AUDITING WITH SIGNALS*

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Abstract

This paper is a first step in the analysis of the use of signals of taxpayer's incomes by tax audit authorities. In a very simple model, we consider the design of the audit strategy when the tax authority can commit to it and has free access to a signal correlated with the taxpayer's true income. We discuss the optimal enforcement policy and compare it with the optimal one when only self-reported income is considered. Our main result is that the well-known regressive bias of revenue-maximizing audit rules may be convert in a progressive one when signals are used.

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1.- INTRODUCTION

Reducing the income tax gap - the difference between actual income taxes owed and those voluntary paid - and maintaining high levels of compliance are among the biggest challenges for tax authorities.[1] In order to reduce noncompliance, tax authorities design audit strategies. Taxpayers' returns that, when audited, are discovered to be fraudulent suffer a penalty in addition to the taxes owed. This mechanism increases the compliance both directly (the collection effect) and indirectly (through the dissuasion effect). Obviously, the most important aspect of the design of audit strategies is how to select taxpayers. Audit strategies can be based in random selection, but in general some data are used to direct this selection in order to screen for returns that have a higher potential audit revenue.

The theoretical literature has hitherto devoted attention to the self-reported income as the main element for the audit selection.[2] However, in practice, tax authorities have access to (potentially) hundreds of different information sources pertaining to the transactions and consumption of taxpayers. Accordingly, the challenge in designing a useful audit strategy is to know how to use this information in a useful and manageable way. This paper is a first step in this direction.

We consider a simple model in which the tax administration has access to statistical information, that we refer to as a *signal*, correlated with the taxpayer's true income. We analyze the use of this signal as a deterrent device. As expected, informative signals will always be used. We show how the signal is used and show that audit signals may convert the regressive bias, that is a well-known characteristic of revenue-maximizing audit rules, into a progressive bias.

In our knowledge, only Scotchmer (1987) has previously dealt with this issue. In her model she considers the use of taxpayer characteristics *known to the taxpayers* (such

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[1] The IRS estimates that taxpayers fail to voluntary pay in excess of $100 billion annually in taxes due to income from legal sources. IRS' data for 1992 suggest that taxpayers voluntary pay 83 percent of the income taxes owed - 87 percent after enforcement (General Account Office, 1995a).

[2] See Cowell (1990) for a general analysis of the economics of tax evasion, and Andreoni, Erard, and Feinstein (1998) for a recent review of the literature on tax compliance.

as age, sex or occupation).[3] She proves that the use of such signals, that allows the tax authority to divide taxpayers in audit classes, tend to promote vertical equity at the cost of reducing horizontal equity.

Here we take the view that, within an audit class, the tax audit authority has access to lots of information other than the self-reported income that can be valuable to select individuals for audit. We resume this information in a single random variable correlated to the true income and assume that the realization of this random variable *is not known to the taxpayer*. To motivate this modelization choice, let us note that in practice, tax administrations tend to adopt a statistical approach to the problem of how to use the information at hand, as manifested in the DIF score method used in the USA.[4] Other countries follow a similar practice. One reason for summarizing the information (in our case in one single signal) is related to cost containment and manageability arguments. Another advantage of using aggregate information instead of the information from each source is related to the fact that as taxpayers become aware of the criteria used by the tax administration, they tend to modify their behavior strategically. The use of an aggregated signal can be understood as a try to handle this behavior by hiding the exact equation that generates the statistical score. In other words, using the information about different items in an aggregate way avoids inducing distortions in taxpayers' behavior among these

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[3] The compliance rate seems to be very different across occupations, for example. The Taxpayer Compliance Measurement Program (US-IRS, 1983) estimates that in 1981 tax compliance was 93.9% for wages and salaries, 86.3% for interest, 83.7% for dividends, 61.2% for royalties, 59.4% for capital gains, 50.3% for non-farm proprietors, 47.0% for small businesses, 37.2% for rents, 20.7% for informal suppliers, and -18% for farmers. This is the reason why certain professions are more targeted for audit.

[4] The IRS audit strategy establishes about 40 audit sources, which are programs and techniques used to select potentially non-compliant returns. Audit sources include, among others, suspected tax shelters and IRS and non-IRS referrals. The discriminant index function (DIF) is one of the majors tools to audit. It is a computer generated score designed to predict returns most likely to result in additional taxes owed if audited (General Account Office, 1998). Two-thirds of audited tax returns are picked by IRS computers using the DIF. Other audits are selected through referrals from state and local governments, from criminal cases and by special enforcement programs (Anderson, 1999).

items.[5] The only information that taxpayers have is that their true income is correlated to the realization of the signal.

There are some empirical analyses that support the existence and the use of signals in tax auditing. Alm, Bahl, and Murray (1993) estimate a model of audit selection and income tax evasion in Jamaica. Their results strongly support the systematic nature of the tax authority behavior, "as illustrated by the variety of reported items that influence the probability of an audit". More recently, Erard and Feinstein (1994), after dividing the taxpayers' population by income sources, allow for the possibility that the tax authority, before the audit decision, observes a signal of the taxpayer's evasion that is not recorded in the data. Their results indicate that the IRS "does appear to possess information that is correlated with evasion behavior and not derived from tax returns."

To be more precise, we consider a tax authority allocating resources in an attempt to identify the taxpayers who underreport and to increase voluntary compliance. The audit strategy takes into account both the income tax return and a signal related to the taxpayer's true income level. We assume that the tax authority can commit to any pre-specified audit policy (the principal-agent approach), and look for the optimal audit policy (for an audit class). For simplicity, we assume that signals are free. Moreover, we develop the analysis for a situation in which three income levels are possible. We obtain the optimal audit schedule for each possible budget size, the informational content of the pre-audit information, and the parameters of the distribution of incomes.

For a given report, the optimal auditing policy involves directing the auditing first to those taxpayers whose signal is more indicative of high income. Clearly, auditing "high-signal taxpayers" is optimal ex-post, i.e., once the reports have been made. The interest of our result is that we prove that it is also optimal to announce this property ex-ante.

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[5] For example, imagine that the fact that taxpayers report a variety of lines items is taken into account. Martinez-Vazquez and Rider (1995) consider the effect of different exogenous audit policies contingent to the different line items report, and estimate a model of tax evasion in which taxpayers choose how much to report in each line item. They find that the evasion in one mode depends on the possibilities for evasion on the other modes, in a similar way as a consumer decides on the diversification of its portfolio. However, in the taxpayers choice problem, the risk associated to the different items depend on the audit policy. Hence, to be very specific on the way the information will be used for audit will induce an important distortion in the taxpayer behavior.

The optimal policy in our framework shares two characteristics with the policy that is optimal when no signal is available. First, auditing pressure over any taxpayer is decreasing with the reported income. Second, the income reported is not decreasing with true income. That is, higher income levels cannot lead to lower reports.

On the other hand, our results contrast with the previous literature in two aspects. First, the actual auditing probabilities can be increasing with the income. Although the probability of auditing (high signals) does not increase with the reported income, as in the previous models, the auditing pressure (for a given report) is increasing in the income. When the second effect is stronger than the first one, the tax administration audits more often taxpayers with higher income.

Second and more important, the conclusion that revenue-maximizing income rules, within an income class, lead to more regressive effective taxes than the nominal taxes can be converted.[6] Our analysis shows that when the tax authority directs the audit using signals about taxpayers' true income, it may be the case that effective taxes are more progressive than nominal ones. The use of signals is worthy not only because it makes the audits more effective, but also because it can reduce the regressive bias associated with the possibility of fraud.

The paper is organized as follows. In Section 2, we present the model. Then, in Section 3, we analyze the taxpayer behavior for any audit schedule. In Section 4, we obtain some characteristics of the optimal policy. In Section 5, we study the optimal audit policy for the tax authority as a function of the budget, the informational content of the signal, and some characteristics of the income distribution. The relation between auditing and progressivity of the effective taxes is discussed in Section 6. Section 7 concludes. The proofs are relegated to the Appendix.

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[6] See, e.g., Chandler and Wilde (1998), Cremer et al (1990) and Sánchez and Sobel (1993). Macho-Stadler and Pérez-Castrillo (1997) obtain the same result in a model with several audit classes where the main difference between audit classes is the probability that a taxpayer's true income is discovered when audited. As mentioned before, a notable exception is Scotchmer (1987) who proves that progressibity can be improved *across* audit classes. We comment on this paper at length in Section 6.

2.- THE MODEL

There exists a continuum of taxpayers characterized by their true tax liability *i*. The true tax liability *i* can take three different values, $i \in \{0, m, 1\}$, i.e., we normalize the tax liability of the lower income level to zero, the high income liability to 1, and $m \in (0, 1)$.[7] The tax authority knows the distribution of tax liability, but it does not know the true tax liability of each particular taxpayer.

Taxpayers fill a tax return and pay the income tax liability corresponding to the return. We will denote a taxpayer's reported tax liability by *r*. If a taxpayer is audited, and his/her true income is discovered, he/she has to pay the evaded tax liability (if any), $(i - r)$. On top of the tax due, an evader must pay a penalty. We assume that the marginal penalty rate is constant, i.e., penalties equal *p* times the level of tax evasion.

Taxpayers are risk neutral. They choose how much income to report in order to maximize their expected net income. Under risk neutrality this is equivalent to saying that taxpayers minimize their expected payment. We denote by $r(i)$ the report made by a taxpayer with true tax liability *i*, $r(i) \in \{0, m, 1\}$. We take the convention that a taxpayer sends the highest report among the ones between which he/she is indifferent.

The tax authority maximizes the revenue collected through taxes and penalties, for a given budget *B*. The tax authority can send officials to audit a taxpayer. We suppose that this audit is so effective that it finds out the true tax liability of the taxpayer in a verifiable way.[8] We denote by $c > 0$ the cost of auditing one taxpayer. To select returns for audit, the tax authority can use non-verifiable signals about the taxpayer's true income. We assume that this signal is costless.[9] The signal can take two values, $s \in$

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[7] In other words, if *y* represents the true income and *t*(.) is the tax function, $i = t(y)$. Note that we do not make any assumption on the shape of the tax function.

[8] This is the audit technique that is typically considered in the literature on tax auditing, see, for example, Sánchez and Sobel (1993). See also Macho-Stadler and Pérez-Castrillo (1997) for a departure from this assumption.

[9] Similarly, we could assume that the observation of a signal implies mainly a (not too large) fixed cost, independent of the number of audits (like buying a computer to deal with the data).

$\{d, u\}$, where d stands for "down" and u stands for "up". The probability of the realization of the signal depends upon the true income of the taxpayer:

$$\text{Prob}(s = u \mid i) = a_i .$$

We assume $a_0 \leq a_m \leq a_1$. This guarantees that the Monotone Likelihood Ratio Property is satisfied. When $a_0 < a_m < a_1$ the signal is positively correlated with the income, so that a higher probability of the "high" (up) signal corresponds to higher tax liability. For $a_0 = a_m = a_1$ the signal is uninformative. Finally, note that even if for example $a_0 = 0$ and $a_1 = 1$, the signal is very useful to allow the auditor to identify low-income taxpayers and high-income taxpayers, but an audit is still necessary to provide the evidence.

The tax authority chooses the auditing function, which depends on the taxpayer's reported income r and the signal received s. We assume that the tax authority can commit to this probability-of-audit function. Let us denote it by p_{rs}.

3.- THE TAXPAYERS' BEHAVIOR

We analyze now the taxpayers' behavior depending on the audit function. In order to decide whether to report truthfully or to understate their income (there are no bonuses for over-reporting), taxpayers minimize the expected taxes and fines to be paid. Let $r(i)$ be the report function. For convenience, we denote $p(i, r) \equiv (1-a_i)\, p_{rd} + a_i\, p_{ru}$ the auditing pressure on a taxpayer earning income i who reports r. Notice that the behavior of a taxpayer with income i, the expected taxes, and the auditing costs depend on p_{rd} and p_{ru} only through $p(i, r)$.

A taxpayer with true tax liability $i = 0$ does not have any possibility for under-reporting, therefore he/she always reports $r(0) = 0$.

A taxpayer with tax liability $i = m$ has only one possible underreporting strategy: to report $r = 0$. He/she reports truthfully (i.e., $r(m) = m$) if and only if:

$$m \leq p(m, 0)\,(1+p)\, m \;\Leftrightarrow\; p(m, 0) \geq 1/(1+p). \qquad (1)$$

Otherwise, he/she reports $r(m) = 0$.

A taxpayer with tax liability i = 1 has two underreporting strategies. He/she can choose $r = m$ or $r = 0$. He/she prefers reporting $r = 1$ than $r = 0$ if the payment when reporting his/her true rent is lower or equal than his/her expected payment when reporting $r = 0$, that is, if

$$1 \leq p(1, 0)\,(1+p) \quad \Leftrightarrow \quad p(1, 0) \geq 1/(1+p). \tag{2}$$

Similarly, the high-income taxpayer prefers reporting $r = 1$ to $r = m$ if and only if:

$$p(1, m) \geq 1/(1+p). \tag{3}$$

Finally, a taxpayer with tax liability $i = 1$ reports $r = m$ rather than $r = 0$ if and only if:

$$m + p(1, m)\,(1+p)\,(1 - m) \leq p(1, 0)\,(1+p). \tag{4}$$

Remark that if $1/(1+p) \leq p(1, r)$, then a high-income taxpayer will not report $r < 1$.

An important feature due to the existence of signals is that the enforcement pressure perceived by a taxpayer does not only depend on the audit policy, but also on his/her true income level. Two taxpayers with different true income facing the same audit probabilities p_{ru} and p_{rd} perceive different audit pressure when they report r. More precisely, for $p_{ru} > p_{rd}$, $p(i, r)$ is increasing in i. Moreover, the difference in the perception of the two taxpayers increases with the informational content of the signal (i.e., with a_i). The previous feature also implies that changes in the audit policy affect the taxpayers' behavior differently depending on their true income.

4.- SOME CHARACTERISTICS OF THE ENFORCEMENT POLICY

We now derive some characteristics of the optimal enforcement policy. Note that auditing a high report is a waste of resources. Therefore, we look for the optimal auditing strategy when the report is either $r = m$ or $r = 0$.

The proof of the next three lemmas is given in the Appendix. The first two lemmas generalize properties that are satisfied for the optimal auditing policy when signals are not available.

Lemma 1. *When looking for the optimal auditing policy, there is no loss of generality in restricting the analysis to audit functions that satisfy:*

(i) $p(1, m) \leq 1/(1+p)$, *and*

(ii) $Min\ \{p(1, 0), p(m, 0)\} \leq 1/(1+p)$.

Lemma 1 sets upper bounds to the audit probabilities in optimal auditing policies. For example, in (i), since an enforcement pressure that satisfies $p(1, m) = 1/(1+p)$ induces high-income taxpayers not to report $r = m$, setting the pressure above this level is a waste of resources. Similarly, a policy that satisfies the equation in (ii) with equality suffices to ensure that high and medium-income taxpayers do not report $r = 0$.

Lemma 2. *When looking for the optimal auditing policy, there is no loss of generality in restricting the analysis to audit functions that satisfy:* $p(1, m)$ £ $p(1, 0)$.

Lemma 2 states that the global enforcement pressure over taxpayers reporting $r = 0$ cannot be inferior to the one exerted over taxpayers reporting $r = m$. It gives us some kind of non-decreasing property of the audit policy on the reports. This characteristic is common in adverse selection problems since the most profitable deviation is to report the lower income level.

The following lemma provides a very intuitive property of the optimal enforcement policy when signals are available: it is optimal to start auditing the taxpayers whose signal is "up", before spending resources in auditing taxpayers whose signal is "down".

Lemma 3. *When looking for the optimal auditing policy, there is no loss of generality in restricting the analysis to audit functions that satisfy:*

$p_{id} > 0$ *only if* $p_{iu} = 1$, *for* $i = 0, m$.

One important consequence of Lemma 3 is that, since the signal "up" appears with higher probability the higher the true income is, the auditing pressure is increasing with income. In particular, note that $r(1) = 0$ only if $r(m) = 0$. Therefore, *the report is increasing with the true income*.

From now on, for simplicity of exposition we will concentrate on a particular example concerning the distribution of the population of taxpayers and the probabilities of receiving the signal "up". Also for simplicity of exposition, we will assume that the penalty rate is high enough. For more general forms, the qualitative results remain unchanged, but the expressions become cumbersome.

Assumption 1. (i) The distribution of the population is uniform. **(ii)** $a_0 = (1–a)$, $a_m = 1/2$, and $a_1 = a$, with $a \in [1/2, 1]$. **(iii)** $p \geq 1$.

Let us explain (iii) in Assumption 1. By Lemma 3, if $p_{md} > 0$ then $p_{mu} = 1$ and therefore $p(1, m) = (1–a)\, p_{md} + a\, p_{mu} = (1–a)\, p_{md} + a > a \geq 1/2$. Suppose now that $p \geq 1$. Then $1/(1+p) \leq 1/2$. We can conclude that $p(1, m) = (1–a)\, p_{md} + a\, p_{mu} > 1/(1+p)$, which contradicts (i) of Lemma 1. Therefore, $p \geq 1$ implies that $p_{md} = 0$. Similarly, if $p_{0d} > 0$ and $p \geq 1$, then we obtain a contradiction of (ii) of Lemma 1. To sum up, $p \geq 1$ implies $p_{0d} = p_{md} = 0$. This allows us to concentrate on the decision on p_{0u} and p_{mu}.

Using Assumption 1 and the previous results, we can simplify the expressions concerning the taxpayers' reporting behavior:

$$r(m) = m \qquad \text{if } p_{0u} \geq \frac{2}{1+p}\text{. Otherwise, } r(m) = 0. \tag{5}$$

$$r(1) = 1 \qquad \text{if } p_{mu} \geq \frac{1}{a}\frac{1}{(1+p)} \text{ and } p_{0u} \geq \frac{1}{a}\frac{1}{(1+p)}. \tag{6}$$

$$r(1) = m \qquad \text{if } p_{mu} < \frac{1}{a}\frac{1}{(1+p)} \text{ and } p_{0u} \geq \frac{1}{a}\frac{1}{(1+p)} m + p_{mu}(1-m). \tag{7}$$

$$r(1) = 0 \qquad \text{if } p_{0u} < \frac{1}{a}\frac{1}{(1+p)} m + p_{mu}(1-m). \tag{8}$$

[10]

5.- TAX COMPLIANCE AND OPTIMAL ENFORCEMENT

Anticipating the optimal taxpayer behavior for any audit policy, the tax authority decides the optimal audit schedule. We solve this decision problem in two steps. First, we fix a specific profile of taxpayer behavior (for instance, $r(m) = m$, $r(1) = m$) and we identify the optimal audit policy sustaining this profile given the budget B. The second step consists in comparing the tax authority revenue in each profile of taxpayers' behavior and identifying the optimal audit policy as a function of the available budget.

In our model, the relevant taxpayers' behavior profiles are the following:

(i) Full non-compliance: $r(m) = 0$, $r(1) = 0$. We will refer to it as Profile A.

(ii) Low Compliance: $r(m) = 0$, $r(1) = m$. Profile B.

(iii) Only high-income taxpayers evade: $r(m) = m$, $r(1) = m$. Profile C.

(iv) Only medium-income taxpayers evade: $r(m) = 0$, $r(1) = 1$. Profile D.

(v) Full compliance: $r(m) = m$, $r(1) = 1$. Profile E.

In principle, there is another possible profile: the case with $r(m) = m$ and $r(1) = 0$. But, as we have discussed after Lemma 3, this is never possible.

It is not always possible to achieve a certain taxpayers' behavior. For example, suppose that we analyze a profile where the high income taxpayers do not report $r = 0$. Then, the probability p_{0u} must be higher than $m/\alpha(1+\pi)$. Moreover, at least the low income taxpayers are reporting $r = 0$. Therefore, a minimum budget is necessary to induce this pattern of taxpayers' behavior.

Lemma 4 states the minimum budget that is necessary to implement each possible profile of behavior. It also compares the different values of the minimum budgets. Since all the enforcement parameters enter in the same way in the revenue function, for convenience, we denote $b = (B/c)(1+\pi)$. An increase in b can be interpreted as an increase in the budget B, a decrease in the audit cost c, or an increase in the penalty associated to evasion π. We will refer to b as "the enforcement strength". We denote by b_P the minimum enforcement strength necessary to induce the taxpayers to adopt a behavior profile P.

Lemma 4.

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[10] Note that it is not necessary to check the constraint $p_{0u} < 1 / \alpha(1+\pi)$, since it is immediately satisfied when $p_{mu} \leq 1 / \alpha(1+\pi)$, which always holds by Lemma 1 (i).

(i) $b_A = 0$; $b_B = \frac{(3-2a)}{6}\frac{1}{a}m$; $b_C = \frac{2}{3}(1-a)$; $b_D = \frac{(3-2a)}{6}\frac{1}{a}$; $b_E = \frac{(1+4a-4a^2)}{6a}$.

(i) $b_E > b_D > b_C > b_A$, *and* $b_D > b_B > b_A$ *for* $a \in \,]1/2,1[$.

(iii) $b_C < b_B$ *if and only if* $m > \frac{4a(1-a)}{3-2a}$.[11]

(iv) $b_E = b_D$ *for* $a = 1/2$ *and* $a = 1$; *and* $b_C = 0$ *for* $a = 1$.

Note, first, that the minimum enforcement strength that allows the tax authority to reach the different profiles is decreasing in a. This characteristic is easy to understand: the more informative the signal is, the more effective is the audit. Note also that the minimum enforcement needed to reach a profile is independent of the tax liability *m* except for Profile B. This is so because the probability that induces high income taxpayers to report $r(1) = m$ and not $r(1) = 0$ depends on the level of *m*.

Next lemma states the optimal audit policy for each possible profile of behavior, when the budget allows to implement such a taxpayers' behavior.

Lemma 5. *The optimal audit policy in each profile is:*

Profile A ($r(m) = 0$, $r(1) = 0$): $p_{0u} = 2B/c$ *and* $p_{mu} = 0$.

Profile B ($r(m) = 0$, $r(1) = m$): *(a) If* $m < \frac{3-2a}{4-2a}$, *then:*

$$p_{0u} = \frac{1}{3-(3-2a)m}\left[\frac{2}{(1+p)}m + 6(1-m)\frac{B}{c}\right] \text{ and } p_{mu} = \frac{1}{3-(3-2a)m}\left[6\frac{B}{c} - \frac{(3-2a)}{a(1+p)}m\right].$$

(b) If $m \geq \frac{3-2a}{4-2a}$, *then:* $p_{0u} = \frac{6}{(3-2a)}\frac{B}{c}$ *and* $p_{mu} = 0$.

Profile C ($r(m) = m$, $r(1) = m$): $p_{0u} = 2/(1+p)$, *and* $p_{mu} = \frac{6}{(1+2a)}\frac{B}{c} - \frac{4(1-a)}{(1+2a)}\frac{1}{(1+p)}$

Profile D ($r(m) = 0$, $r(1) = 1$): $p_{0u} = \frac{6}{(3-2a)}\frac{B}{c}$ *and* $p_{mu} = \frac{1}{a}\frac{1}{(1+p)}$.

[11] Notice that $4a(1-a)/(3-2a)$ is always inferior to $(3-2a)/(4-2a)$ for $a \in [1/2,1]$.

Profile E ($r(m) = m$, $r(1) = 1$): $p_{0u} = \frac{2}{1+p}$ *and* $p_{mu} = \frac{1}{a}\frac{1}{(1+p)}$.

Notice that Profile B has two sub-profiles depending on the comparison between m and a function of a.[12] The reason is that, in this case, the tax authority can decide either to audit only reports $r = 0$ or to audit both reports, $r = 0$ and $r = m$. The revenue of both strategies depends on the tax liability m and on the informative content of the signal. If m is high enough, high-income taxpayers evade a low amount (they evade $1–m$) and medium income taxpayers evade a lot. Hence, it is more profitable to concentrate audit efforts on uncovering medium income taxpayers, that is, auditing reports $r = 0$. Similarly, if m is low, it is more convenient to increase the compliance of high-income taxpayers. Therefore, p_{mu} is as high as possible, and p_{0u} keeps the level ensuring that high income taxpayers do not have incentives to report $r = 0$.

As Lemma 2 implied, $p_{mu} \leq p_{0u}$ in every profile. This corresponds to the classical result that the pressure can not be increasing with the report in order to discourage high-income taxpayers to make the lowest reports. However, it is important to notice that, in our model, this fact does not imply the actual audit probability to be non-increasing with the income. To clarify this point, remember that $p(i, r(i))$ is the actual probability of auditing taxpayer i in a particular profile. It is easy to check that, for $a > 1/2$, $p(m, r(m)) > p(0, r(0))$ in every profile except in C (where $p(m, r(m)) > p(0, r(0))$ only for high enough a and b). In particular, in profile E, $p(m, r(m)) > p(0, r(0))$ is compatible with $p_{r(m)u} = p_{mu} < p_{0u}$. Also, $p(1, r(1)) > p(m, r(m))$ in profiles A and C. Therefore, when the tax administration makes use of signals to direct its audit strategy, it is possible to see an audit probability increasing with the income in a situation in which the tax administration commits to its audit strategy.

To characterize the optimal audit policy, we write the tax authority revenue (the sum of the voluntary compliance plus taxes and penalties collected after the audit) as a function of the parameters of the model.

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[12] Notice that $(3 – 2a) / (4 – 2a)$ is a decreasing function of a that takes values in the interval (1/2, 2/3).

Corollary 1. *The total expected ex-post collection by the tax authority as a function of the behavior induced on the taxpayers and the enforcement tools are:*

<u>*Profile A*</u>. $M_A(b) = \frac{1}{3}(m + 2\alpha)b$.

<u>*Profile B*</u>. *If* $m < \frac{3-2\alpha}{4-2\alpha}$, $M_B(b) = \frac{m+2\alpha}{3-(3-2\alpha)m}\left[b(1-m) + \frac{m}{3}\right]$.

If $m \geq \frac{3-2\alpha}{4-2\alpha}$, $M_B(b) = \frac{1}{(3-2\alpha)}mb + \frac{1}{3}m$.

<u>*Profile C.*</u> $M_C(b) = \frac{2}{3}m + (1-m)\frac{2\alpha}{1+2\alpha}\left(b - \frac{2}{3}(1-\alpha)\right)$.

<u>*Profile D.*</u> $M_D(b) = \frac{1}{(3-2\alpha)}mb + \frac{1}{3}$.

<u>*Profile E*</u>. $M_E(b) = \frac{1}{3}m + \frac{1}{3}$.

It is easy to check that the functions M_K are increasing in m and α, for $K = A, B, C, D, E$. That is, expected revenue always increases both with the tax liability and with the informational content of the signal.

Once we have calculated the optimal policy and revenues for each possible behavior profile, we now look at the choice of which profile is preferred by the tax authority. Lemma 4 states that only Profile A is sustainable for low b. When b increases, the tax authority can induce other taxpayers' behaviors. Proposition 4 states the optimal choice by the tax authority.

Proposition 1.

(a) *Profile A is optimal if* $b < \min\{b_B, b_C\}$.

(b) *Profile B is optimal if* $b \in [b_B, b_D)$ *and* $m < m^\circ(b)$ *for* $b \geq b_C$.

(c) *Profile C is optimal if* $b \in [b_C, b_E)$ *and* $m \geq m^\circ(b)$.

(d) *Profile D is optimal if* $b \in [b_D, b_E)$ *and* $m < m^\circ(b)$.

(e) *Profile E is optimal if* $b \geq b_E$.

Where $m°(b)$ is an increasing (and discontinuous at the point $b = b_D$) function defined in the interval $[b_C, b_E)$.

Before explaining the characteristics of the optimal auditing policy and the taxpayers' behavior, we present the extreme cases. The benchmark, i.e., the case where no signal is available, corresponds to the situation with $a = 1/2$, that is represented in Figure 1. In this case, by Lemma 5 and Proposition 1, the optimal audit policy is the following function of m: for $m < 1/2$, the optimal policy lies in Profile A ($r(m) = 0$, $r(1) = 0$) until the budget is high enough (formally, $b \geq (2/3)\, m$) to reach Profile B ($r(m) = 0$, $r(1) = m$); for $m > 1/2$, the optimal audit policy lies in Profile A until the budget ($b \geq 1/3$) allows to reach Profile C ($r(m) = m$, $r(1) = m$). In both cases, the optimal policy for $b \geq 2/3$ is Profile E.

[Insert Figure 1 about here]

Note that compliance in the benchmark is always increasing on b for every group of taxpayers. Also, in this case, the tax authority does not audit medium-income reports unless there is no fraudulent taxpayer reporting $r = 0$.[13] This is so because a change in the audit probability affects all taxpayers in the same way.

In the other extreme case, that is, when $a = 1$, low-income taxpayers never have associated the signal "up". Therefore, Profile C can be reached with a zero budget. Hence, starting with a low or zero budget, the optimal policy consists in inducing medium and high-income taxpayers to report m (formally, $r(m) = m$, $r(1) = m$), until b is high enough to allow full compliance.

The results presented in Proposition 1 are summarized in Figure 2.

[Insert Figure 2 about here]

We now comment on the most interesting properties of the optimal auditing policy and the induced taxpayers' behavior, postponing to the next section the discussion about progressivity. The first important fact is that, when signals are available, it is not necessarily true that taxpayers' compliance increases with b. For intermediate values of m, the optimal

[13] This property does not hold in Profiles B (b) and D, profiles that are sometimes optimal when signals are available.

policy moves from Profile C to Profile D as *b* goes from levels lower than b_D to levels higher than b_D. This change in profile implies that medium-income taxpayers decrease their report (they report $r(m) = m$ in Profile C while they report $r(m) = 0$ in Profile D), and they lower their compliance as the enforcement strength increases. Therefore, in our framework, an increase in the budget, a decrease in the audit cost, or an increase in the penalty level may lead to a lower compliance by a group of taxpayers. The reason for this property is that, when the enforcement strength is lower than b_D and *m* is high enough, inducing both groups of taxpayers to report $r = m$ is optimal. On the other hand, for low values of *m*, it is worthwhile for the tax authority to induce truthful revelation by high-income taxpayers as soon as the enforcement tools allows it (i.e., when the enforcement strength is higher than b_D). The cheapest way to induce this behavior is to audit with probability $p = 1/[a(1+p)]$ the taxpayers reporting either $r = 0$ or $r = m$. But, when the signal is valuable (i.e., when $a > 1/2$), this probability is not high enough for the medium-income taxpayers to report honestly, hence $r(m) = 0$. Consequently, for intermediate values of *m*, medium-income taxpayers may perceive a lower audit pressure as *b* increases. Although this is an interesting result, we suspect that it might be due to the discrete nature of our model.

Some other characteristics of Figure 2 can be explained. When the medium level of tax liability *m* is high, the tax authority is more concerned by evasion of medium-income taxpayers than by inducing full compliance of high-income taxpayers. Therefore, Profile D is never optimal. On the contrary, when *m* is low, the tax authority prefers devoting resources to induce high income taxpayers to pay as much as possible (through penalties) than inducing them to report *m*. Therefore Profile C is never optimal.

6.- AUDITING AND PROGRESSIVITY

From a normative point of view, a very important aspect of the audit policy is the relationship between progressivity of the nominal taxes and progressivity of the effective taxes. Cremer et al (1990), Sánchez and Sobel (1993), Macho-Stadler and Pérez-Castrillo (1997), and Chandler and Wilde (1998), among others, point out that effective taxes tend to be more regressive than nominal ones. The reason for this result is that the possibilities for underreporting increase with the level of income. The optimal auditing policy must first devote resources to the audit of low reports, since this is the most profitable deviation for any taxpayer. But then, low-income taxpayers do not have any room for underreporting, while high-income taxpayers are not fully compliant since they can report medium incomes not audited.

The previous argument is less clear-cut when a signal independent of the report but correlated with true income is available. In this case, the same report by two different taxpayers triggers two different audit pressures since a higher income involves a higher probability of inducing a signal "up". We claim that, in some sense, the existence of the signal helps the progressivity of the effective tax scheme.

To explain our previous claim, let us denote by $T(i)$ the expected payment (taxes plus fines) made by a taxpayer with true tax liability i facing the optimal audit policy. Also, denote by $g(i)$ the "burden ratio" of this taxpayer, i.e., $g(i) = T(i)/i$. In the literature of optimal auditing without signals, $g(i)$ is a function weakly decreasing in i. For example, in Sánchez and Sobel (1993), $g(i) = 1$ for every $i < a$, where a is a certain cut-off value, and $g(i) = a/i$ for $i \geq a$ (strictly decreasing in i). Hence, effective taxes are more regressive than nominal ones.

In our simple model, with only three income levels, the relevant comparison is between $g(1)$ and $g(m)$. In the benchmark, with $\alpha = 1/2$, only three profiles are possible as a function of m. For $m < 1/2$, the optimal policy either lies in Profile A, with $g(1) = g(m) = b$; or in Profile B, with $g(1) = g(m) < 1$; or in Profile E, with $g(1) = g(m) = 1$. For $m > 1/2$, as b increases, the induced behavior goes from Profile A, to Profile C, where $g(1) < g(m) = 1$ and to Profile E. Hence, $g(i)$ is weakly decreasing. However, the result may change if $\alpha > 1/2$. In Profile A and in Profile B, for $\alpha > 1/2$, it can be checked that $1 > g(1) > g(m)$. In addition, Profile D can be optimal, and there $g(1) = 1 > g(m)$. Therefore, considering the effective compliance of high and medium-income taxpayers, effective taxes may be more progressive than nominal taxes.

The previous conclusion is reinforced if we look at the behavior of the ratio $g(1)/g(m)$ in each profile as α changes. In profiles A, B, and C this ratio increases with α. Hence progressivity increases with α. Only in Profile D, $g(1)/g(m)$ decreases with α. This is reasonable since in this profile high income taxpayers fully comply $g(1) = 1$, so increases in a allows to devote more resources to improve compliance by medium-income taxpayers, who are the only evaders.

We also know that changes in a affect the minimum enforcement strength needed to reach each profile. Sometimes, an increase in a allows going from one profile to another in the direction of increasing progressivity. For example, an increase in a can drive the optimal audit from Profile C to Profile D. However, in other circumstances, increases in a may lead to higher compliance to every taxpayer but towards a more regressive induced behavior. This happens, for example, when the increase in a allows to go from Profile A to Profile C.

Scotchmer (1987) also shows that the use of information other than the reported income can lead to effective taxes more progressive than stipulated by the tax code. In her paper, the information that the tax authority holds allows it to define different taxpayers' audit classes. The tax authority finds it lucrative to condition the probability of audit on audit class, as well as on reported income. Each taxpayer is aware of the information in hands of the administration, and hence of the income class he/she belongs to. Within an income class, the tax authority will audit taxpayers with low income reports with higher probability than it audits high-report taxpayers, and this introduces a regressive bias in the effective tax code. On the other hand, given that the audit probability depends on the audit class, the regressive bias is of limited importance if the audit class is a good predictor of true income. Effective taxes can actually more progressive than nominal taxes. We think that our analysis is complementary to Scotchmer (1987)'s. They both show that acquiring information (signals) about taxpayers can help to direct the audit not only to improve compliance but also to reduce the regressive bias due to the possibility of fraud.

7.- CONCLUSION

In this paper, we present a model of optimal tax auditing that takes into account the existence of external signals correlated with taxpayers’ true income. The limitations of our exercise are clear. First, it suffers from the same criticisms as the papers on optimal auditing that follows the principal-agent approach: we assume commitment by the tax authority to an audit policy that is not ex-post optimal, taxpayers are assumed to be risk neutral, the taxpayers income is exogenous, and the audit technology is particular.

Second, we carry out the analysis in a framework with only three possible incomes. Moreover, most of the results are obtained under that assumption that the population is homogeneously distributed over the set of incomes. We can extend the analysis to more general distributions of the population of taxpayers and the results are qualitatively the same. However, it is clear that we are far from being able to provide a general analysis of tax compliance in the presence of signals. The technical difficulties associated to a general model become apparent in one takes into account the technical complication of the model without signals and the additional difficulty added by signals, which imply that a given audit probability induces different pressures depending on the taxpayers' true income.

In spite of the previous limitations, we think that the analysis carried out here help to enlighten some relevant positive and normative aspects of the optimal auditing policy and the taxpayers' compliance behavior. Two results obtained in the literature without signals seem robust to the introduction of them. On the one hand, facing the optimal audit policy, a taxpayer does not suffer higher audit pressure as he/she increases his/her report. On the other hand, reports are weakly increasing with income. However, the result that tax compliance is (weakly) increasing in the budget for every taxpayer does not seem to be robust to the introduction of signals. Moreover, the actual audit probability can be increasing with the income when the tax administration makes use of signals to direct its audit strategy.

A more interesting result from a normative point of view is the relationship between the informational content of the signal and the progressivity of effective taxes. Our analysis shows that effective taxes may be more progressive than nominal ones. Moreover, it suggests that effective progressivity, in general, increases with the informational content of the signal. This is an appealing property that, we think, deserves further research. A tax authority could be interested in acquiring signals about taxpayers (even if this is costly), not only because this increases tax revenues, but also because it improves the progressivity of the effective tax system.

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APPENDIX

Proof of Lemma 1. If $p(1, m) > 1 / (1+p)$, then $r(1) \neq m$, so no penalty is collected through the audit of the returns that report $r = m$. This is also true for $p(1, m) = 1 / (1+p)$, at a lower or equal cost. Similarly, if Min $\{p(1, 0), p(m, 0)\} > 1 / (1+p)$, there will be no evader reporting $r = 0$. The behavior of every taxpayer and the collected taxes and penalties will be the same at lower auditing cost if the tax authority decreases p_{0d} and/or p_{0u} until the strict inequality becomes an equality. Q.E.D.

Proof of Lemma 2. If $p(1, m) > p(1, 0)$, then there are two possibilities. The first possibility is that both sides of the equation are higher or equal than $1 / (1+p)$, so $r(1) = 1$, and then setting $p(1, m) = 1/(1+p)$ induces the same behavior at a lower or equal cost. The second possibility is that $p(1, 0) < 1 / (1+p)$, then $[1 / (1+p)]\, m + p(1, m)\,(1–m) > p(1, 0)$, which implies that (4) is not satisfied, and so $r(1) = 0$. Consequently, it is still possible to decrease $p(1, m)$ until the inequality becomes equality, the taxpayers' behavior is not altered by the change and the auditing cost is lower or equal. Q.E.D.

Proof of Lemma 3. Suppose that some taxpayers are reporting r, for $r = 0, m$. We denote by l_r the rate of taxpayers whose signal is "up" among the taxpayers reporting r. Similarly, we denote by m_r the rate of taxpayers whose signal is "up" among the taxpayers earning an income level strictly higher than r and reporting r. Notice that $a_r \leq l_r \leq m_r$.

For $i = m$, suppose that $p_{md} > 0$ and $p_{mu} < 1$. We show that we can change the policy not altering taxpayers' revenue and without increasing the auditing cost. Consider an increase in p_{mu} and a decrease in p_{md} so that $p(1, m)$ does not change. That is, $\Delta p_{mu} = d > 0$ and $\Delta p_{md} = -\,(a_1/(1–a_1))\, d$ (with d small enough). High-income taxpayers' behavior and the revenue from these taxpayers do not change with the new policy, since $p(1, m)$ is the same. On the other hand, the change in costs is proportional to:

$$l_m \Delta p_{mu} + (1 - l_m)\Delta p_{md} = \left[l_m - \frac{a_1}{(1 - a_1)}(1 - l_m) \right] d \leq 0$$

since, as it is easy to see, $l_m \leq a_1$. Notice that we can go on with such changes in the policy until either $p_{mu} = 1$ or $p_{md} = 0$.

For $i = 0$, suppose that $p_{0d} > 0$ and $p_{0u} < 1$. We distinguish between three possibilities:

(a) Suppose that $r(1) \neq 0$. Then, consider an increase in p_{0u} and a decrease in p_{0d} so that $p(m,0)$ does not change. That is, $\Delta p_{0u} = d > 0$ and $\Delta p_{0d} = -(a_m/(1-a_m))\, d$ (with d small enough). Notice that $p(1,0)$ does not decrease with the change, given that $a_1 \geq a_m$:

$$\Delta p(1,0) = a_1 \Delta p_{0u} + (1-a_1)\Delta p_{0d} = \left[a_1 - \frac{a_m}{(1-a_m)}(1-a_1) \right] d \geq 0.$$

This implies that high-income taxpayers keep not reporting $r = m$ after the change, i.e., they do not change their behavior. Also, the behavior of, and the revenues from, medium-income taxpayers is the same. Finally, the change in cost of the auditing policy is proportional to:

$$l_0 \Delta p_{0u} + (1-l_0)\Delta p_{0d} = \left[l_0 - \frac{a_m}{(1-a_m)}(1-l_0) \right] d \leq 0$$

since $l_0 \leq a_m$, given that $r(1) \neq 0$. Therefore, the change in the policy is not damaging. Notice that we can proceed until either $p_{0u} = 1$ or $p_{0d} = 0$.

(b) Suppose that $r(1) = 0$ and $r(m) = 0$. Then, consider an increase in p_{0u} and a decrease in p_{0d} that keeps the revenues constant, i.e., $\Delta p_{0u} = d > 0$ and $\Delta p_{0d} = -(m_0/(1-m_0))\, d$. Notice, first, that since $l_0 \leq m_0$, it is easy to show (using the same method as in (iia)) that the auditing cost does not increase. Also, given that $a_m \leq m_0 \leq a_1$, $p(1, 0)$ does not decrease while $p(m, 0)$ does not increase. Therefore, medium-income taxpayers will not alter their behavior with the change. As to high-income taxpayers, there are two possibilities. Either we can go on with the modification in the audit probabilities until $p_{0u} = 1$ or $p_{0d} = 0$, with high-income taxpayers reporting $r = 0$, and then it is proved. Or there is a certain point at which they want to change their report to $r'(1) = 1$ or $r'(1) = m$. Suppose that $r'(1) = m$ (similarly for the other case). Denote by $p'(1, 0) \geq p(1, 0)$ the auditing pressure such that high-income taxpayers are indifferent between reporting $r = 0$ and $r = m$, given the pressure $p(1, m)$ on the reports $r = m$. We proceed with the increase of p_{0u} and the decrease of p_{0d} until we reach $p'(1, 0)$. A this point, high-income taxpayers change their report, with the same revenue for the tax administration, since the taxpayers are indifferent. Moreover, the cost of auditing these taxpayers does not increase, since

$p'(1, 0) \geq p(1, m)$ if they are reporting $r = m$. Therefore, up to this point, the change in the policy is not damaging. Once we have proceed with the change in the policy until we reach $p'(1, 0)$, we are in a situation as in (iia), and we can proceed as described there, until either $p_{0u} = 1$ or $p_{0d} = 0$.

(c) Suppose that $r(1) = 0$ and $r(m) = m$. This means that $p(m, 0) \geq 1/(1+\rho)$. First, decrease p_{mu} and/or p_{md} until we reach a situation in which $p'(1, m)$ is so low that high-income taxpayers are indifferent between reporting $r = 0$ and $r = m$. If these taxpayers prefer reporting $r = 0$ even for $p_{mu} = p_{md} = 0$, then we fix $p'(1, m) = 0$. In both cases, $p'(1, m) \leq p(1, 0)$. Second, increase p_{0u} and decrease p_{0d} so that $p(m, 0)$ does not change, until either $p_{0u} = 1$ or $p_{0d} = 0$. At the end of this process, $p'(1, 0) \geq p'(m, 0) = p(m, 0) \geq 1/(1+\rho)$. Therefore, high-income taxpayers will end up reporting $r = m$. In short, revenues from high-income taxpayers do not decrease, since previous to the changes in p_{0u} and p_{0d} high-income taxpayers were indifferent or preferred strictly reporting $r = 0$ than $r = m$; the cost of auditing these consumers does not increase, since $p'(1, m) \leq p(1, 0)$; medium income taxpayers keep reporting their true income, and do not suffer more auditing than before; and low-income taxpayers suffer less or equal inspection, given that $\alpha_0 \leq \alpha_m$. Therefore, the change in the policy is not damaging. Q.E.D.

Proof of Lemma 4 and 5 and Corollary 1. For convenience, we present the proofs together. We first proceed profile by profile.

Profile A: $r(m) = 0$, $r(1) = 0$. The only parameters that matters is p_{0u}, the whole population is reporting $r = 0$. Half of them receive the signal u. Then, with a budget B, the tax authority can credibly commit to a probability: $p_{0u} = 2B/c$. The tax authority only collects revenues through penalties, to the amount: $M_A(B) = \frac{1}{3}(1+\rho)(m+2\alpha)\frac{B}{c}$.

The minimum amount of money needed to be in this profile is: $B_A = 0$.

Profile B: $r(m) = 0$, $r(1) = m$. For the taxpayers to report according to this profile, the audit probabilities must satisfy:

$$p_{0u} \in \left[\frac{1}{\alpha}\frac{1}{(1+\rho)}m + p_{mu}(1-m),\ \frac{2}{1+\rho}\right) \text{ and } p_{mu} < \frac{1}{\alpha}\frac{1}{(1+\rho)}.$$

The minimum budget B_B necessary to be in this profile corresponds to $p_{mu} = 0$ and $p_{0u} = m / a\,(1+p)$. The number of taxpayers audited, because they are reporting $r = 0$ and their signal is u, is (1/6) $(3 - 2a)$. Therefore: $B_B = \frac{(3-2a)}{6}\frac{1}{a}\frac{1}{(1+p)}mc$.

The possible auditing policies are determined by the budget constraints:

$$\frac{(3-2a)}{6}p_{0u}c + \frac{1}{3}a p_{mu}c = B \text{ , i.e., } p_{mu} = \frac{3}{a}\frac{B}{c} - \frac{(3-2a)}{2a}p_{0u} .$$

Therefore, we can write the revenues as a function of p_{0u}:

$$M_B(B, p_{0u}) = \frac{1}{3}m + \frac{B}{c}(1+p)(1-m) + \frac{1}{6}\left[m - (3-2a)(1-m)\right]p_{0u}(1+p) .$$

Note that the revenue function is increasing (resp. decreasing) in p_{0u} if and only if $(4 - 2a)\, m \geq 3 - 2a$ (resp. $(4 - 2a)\, m \leq 3 - 2a$). Therefore, the optimal audit policy depends upon the value of m:

(a) If $m < \frac{3-2a}{4-2a}$: $p_{0u} = \frac{1}{a}\frac{1}{(1+p)}m + p_{mu}(1-m)$ and $p_{mu} = \frac{3}{a}\frac{1}{(1+p)}\frac{B}{c} + \frac{(3-2a)}{2a}p_{0u}$

$$M_B(B) = \frac{m+2a}{3-(3-2a)m}\left[\frac{B}{c}(1+p)(1-\mathrm{m}) + \frac{m}{3}\right]$$

(b) If $m \geq \frac{3-2a}{4-2a}$: $p_{0u} = \frac{6}{(3-2a)}\frac{B}{c}$ and $p_{mu} = 0$.

$$M_B(B) = \frac{1}{(3-2a)}(1+p)m\frac{B}{c} + \frac{1}{3}m .$$

<u>Profile C</u>: $r(m) = m$, $r(1) = m$. The audit probabilities that lead the taxpayers to send reports in this profile are:

$$p_{0u} \geq \text{Max}\left\{\frac{1}{a}\frac{1}{(1+p)}m + p_{mu}(1-m),\ \frac{2}{1+p}\right\} \text{ and } p_{mu} < \frac{1}{a}\frac{1}{(1+p)} .$$

The two preceding inequality are equivalent to $p_{mu} < \frac{1}{a}\frac{1}{(1+p)}$ and $p_{0u} \geq \frac{2}{1+p}$.

The minimum budget needed to be in profile C corresponds to p_{mu} = 0 and p_{0u} = 2 / (1+π), and the number of taxpayers audited because they report r = 0 is (1/3) (1–α). Therefore: $B_C = \frac{2}{3}(1-\alpha)\frac{1}{1+\pi}c$.

The optimal audit policy in this profile increases p_{mu} as much as the budget allows for (since increasing p_{0u} is never optimal). The budget constraint is:

$$\frac{1}{3}(1-\alpha)\frac{2}{1+\pi}c+\left[\frac{1}{3}\frac{1}{2}+\frac{1}{3}\alpha\right]p_{mu}c = B, \text{ i.e., } p_{mu} = \frac{6}{(1+2\alpha)}\frac{B}{c}-\frac{4(1-\alpha)}{(1+2\alpha)}\frac{1}{(1+\pi)}.$$

Therefore, the revenue raised by the tax authority is:

$$M_C(B)=\frac{2}{3}m-\frac{4}{3}\frac{\alpha(1-\alpha)}{(1+2\alpha)}(1-m)+2\frac{\alpha}{1+2\alpha}(1-m)(1+\pi)\frac{B}{c}$$

Profile D: $r(m)$ = 0, $r(1)$ = 1. In this profile, the parameters must satisfy the following constraints:

$$p_{0u}\in\left[\frac{1}{\alpha}\frac{1}{(1+\pi)},\ \frac{2}{1+\pi}\right) \text{ and } p_{mu}\geq\frac{1}{\alpha}\frac{1}{(1+\pi)}.$$

Given that no taxpayer is reporting m, p_{mu} is free. Therefore, the minimum budget B_D to reach this profile is: $B_D = \frac{(3-2\alpha)}{6}\frac{1}{\alpha}\frac{1}{(1+\pi)}c$. To calculate the maximum p_{0u} possible with a budget B, we use the budget constraint: $\frac{(3-2\alpha)}{6}p_{0u}c = B$, i.e., $p_{0u} = \frac{6}{(3-2\alpha)}\frac{B}{c}$.

Hence, the revenue raised by the tax authority is: $M_D(B)=\frac{1}{(3-2\alpha)}m(1+\pi)\frac{B}{c}+\frac{1}{3}$.

Profile E: $r(m) = m$, $r(1)$ = 1. The optimal audit is: $p_{0u}=\frac{2}{1+\pi}$ and $p_{mu}=\frac{1}{\alpha}\frac{1}{(1+\pi)}$.

The budget required and the revenue raised in this profile are:

$$B_E=\frac{(1+4\alpha-4\alpha^2)}{6\alpha}\frac{1}{1+\pi}c \quad \text{and} \quad M_E(B)=\frac{1}{3}m+\frac{1}{3}. \qquad \text{Q.E.D.}$$

For the proof of Lemma 4 (ii) – (iv), note first that $b_E > b_D$ if and only if $1 + 4\alpha - 4\alpha^2 > 3 - 2\alpha$, i.e., $2(1-\alpha)(1-2\alpha) < 0$, which is always the case for $\alpha \in (1/2,1)$. They coincide for

a =1/2 and a =1. Also, $b_D > b_C$ if and only if $3 - 2a > 4(1-a)\,a$, i.e., $3 - 6a + 4a^2 > 0$, which always holds. Note that $b_D > b_B$ is immediate. Finally, $B_C < B_B$ if and only if $4(1-a)\,a < (3 - 2a)\,m$. Q.E.D.

Proof of Proposition 1. We prove it through a series of claims.

(i) Profile A is not optimal if $b \geq \min\{b_B, b_C\}$.

$M_B - M_A \geq \frac{m + 2a}{3-(3-2a)m}\left[b(1-m)+\frac{m}{3}\right]-\frac{(m+2a)b}{3} \geq 0$ if and only if $b < 2a$, which is always true. Also, $M_C - M_A > 0$ if and only if $b < \frac{m(2+8a-4a^2)-4a(1-a)}{m(1+8a)-4a(1-a)} \equiv b°$. For our purposes, it is enough to prove that $b° > b_E$ when $m \geq 4a(1-a)/(3-2a)$ (i.e., when $b_C < b_B$), which is always true.

(ii) Profile E is optimal if $b \geq b_E$. The proof is straightforward.

(iii) Profile B is never optimal for $b \geq b_C$ and $m \geq (3-2a)/(4-2a)$. M_C ³ M_B for $m \geq (3-2a)/(4-2a)$ if and only if $(3-2a)\,[(1+6a-4a^2)\,m - 4a(1-a)] \geq 3\,b[(1+8a-4a^2)\,m - 2a(3-2a)]$. The term multiplying b is positive for all $m \geq (3-2a)/(4-2a)$. Hence, the inequality is more difficult to hold the bigger is b. It can be checked that the inequality holds for b_D (looking at the expressions in Corollary 1, it is easy to see that $M_D > M_B$ for this range of m).

(iv) For $m < (3-2a)/(4-2a)$, there is a function $\tilde{b}(m;a)$, increasing in m, such that $M_C \geq M_B$ if and only if $b \geq \tilde{b}(m;a)$.

$M_C \geq M_B$ for $m < (3-2a)/(4-2a)$ if and only if $X(m, a) \geq Y(m, a)\,b$, where

$Y(m, a) = -3(1-m)\,[4a(1-a) - m(1+8a-4a^2)]$ and

$X(m, a) = -12\alpha(1-a) + m(6+34a-36a^2+8a^3) - m^2(7+22a-28a^2+8a^3)$.

$X(m, a) < Y(m, a)$ for $m \in [0, (3–2a) / (4–2a)]$ and both functions coincide at the extreme values of m. For $m = 0$ both are negative. For $m = (3–2a) / (4–2a)$ both are positive. Consequently, if $Y(m, a) < 0$, then both functions are negative, $X(m, a) / Y(m, a) > 1$, and the condition does not hold. When $Y(m, a) > 0$ and $X(m, a) < 0$, the condition holds. Finally, for $Y(m, a) > 0$ and $X(m, a) > 0$, the condition can be written as $b < X(m, a) / Y(m, a)$. After tedious calculations, it can be shown that $X(m, a) / Y(m, a)$ is increasing in m. Hence, for low values of m, Profile B is superior. Defining $X(m, a) / Y(m, a)$ as $\tilde{b}(m;a)$ we have proven the claim (given that this function is increasing in m).

(v) Profile B is never optimal for $b \geq b_D$. It is straightforward that $M_D \geq M_B$ for $m \geq (3–2a) / (4–2a)$. $M_D \geq M_B$ for $m < (3–2a) / (4–2a)$ if and only if $6a[3–2a–2(2–a)m]b < (3–2a)(3–3m–m^2)$. The term multiplying b is positive for all $m < (3–2a) / (4–2a)$. Hence, the inequality is more difficult to hold the bigger is b. After tedious calculations, it can be checked that the inequality holds for b_E.

(vi) There is a function $\hat{b}(m;a)$, increasing in m, such that $M_C > M_D$ iff $b > \hat{b}(m;a)$. $M_D \geq M_C$ if and only if $b\,Z(m, a) \geq W(m, a)$, where: $Z(m, a) = 3[m(1+8a–4a^2)–2a(3–2a)]$ and $W(m, a) = (3–2a)[m(2+8a–4a^2)–1–2a(3–2a)]$. For $m < 2a(3–2a)/(1+8a–4a^2)$, $Z(m, a)$ and $W(m, a)$ are negative. For b_E the inequality holds. For $m \in [2a(3–2a)/(1+8a–4a^2), (1+2a(3–2a)/(2+8a–4a^2)]$, $Z(m, a)$ is positive and $W(m, a)$ negative and the condition holds. For $m > (1+2a(3–2a)/(2+8a–4a^2)$, $Z(m, a)$ and $W(m, a)$ are positive. We define $\hat{b}(m;a)$ as $W(m, a) / Z(m, a)$, a function which is increasing in m, and we obtain the result.

To conclude the proof of Proposition 1, we only have to define the function $m°(b)$ from $[b_C, b_E)$ to $[0, 1]$ as the inverse of the function $\tilde{b}(m;a)$ in the interval $[b_C, b_D)$ and as the inverse of $\hat{b}(m;a)$ for the levels of m in the interval $[b_D, b_E)$. Q.E.D.

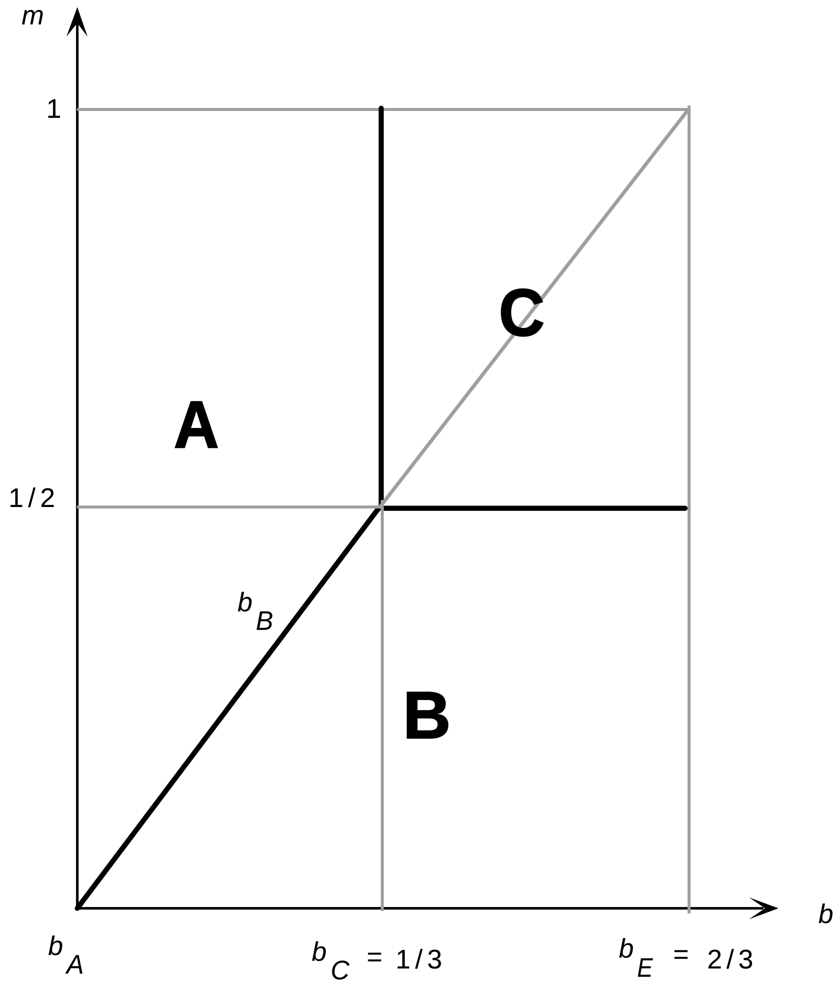

Figure 1

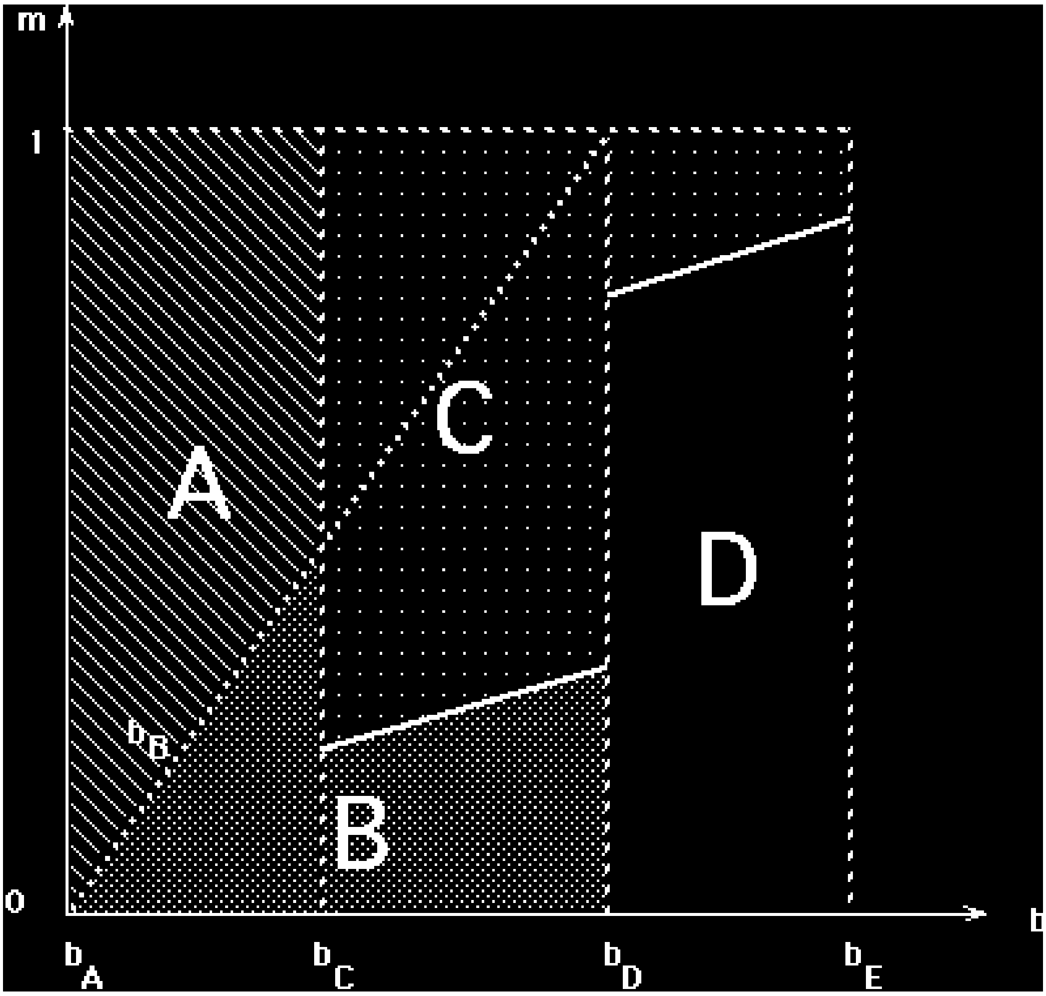

Figure 2