $\{c_n\}$ and types $\{i_n\}$ with $c_n \notin \Omega(c^{RS}(i_n), \tilde{\varepsilon})$ and a sequence $\{\overline{\varepsilon}_n\}$ with $\overline{\varepsilon}_n \to 0$ such that

$$\begin{aligned} V(i_n,c_n) &\geq V^{RS}(i_n) - \overline{\varepsilon}_n, \\ V(i_n-1,c_n) &\leq V^{RS}(i_n-1) + \overline{\varepsilon}_n \text{ if } i_n > 1, \\ U(i_n,c_n) &\geq U^0 - \overline{\varepsilon}_n. \end{aligned} \tag{16}$$

By the finiteness of I we can select a subsequence where all types i_n are identical. Assume that this is satisfied for the original sequence and a type $i = i_n$. Moreover, for all sufficiently small values of $\overline{\varepsilon}_n$, (16) implies from (A.1) that all c_n belong to some compact set denoted by $\bar{C}$. Hence, we can select a subsequence where contracts converge to some $\bar{c} \in \bar{C} \backslash \Omega(c^{RS}(i), \tilde{\varepsilon})$. As (16) is satisfied along this sequence, this contradicts the construction of $c^{RS}(i)$. **Q.E.D.**

The following result is now implied by the definition of $c^{RS}(i)$ and the continuity of payoff functions.

Claim 2. *For any $\tilde{\varepsilon} > 0$ we can find $\overline{\varepsilon}_2(\tilde{\varepsilon}) > 0$ such that the following implications hold for any $c \in C$:*
i) For $i > 1$, $U(i,c) > U^0 + \tilde{\varepsilon}$ implies $V(i,c) < V^{RS}(i) - \overline{\varepsilon}_2(\tilde{\varepsilon})$ or $V(i-1,c) > V^{RS}(i) + \overline{\varepsilon}_2(\tilde{\varepsilon})$.
ii) For $i = 1$, $U(1,c) > U^0 + \tilde{\varepsilon}$ implies $V(1,c) < V^{RS}(1) - \overline{\varepsilon}_2(\tilde{\varepsilon})$.

We now proceed stepwise to reduce the set of equilibrium allocations.

Claim 3. *For any* $\tilde{\varepsilon} > 0$ *there exists* $\bar{\delta}_1(\tilde{\varepsilon}) < 1$ *such that for all* $\delta > \bar{\delta}_1(\tilde{\varepsilon})$*,* $i > 1$*, and* $\psi \in \Psi_\delta(M)$ *it holds that* $\sum_{c\in C(i)}(\alpha^W(i,c) + \alpha^F(i,c)) \leq \tilde{\varepsilon}$*, where* $C(i) = \{c \mid V(i-1,c) > V^{RS}(i-1) + \tilde{\varepsilon}\}$*. Moreover, for all* $i \in I$*,* $V(i,c) < V^{RS}(i) - \tilde{\varepsilon}$ *implies* $\alpha^W(i,c) + \alpha^F(i,c) = 0$*.*

Proof. The assertion follows directly from Proposition 1 and its proof. (The argument regarding the adjacent type $i-1$ is identical to that used in Assertion 2 of Claim 5). **Q.E.D.**

Claim 4. *For any* $\tilde{\varepsilon} > 0$ *there exists* $\bar{\delta}_2(\tilde{\varepsilon}) < 1$ *such that for all* $\delta > \bar{\delta}_2(\tilde{\varepsilon})$*,* $i \in I$*, and* $\psi \in \Psi_\delta(M)$*, it holds that* $\sum_{c\in C(i)}(\alpha^W(i,c) + \alpha^F(i,c)) \leq \tilde{\varepsilon}$*, where* $C(i) = \{c \mid U(i,c) < U^0 - \tilde{\varepsilon}\}$*.*

Proof. We argue to a contradiction. By the finiteness of I we can then assume that there exists a type i, a sequence $\psi_\delta \in \Psi_\delta(M)$, where $\delta \to 1$, and a sequence of sets $C_\delta \subseteq B_\delta(i)$ satisfying $\sum_{c\in C_\delta}(\alpha^W_\delta(i,c) + \alpha^F_\delta(i,c)) \geq \tilde{\varepsilon}$ and $U(i,c) < U^0 - \tilde{\varepsilon}$ for all $c \in C_\delta$. Recall next that the expected payoff of firms equals U^0. By $\mu_\delta(i) > \mu^B$, which was proved in Proposition 1 for sufficiently high δ, it follows from inspection of (2) that there exist a sequence of types i_δ and of sets of contracts C'_δ, and a threshold $\bar{\varepsilon}_3 > 0$, such that $U(i_\delta, c) > U^0 + \bar{\varepsilon}_3$ for all $c \in C'_\delta$, while $\sum_{c\in C'_\delta}(\alpha^W_\delta(i_\delta,c) + \alpha^F_\delta(i_\delta,c)) > \bar{\varepsilon}_3$.

We use now Claims 2-3 to show that this can not be the case for high values of δ. By Claim 2 we know that $U(i_\delta, c) > U^0 + \bar{\varepsilon}_1$ implies either $V(i_\delta,c) < V^{RS}(i) - \bar{\varepsilon}_2(\bar{\varepsilon}_3)$ or $V(i_\delta - 1, c) > V^{RS}(i_\delta - 1) + \bar{\varepsilon}_2(\bar{\varepsilon}_3)$. (Of course, for $i_\delta = 1$ only the first possibility is relevant.) Hence, we can derive a sequence of sets C''_δ with $\sum_{c\in C''_\delta}(\alpha^W_\delta(i_\delta,c) + \alpha^F_\delta(i_\delta,c)) > \bar{\varepsilon}_3/2$ where either $V(i_\delta,c) < V^{RS}(i) - \bar{\varepsilon}_2(\bar{\varepsilon}_3)$ or $V(i_\delta - 1, c) > V^{RS}(i_\delta - 1) + \bar{\varepsilon}_2(\bar{\varepsilon}_3)$. Both possibilities must, however, contradict Claim 3. Precisely, this is the case if we choose $\delta > \bar{\delta}_1(\bar{\varepsilon}_4)$ with $\bar{\varepsilon}_4 = min\{\bar{\varepsilon}_2(\bar{\varepsilon}_3), \bar{\varepsilon}_3/2\}$. **Q.E.D.**

The following assertion combines Claim 1 with Claims 3-4.

Claim 5. *For any* $\tilde{\varepsilon} > 0$ *there exists* $\bar{\delta}_3(\tilde{\varepsilon}) < 1$ *such that for all* $\delta > \bar{\delta}_3(\tilde{\varepsilon})$*,* $i \in I$*, and* $\psi \in \Psi_\delta(M)$*, it holds that* $\sum_{c\in C\setminus\Omega(c^{RS}(i),\tilde{\varepsilon})}(\alpha^W(i,c) + \alpha^F(i,c)) < \tilde{\varepsilon}$*.*

Proof. Given some ε_1, it holds by Claims 3-4 for all $\delta > max\{\bar{\delta}_1(\varepsilon_1/2), \bar{\delta}_2(\varepsilon_1/2)\}$, $i \in I$, and $\psi \in \Psi_\delta(M)$ that $\sum_{c\in C(i)}(\alpha^W(i,c) + \alpha^F(i,c)) < \varepsilon_1$, where $C(i)$ comprises all contracts $c \in C$ satisfying any of the following conditions:

- For $i > 1$, $V(i-1,c) > V^{RS}(i-1) + \varepsilon_1$.
- $V(i,c) < V^{RS}(i) - \varepsilon_1$.
- $U(i,c) < U^0 - \varepsilon_1$.

Given $\tilde{\varepsilon}$ we can derive next from Claim 1 the boundary $\bar{\varepsilon}_1(\tilde{\varepsilon}) > 0$. The claim follows then by choosing $\varepsilon_1/2 = \bar{\varepsilon}_1(\tilde{\varepsilon})$ and defining $\bar{\delta}_3(\tilde{\varepsilon}) = max\{\bar{\delta}_1(\varepsilon_1/2), \bar{\delta}_2(\varepsilon_1/2)\}$. **Q.E.D.**

While we have so far restricted attention to allocations, observe that the assertion in the Proposition makes a claim on the resulting *distribution* of contracts, as defined in (1). Recall now from the proof of Proposition 1 that in any considered equilibrium it holds for high δ and for all $i \in I$ that $\alpha^W(i,c) + \alpha^F(i,c) > \alpha^B > 0$. The claim in the proposition follows then directly from the definition (1) and Claim 5. **Q.E.D.**

Appendix C: Proof of Proposition 3

For high δ we will prove existence of an equilibrium where agents adopt the following strategies:

- All workers enter, i.e. $E^W(i) = W^0(i)$, while $E^F = W^0$.

- In Γ^F firms choose between the following two strategies. They may either randomize over a set of menus of which each ensures that the match is successful with both types. Or they offer a contract which is only accepted by the high type. The latter strategy is chosen with probability $1 - \rho^F$.

- In Γ^W the high type offers $c^{RS}(2)$ with probability one, which is accepted. The low type offers $c^{RS}(1)$ only with probability ρ^W, while with probability $1 - \rho^W$ the match is dissolved unsuccessfully.

Given $E^W(i) = W^0(i)$ and the strategies in the contractual games, we obtain for the distribution in the market

$$\mu(1) = \frac{\mu^0(1)}{\mu^0(1) + \mu^0(2)\left[(1-b)\rho^W + b\rho^F\right]}. \tag{17}$$

Moreover, observe that stocks in the markets are fully specified if we determine additionally m.

We proceed now in three steps. First, we set up a fixed-point problem in the three variables (ρ^F, ρ^W, m) and show that this has a solution for any δ. The specified solution will also determine the contracts offered in Γ^F such that the equilibrium candidate is fully specified. As the conditions imposed for the fixed-point problem do not already imply that strategies are optimal, we show in a second step that this is indeed the case for high values of δ. Finally, we argue that the selected sequence of equilibria satisfies the asserted requirements on f and W.

To set up the fixed-point problem, we define first some programs and derive auxiliary results.

Programs and auxiliary results

Define the following program $P(V^R(1), V^R(2), \mu)$: For a given distribution μ, choose a pair $(c(1), c(2)) \in C^2$ to maximize

$$\mu(1)(v(1, y(1)) + t(1)) + \mu(2)(v(2, y(2)) + t(2))$$

subject to the following constraints: $IC(1)$ with $V(1, c(1)) \geq V(1, c(2))$, $IR(1)$ with $V(1, c(1)) \geq V^R(1)$, and $IR(2)$ with $V(2, c(2)) \geq V^R(2)$. (Observe that we do not consider incentive compatibility for the high type. Moreover, the menu must specify an acceptable contract for either type.)[42] By (A.1)-(A.2) a solution always exists, while by optimality $IR(2)$ becomes binding. We introduce the following notation. The realized value is denoted by $U^P(\cdot)$. The program may have more than one solution which may also implement different utilities for the low type. Denote by $\mathbf{V}^P(\cdot)$ the convex set of *lotteries* over the low type's utilities.

Claim 1. *$\mathbf{V}^P(\cdot)$ is USC, while $U^P(\cdot)$ is continuous (all in the parameters $(V^R(1), V^R(2), \mu)$). Moreover, $\inf \mathbf{V}^P(\cdot)$ and $\sup \mathbf{V}^P(\cdot)$ are constant in $V^R(1)$ or increasing with slope one.*[43]

Proof. Denote the set of solutions by $\mathbf{C}^P(\cdot)$. By optimality, it must hold that $t(2) = V^R(2) - v(2, y(2))$ and $y(1) = y^*(1)$. Moreover, as one of the constraints for $i = 1$ becomes binding, we obtain $t(1) = \min\left\{V^R(1), V(1, c(2))\right\} - v(1, y^*(1))$. The residual program is continuous in $y(2)$ and $(V^R(1), V^R(2), \mu)$, which together with (A.3) proves continuity of $U^P(\cdot)$ and USC of $\mathbf{C}^P(\cdot)$ by the maximum theorem. USC of $\mathbf{V}^P(\cdot)$ follows from the continuity of

[42] In what follows, it will be ensured that $\mu(1) > 0$, while the program naturally extends to the case where $\mu(2) = 0$.

[43] For concreteness, define the slope as the right-side derivative.

payoff functions.[44] The assertions regarding $\inf \mathbf{V}^P(\cdot)$ and $\sup \mathbf{V}^P(\cdot)$ are immediate from the construction of the program. **Q.E.D.**

Define next the program $P^-(V^R(1), V^R(2), \mu)$, where $c(2) \in C$ is chosen to maximize

$$\mu(1)U^0 + \mu(2)(v(2, y(2)) + t(2))$$

subject to $IC(1)$ and $IR(2)$. A solution exists by (A.1)-(A.2), where by optimality $V(2, c(2)) = V^R(2)$. Denote the realized utility by $U^-(\cdot)$.

Claim 2. *$U^-(\cdot)$ is continuous.*

Proof. The proof is analogous to that of continuity of $U^P(\cdot)$. **Q.E.D.**

We define next an auxiliary fixed-point problem which is used below. Given (ρ^F, ρ^W), m , and $V^{RS}(2)$, denote by $T(V^R(2), \rho^F, \rho^W, m)$ the set of all values $V^R(1)$ which satisfy the following conditions:

$$V^P(1) \in \mathbf{V}^P(V^R(1), V^R(2), \mu) \tag{18}$$

$$V^R(1) = max \left\{ \frac{\delta(1-m)\left[b\rho^F V^P(1) + (1-b)V^{RS}(1)\right]}{1 - \delta[1 - (1-m)(b\rho^F + 1 - b)]}, \frac{\delta(1-m)b\rho^F V^P(1)}{1 - \delta\left[1 - (1-m)b\rho^F\right]} \right\} \tag{19}$$

In words, given the choice of $V^R(1)$, the expected value $V^P(1)$ is realized if firms choose to make an acceptable offer to both types, while given $V^P(1)$, the value $V^R(1)$ represents the low type's reservation value if he optimally chooses between making an acceptable offer of $c^{RS}(1)$ in Γ^W or not. Observe in particular that for $\rho^F = 0$ the actual realization of $V^P(1)$ does not enter into (19).

Claim 3. *$T(\cdot)$ is non-empty, convex, and USC.*

Proof. Observe that (19) defines $V^R(1)$ as a nondecreasing and continuous function of $V^P(1)$, which has a slope strictly smaller than one. Given the results of Claim 2, existence of a fixed-point and convexity of $T(\cdot)$ follow from a simple graphical argument. Finally, continuity of the expression in (19) and USC of $\mathbf{V}^P(\cdot)$ establish USC of $T(\cdot)$. **Q.E.D.**

Fixed-point argument

Recall the definitions of f and g as functions of δ and m. Fix now a triple (ρ^F, ρ^W, m), where each element is restricted to $[0, 1]$, and calculate the respective values of μ, g, and f. Define next $V^R(2) = gV^{RS}(2)$. We define now a mapping $\varphi : [0, 1]^3 \to [0, 1]^3$ in a series of steps.

- Take some $V^R(1) \in T(V^R(2), \rho^F, \rho^W, m)$. Note next that the choice of $V^R(1)$ also defines uniquely a value $U^P(V^R(1), V^R(2), \mu)$ and a value $U^-(V^R(1), V^R(2), \mu)$.

- Keeping (ρ^F, ρ^W, m) fixed, define for any choice of $V^R(1)$ the set $\hat{\varphi}(\rho^F, \rho^W, m, V^R(1))$ of all triples $(\hat{\rho}^F, \hat{\rho}^W, \hat{m})$, which simultaneously satisfy the following three conditions:

1. $\hat{m}$ is uniquely defined by

$$\begin{aligned} f(\hat{m})max\left\{U^-(\cdot), U^P(\cdot)\right\} &= U^0 \text{ if } f(1)max\left\{U^-(\cdot), U^P(\cdot)\right\} \geq U^0, \\ \hat{m} &= 1 \text{ otherwise.} \end{aligned} \tag{20}$$

[44] Observe that $\mathbf{C}^P(\cdot)$ may not be convex. For this reason we allow firms to randomize so as to realize any (expected) utility in the convex set $\mathbf{V}^P(\cdot)$. Alternatively, it can be checked that (A.1)-(A.2) together with $d^2v(j, y)/dy^2 \leq d^2v(i, y)/dy^2$ for $j > i$ convexifies the set of solutions.

2. $\hat{\rho}^F$ satisfies

$$\hat{\rho}^F = \begin{cases} 0 & \text{if } U^-(\cdot) > U^P(\cdot) \\ 1 & U^-(\cdot) < U^P(\cdot) \\ \in [0,1] & U^-(\cdot) = U^P(\cdot) \end{cases} . \tag{21}$$

3. $\hat{\rho}^W$ satisfies

$$\hat{\rho}^W = \begin{cases} 0 & \text{if } V^{RS}(1) < V^R(1) \\ 1 & V^{RS}(1) > V^R(1) \\ \in [0,1] & V^{RS}(1) = V^R(1) \end{cases} \tag{22}$$

Observe first that by uniqueness of $\hat{m}$ and by (21)-(22) the set $\hat{\varphi}(\cdot)$ is convex. Define next

$$\varphi(\rho^F, \rho^W, m) = \bigcup_{V^R(1) \in T(V^R(2), \rho^F, \rho^W, m)} \hat{\varphi}(\rho^F, \rho^W, m, V^R(1)).$$

By convexity of $T(\cdot)$ and continuity of $U^P(\cdot)$ and $U^-(\cdot)$, $\varphi(\cdot)$ is convex. We show next that it is also USC in (ρ^F, ρ^W, m). This follows again from continuity of $U^P(\cdot)$ and $U^-(\cdot)$ and from USC of $T(\cdot)$.

By applying Kakutani's theorem, the equation $(\rho^F, \rho^W, m) \in \varphi(\rho^F, \rho^W, m)$ has thus a fixed-point. We choose one for given δ and denote the respective values by ρ^F_δ, ρ^W_δ, and m_δ. Observe that with this choice we have also determined the following variables: f_δ, g_δ, μ_δ, $V^P_\delta(1)$, U^P_δ, U^-_δ.

Properties for high δ

The fixed-point result so far neither implies existence of an equilibrium, nor that the asserted characteristics are satisfied. This will follow for high δ from a series of claims.

Claim 4. *It holds that* $f_\delta \to 1$.

Proof. We argue by contradiction, which implies existence of a subsequence where $f_{\delta_n} \to \bar{f} < 1$. Take the original sequence as the subsequence and observe that this implies existence of some $\hat{\delta}_0 < 1$ such that $m_\delta < 1$ for all $\delta > \hat{\delta}_0$, while also $g_\delta \to 1$. By construction this implies next that $V^R_\delta(2) \to V^{RS}(2)$, while from (19) a lower boundary for $V^R_\delta(1)$ converges to $V^{RS}(1)$. To obtain a contradiction, we derive next a series of implications following from the assumption that $f_\delta \to \bar{f} < 1$.

We claim first that there exist $\bar{\varepsilon}_1 > 0, \hat{\delta}_1 < 1$ such that for $\delta > \hat{\delta}_1$ it holds that $\mu_\delta(2) > \bar{\varepsilon}_1$, while also $\rho^F_\delta > 0$. To see that this holds, recall first that (20) is solved by some $m_\delta < 1$ for all $\delta > \hat{\delta}_0$. By $U^0 > 0$ and $\bar{f} < 1$, firms must realize strictly more than U^0 in Γ^F, which by construction of $c^{RS}(1)$ and the property of $V^R_\delta(1)$ can only be the case with the high type, implying indeed that $\mu_\delta(2)$ must remain bounded away from zero. In case the claim for ρ^F_δ does not hold, we can select a subsequence of high values δ_n where $\rho^F_{\delta_n} = 0$, implying from (19) that $V^R_{\delta_n}(1) \to V^{RS}(1)$. By construction of the RS family of contracts this ensures that $U^-_\delta \to U^0$ such that (20) cannot be satisfied. Hence, we have shown that $\mu_\delta(2) > \bar{\varepsilon}_1$, while also $\rho^F_\delta > 0$ holds for $\delta > \hat{\delta}_1$.

We claim next that there exist $\bar{\varepsilon}_2 > 0, \hat{\delta}_2 < 1$ such that for $\delta > \hat{\delta}_2$ it holds that $V^P_\delta(1) > V^{RS}(1) + \bar{\varepsilon}_2$. To see this, recall first that for $\delta > \hat{\delta}_1$ it holds from $\rho^F_\delta > 0$ that $f_\delta U^P_\delta = U^0$. By $f_\delta \to \bar{f} < 1$, the properties of $V^R_\delta(i)$, and construction of $c^{RS}(1)$, this can only be the case if $y^{RS}(2) > y^*(2)$ and contracts for $i = 2$ in Γ^F are sufficiently less distorted. The assertion $V^P_\delta(1) > V^{RS}(1) + \bar{\varepsilon}_2$ follows then immediately from (A.2) and incentive compatibility for $i = 1$.

We claim next that there exist $\bar{\varepsilon}_3 > 0, \hat{\delta}_3 < 1$ such that for $\delta > \hat{\delta}_3$ it holds that $\rho^F_\delta > \bar{\varepsilon}_3$. By $\mu_\delta(2) > \bar{\varepsilon}_1$ for $\delta > \hat{\delta}_1$ this is indeed the case if we can show that $\rho^W_\delta = 0$ holds for sufficiently high values of δ, which would follow from (22) in case $V_\delta^R(1) > V^{RS}(1)$. To prove the last assertion we argue to a contradiction and assume that this does not hold for high δ along some subsequence. By $V_{\delta_n}^P(1) > V^{RS}(1) + \bar{\varepsilon}_2$ for $\delta_n > \hat{\delta}_2$ and $g_{\delta_n} \to 1$, this implies $\rho_{\delta_n}^F \to 0$, such that for high δ it follows from (21) that $f_{\delta_n} U_{\delta_n}^- = U^0$. By $V_{\delta_n}^R(1) \leq V^{RS}(1)$, $f_{\delta_n} \to \bar{f} < 1$, and $U^0 > 0$ this leads again to a contradiction given the construction of $c^{RS}(2)$. Hence, we have shown that $V_\delta^R(1) > V^{RS}(1)$ must hold for high δ, implying $\rho_\delta^F > \bar{\varepsilon}_3$.

We are now in a position to complete the proof of Claim 4. Given $\rho_\delta^F > \bar{\varepsilon}_3$ and $V_\delta^P(1) > V^{RS}(1) + \bar{\varepsilon}_2$ for $\delta > \left\{\hat{\delta}_2, \hat{\delta}_3\right\}$, we know from $g_\delta \to 1$ that $V_\delta^R(1) - V_\delta^P(1) \to 0$. We can next apply an argument as used repeatedly in the proof of Proposition 1. By construction of $c^{RS}(1)$ we can conclude from $V_\delta^P(1) > V^{RS}(1) + \bar{\varepsilon}_2$ that the firm realizes a strictly negative utility with $i = 1$. As $\mu_\delta(1) > \mu^0(1)$ holds by construction and as $V_\delta^R(1) - V_\delta^P(1) \to 0$, it follows from (A.2) that firms are strictly better off for high δ by restricting an offer to $i = 2$. Formally, we can find some $\hat{\delta}_4 < 1$ such that for $\delta > \hat{\delta}_4$ it holds that $U_\delta^- > U_\delta^P$.[45] For $\delta > max\left\{\hat{\delta}_0, \hat{\delta}_1, \hat{\delta}_2, \hat{\delta}_3, \hat{\delta}_4\right\}$ we have thus arrived at a contradiction as $U_\delta^- > U_\delta^P$ and $\rho_\delta^F > 0$ contradict (22). By contradiction we have therefore established that $f_\delta \to 1$. **Q.E.D.**

Claim 5. *There exist* $\varepsilon > 0$, $\bar{\delta}_1 < 1$ *such that for* $\delta > \bar{\delta}_1$ *it holds that* $\mu_\delta(1) < 1 - \varepsilon$.

Proof. We argue to a contradiction and assume existence of a subsequence where $\mu_{\delta_n}(1) \to 1$. Take the original sequence as the subsequence. From (17) this implies $\rho_\delta^W < 1$ for high δ, which by (22) is only the case if $V_\delta^R(1) \geq V^{RS}(1)$. For this to hold, however, it must be the case that $\rho_\delta^F > 0$ and that $V_\delta^P(1) > V_\delta^R(1)$, i.e. that $IR(1)$ is not binding in a solution to $P(\cdot)$ for high δ. By optimality this implies that $IC(1)$ binds and that the respective value of the sorting variable specified for the high type, which we denote by $y_\delta(2)$, is chosen to maximize $\mu_\delta(2)s(2, y_\delta(2)) + \mu_\delta(1)\left[v(2, y_\delta(2)) - v(1, y_\delta(2))\right]$. By (A.2) this implies $y_\delta(2) \to \infty$ as $\mu_\delta(1) \to 1$, which yields for high δ a contradiction to $V_\delta^P(1) > V_\delta^R(1)$. This proves the assertion. **Q.E.D.**

Claim 6. *There exist* $m^B < 1$, $\bar{\delta}_2 < 1$ *such that for* $\delta > \bar{\delta}_2$ *it holds that* $m_\delta < m^B$.

Proof. We argue to a contradiction and assume existence of a subsequence where $m_{\delta_n} \to 1$. Take the original sequence as the subsequence. Consider the strategy for firms to offer in Γ^F only a contract to $i = 2$. By specifying $y = y^{RS}(2)$ and $t = V_\delta^R(2) - V(2, y^{RS}(2))$, incentive compatibility for $i = 1$ is ensured due to $V_\delta^R(1) \geq g_\delta V^{RS}(1)$ and $V_\delta^R(2) = g_\delta V^{RS}(2)$. As firms must not receive more than U^0 under this strategy due the requirement (20), it must hold that $U^0 \geq f_\delta[\mu_\delta(1)U^0 + \mu_\delta(2)(s(2, y^{RS}(2)) - V_\delta^R(2))$, which by $s(2, y^{RS}(2)) = U^0 + V^{RS}(2)$ transforms to the requirement

$$\frac{U^0}{\mu_\delta(2)V^{RS}(2)} \geq \frac{1 - g_\delta}{1 - f_\delta} f_\delta. \tag{23}$$

Observe next that by $m_\delta \to 1$ the expression

$$\frac{1 - g_\delta}{1 - f_\delta} = \frac{1 - \delta(1 - bm_\delta)}{1 - \delta(1 - (1 - b)(1 - m_\delta))}$$

grows beyond any boundary as $\delta \to 1$. As $\mu_\delta(2)$ remains bounded away from zero by Claim 5 and as $f_\delta \to 1$ by Claim 4, this contradicts (23). Hence, m_δ must indeed be bounded away from one for sufficiently high values of δ. **Q.E.D.**

We are finally in a position to complete the existence proof. By Claim 6 it follows that $g_\delta \to 1$, implying from

[45] Recall that $V_\delta^P(1)$ may be the expected outcome if firms randomize. In this case the argument holds for all offers realizing for low types not less than the expected value $V_\delta^P(1)$.

$V_\delta^R(i) \geq g_\delta V^{RS}(i)$ for $i \in I$ that entry is indeed optimal for $\delta > \bar{\delta}_3$ and some $\bar{\delta}_3 < 1$. Observe next that by $m_\delta < 1$ for $\delta > \bar{\delta}_2$ it holds by (20) that $f_\delta max\left\{U^-(\cdot), U^P(\cdot)\right\} = U^0$. Recall also that we have ensured by construction that firms make an optimal choice in Γ^F, while the same holds for the decision of low types to break up a match in Γ^W. By specifying pessimistic beliefs we ensure that offers in Γ^W are optimal.

It thus remains to show that the high type's incentive compatibility constraint, which was neglected in $P(\cdot)$, holds for high δ. To see this, note first that $V_\delta^R(i) \to V^{RS}(i)$, which indeed ensures that $V_\delta^R(2) > V_\delta^R(1) + v(2, y^*(1)) - v(1, y^*(1))$ holds for $\delta > \bar{\delta}_3$ and some $\bar{\delta}_4 < 1$. Existence of an equilibrium is thus ensured for all $\delta > \bar{\delta} = max\left\{\bar{\delta}_1, \bar{\delta}_2, \bar{\delta}_3, \bar{\delta}_4\right\}$.

By Claim 4 the constructed sequence also satisfies $f_\delta \to 1$. It thus remains to show that the measure of workers remains bounded. By (17) we obtain that $(1-b)\rho_\delta^W + b\rho_\delta^F$ is equal to $[\mu^0(1)(1-\mu_\delta(1))]/[\mu_\delta(1)\mu^0(2)]$, which by $\mu_\delta(1) < 1 - \varepsilon$ due to Claim 5 remains for $\delta > \bar{\delta}$ strictly bounded from below by some value $\bar{\rho} > 0$. As also $m_\delta < m^B$ for $\delta > \bar{\delta}$ due to Claim 6, stocks are bounded from above by $W_\delta(2) \leq \bar{W}(2) = E^W(2)/(1-m^B)$ and $W_\delta(1) \leq \bar{W}(1) = E^W(1)/[(1-m^B)\bar{\rho}]$. Choosing $M = \bar{W}(1) + \bar{W}(2)$ completes the proof. **Q.E.D.**

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