Victory and Defeat
in a Model of Behavior in Games and Toward Risk

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The standard expected utility model is augmented by allowing individuals to receive additional utility in states in which they consider themselves victorious and to lose a utility increment in which they consider themselves defeated. The resulting event-dependent expected utility model is used to explain behavior in games and toward risk. In games, players consider themselves defeated when their monetary payoffs are low compared to their opponents' payoffs, and they consider themselves victorious when their payoffs are high, but not too high, compared to their opponents' payoffs. Under these conditions the model can accommodate behavior that has been interpreted elsewhere as inequity aversion, as well as cooperation in the prisoner's dilemma and in public good provision games. In situations of risk, individuals consider themselves victorious (defeated) when they receive an unlikely, avoidable, high (low) outcome. Under these conditions the model can accommodate such behavior as the Allais paradox, boundary effects, and simultaneous gambling and insurance.

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1. Introduction

People like winning and dislike losing, yet this feature has never been incorporated into the study of games. This paper remedies that situation. It is assumed that besides the utility he receives from his monetary payoff, a player gets a positive utility increment in situations in which he considers himself victorious, and suffers a negative utility increment in any situation that he considers a defeat. Natural notions of victory and defeat in strategic situations allow the model to accommodate much of the experimental evidence on failures to behave purely selfishly in games, such as the behavior of responders in the ultimatum game and proposers in the dictator game, as well as allowing for cooperative equilibria in the one-shot prisoner's dilemma and a simple public good provision game. More surprisingly, though, notions of victory and defeat can be extended to decisions toward risk, allowing the model to explain many of the expected utility violations found in the experimental literature, such as the Allais paradox and boundary effects, as well as simultaneous gambling and insurance.[1] Thus, the paper constructs a single model which is able to accommodate behavioral patterns from two disparate branches of the literature.

For games, it is natural for a player to feel victorious or defeated based on how his monetary payoff compares to those of his opponents. It is assumed that the player suffers a defeat if his monetary payoff is small compared to his opponents' monetary payoffs, he experiences victory if his monetary payoff is large, but not too large, relative to his opponents', and experiences neither victory nor defeat otherwise. This assumption enables the model to accommodate all of the same behavior as models of fairness or inequity aversion, as in Bolton (1991), Rabin (1993), Fehr and Schmidt (1999), and Bolton and Ockenfels (2000).[2] Essentially, if a player's monetary payoff is either too high relative to

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[1] Boundary effects reflect the idea that people behave differently when some alternatives have different numbers of outcomes than others, as in Neilson (1992) and Harless and Camerer (1994).

[2] There are several related notions of concern for others, such as reciprocity (e.g. Sugden (1984), Falk and Fischbacher (1999), Croson (1999)), spite (e.g. Saijo and Nakamura (1995), Levine (1998)), and altruism (e.g. Andreoni (1995), Croson (1999)). The model proposed here can handle elements of these notions, but only those elements that coincide with fairness. Rabin (1993), Levine (1998), and Bolton and Ockenfels (2000) assume that a player's opponents' intentions matter, and while intentions could be incorporated into the model to determine whether the player feels victorious or defeated, intentions are not considered here.

his opponents' payoffs to lead to victory or sufficiently low to lead to defeat, he might prefer a different strategy which reduces his monetary payoff but leads to a more equitable payoff vector. By switching strategies, the player gets an extra utility boost by achieving a victory in the first case or avoiding defeat in the second.

For an individual making a decision toward risk, there are no opponents with which to compare payoffs, so different notions of victory and defeat are needed. The analysis here rests on two key assumptions. The first is avoidability - if an outcome is to be considered either a victory or a defeat, the decision-maker must have the opportunity to make some choice which would avoid that outcome. If the outcome is avoidable, then the individual must make a conscious decision *not* to avoid it. The second assumption is that the outcome must be unlikely. So, the individual experiences victory if his payoff is high, that payoff is not a sure thing, and he could have made a choice in which that payoff was impossible. He suffers a defeat if his payoff is low, it is not a sure thing, and he could have avoided the low payoff.

The model itself is based on Karni's (1992) model of event-dependent preferences. Events are subsets of the state space, and event-dependent preferences are similar to state-dependent preferences except that a single state-dependent utility function is used for all states in a given event. The model assumes that there are three events, corresponding to victory, defeat, and neutral outcomes, with the utility function corresponding to the victory event higher than the neutral utility function, which in turn is higher than the utility function corresponding to defeat. Much of the analysis is devoted to assumptions that determine when a state is considered a victory, a defeat, or neutral.

The paper proceeds as follows. Section 2 adapts event-dependent preferences based on Karni (1992) to fit notions of victory and defeat. Section 3 presents the assumptions governing when an outcome in a game is considered a victory, a defeat, or neutral, and shows that these assumptions allow the model to accommodate inequity aversion in an abstract setting. Section 4 illustrates how the model can be used to analyze specific games. Section 5 looks at behavior toward risk, providing assumptions that determine whether a state is considered a victory, a defeat, or neutral, and it also supplies assumptions that guarantee first-order stochastic dominance preference. Section 6 uses the model to explain several behavioral patterns that are inconsistent with expected utility. Sections 4 and 6 also contain comparisons of this model to other, existing models. While there are many existing models that can handle some of the evidence discussed in this paper, there are no other models that fit the evidence on games and the evidence on risk.

Finally, Section 7 offers some conclusions.

2. Event-dependent preferences

In this section, Karni's (1992) model of event-dependent preferences is introduced and extended to accommodate notions of victory and defeat. Let S be a state space with typical element s. Let $E_1,...,E_k$ be a partition of S, so that $\cup E_i = S$ and $E_i \cap E_j = \emptyset$ for $i \neq j$. Each E_i is called an event. Let $x: S \to \mathbb{R}$ be a payoff function that maps states into monetary values. Assume that the set of possible monetary values is bounded, so that payoffs lie in the interval $[x_0, x_M]$. The individual has preferences over probability distributions defined over the state space, and the preferences can be represented by the function V_s, where the subscript distinguishes the preference function from one discussed later. Following Karni (1992) the decision-maker is an expected utility maximizer with event-dependent preferences if there exist functions $u_1,...,u_k$ such that

$$V_s(F_s) = \sum_{i=1}^{k} \int_{E_i} u_i(x(s))dF_s(s), \tag{1}$$

where F_s is a probability distribution defined over states. Karni (1992) provides an axiomatic foundation for this model. The basic difference between event-dependent utility and state-dependent utility is that event-dependent utility is more restrictive. Utility can depend upon the state of the world, but the utility functions must be identical across a subset of the states. The set of utility functions is unique up to an increasing affine transformation, so that V_s and V_s^* represent identical preferences if $u_i^*(x) = au_i(x) + b$ for $i = 1,...,k$ where a is a positive scalar and b is a scalar.

For considerations of victory and defeat, it is assumed that there are just three events, E_D, E_N, and E_V, with corresponding utility functions u_D, u_N, and u_V. The decision maker considers himself victorious when a state in E_V occurs, he feels defeated when a state in E_D occurs, and he experiences neither victory nor defeat when a state in the neutral event, E_N, occurs. Whether the individual identifies a state as a victory, a defeat, or neutral depends on the nature of the decision task at hand. In particular, it depends on the game the individual is playing.[3] For example, the player might feel victorious when his

[3] This might be a game against nature, as discussed in Section 5.

monetary payoff is zero and everyone else's is negative, but the same zero payoff might be considered a defeat when all other players receive positive payoffs. To account for this, it is assumed that the events and their corresponding utility functions depend on the game. Different games lead to different conceptions of victory and defeat.

Formally, let Γ denote a monetary game, which includes the list of players, their strategy spaces, and the monetary payoffs from action combinations.[4] Let A_i denote player i's strategy space.[5] Each player i's task is to choose the optimal strategy in A_i given the strategies chosen by the other players, so that the resulting strategy combination is a Nash equilibrium.[6] A state s is a pure strategy (or action) combination, which determines the payoffs to the different players, and the probability distribution F_s captures any mixing. Assume that for each game Γ there are three events, $E_V(\Gamma)$, $E_D(\Gamma)$, and $E_N(\Gamma)$, and let $u_V(\cdot;\Gamma)$, $u_D(\cdot;\Gamma)$, and $u_N(\cdot;\Gamma)$ be the corresponding utility functions. Much of the remainder of the paper is concerned with how the events and their corresponding utility functions vary with the game. The first such assumption is that the neutral utility function u_N depends only on the payoff and not on the game, so that it is unbiased by feelings of victory or defeat. The preference function, which now depends on the game Γ as well as the probability distribution under consideration, becomes

$$V_s(F_s;\Gamma) = \int_{E_D(\Gamma)} u_D(x(s);\Gamma)dF_s(s) + \int_{E_N(\Gamma)} u_N(x(s))dF_s(s) + \int_{E_V(\Gamma)} u_V(x(s);\Gamma)dF_s(s). \qquad (2)$$

Preferences that have the representation given in (2) are referred to here as *VD preferences*, and (2) is the *VD model*.

Two assumptions govern relationships within and between events. The first states that the individual most prefers a given monetary payoff when he is victorious and least prefers it when he is defeated.

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[4] Crawford (1990) and Chen and Neilson (1999) also analyze monetary games.

[5] I depart from the usual convention of using "S" to denote the strategy space because S denotes the state space.

[6] This framework is adapted to fit decisions under risk in Section 5.

A1 – Ranking of events: For any combination of s and Γ such that $s \in E_V(\Gamma)$, $u_V(x(s);\Gamma) > u_N(x(s))$, and for any combination such that $s \in E_D(\Gamma)$, $u_N(x(s)) > u_D(x(s);\Gamma)$.

Assumption A1 states that in comparison to the neutral event, the outcome x generates more utility when it is a victory and less utility when it is a defeat. This assumption, combined with assumptions about when a state is a victory or a defeat, allows the model to accommodate a wide variety of behavioral patterns. The second assumption states simply that each of the event-dependent utility functions is nondecreasing.

A2 – Monotonicity within events: For any Γ, $u_D(x;\Gamma)$, $u_N(x)$, and $u_V(x;\Gamma)$ are all nondecreasing in x.

It is assumed throughout the remainder of the paper that conditions A1 and A2 are satisfied.

3. Victory and defeat in strategic settings

The analysis begins with strategic situations, in which natural notions of victory and defeat involve a comparison of the player's monetary payoff with his opponents' payoffs. Three assumptions are used to govern whether a state is considered a victory, a defeat, or neutral. To state these, let Γ be an n-player game and let the state s be a pure strategy combination defining the state of the world, as above. Let $x(s)$ be the individual's own (monetary) payoff when the state is s, as before, and let $y(s)$ be the n-1 vector of the other player's (monetary) payoffs. Whether or not a player considers an outcome of the game a victory or a defeat depends on $x(s)$ and $y(s)$. However, since $x(s)$ and $y(s)$ have different dimensions, a further step is needed to compare them.

The function $?:\mathbb{R}^m \to \mathbb{R}$ is an *evaluation rule* if $\min\{y_1,...,y_m\} \leq ?(y) \leq \max\{y_1,...,y_m\}$. So, an evaluation rule takes a vector and returns a value between the highest and lowest components of the vector. The evaluation rule ? is said to be *monotone* if $?(y^*) \geq ?(y)$ whenever $y^* \geq y$ component-wise, i.e. $y_i^* \geq y_i$ for $i = 1,...,m$, so that when the components of the evaluated vector increase, the evaluation increases.[7]

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[7] Sarin (2000) introduces monotone evaluation rules for the study of learning in games. The evaluation rules used here are different from his because here they must be

Examples of monotone evaluation rules are any weighted average with fixed weights and any order statistic such as the minimum, maximum, or median.

The events in which a player experiences victory or defeat are characterized using monotone evaluation rules. The availability of different evaluation rules allows for considerable flexibility in determining what is a victory and what is a defeat. For example, an individual might consider it a victory if his own monetary payoff is above the average of his opponent's payoffs, and this can be characterized as $s \in E_V(G)$ if $x(s) \geq ?(y(s)) = \Sigma y_i(s)/(n-1)$. For a second example, an individual might consider it a victory if his own monetary payoff is above everyone else's, in which case $s \in E_V(G)$ if $x(s) \geq ?(y(s)) = \max\{y_1(s),...,y_{n-1}(s)\}$. Finally, a player might consider it a defeat if his is the lowest of the payoffs, in which case $s \in E_D(G)$ if $x(s) \leq ?(y) = \min\{y_1(s),\ldots,y_{n-1}(s)\}$. All of these examples fit within the structure imposed by the next set of assumptions.

The first assumption governs when a strategy combination is considered a victory for the player under consideration.

> **S1 – Minimum victory threshold:** There exists a monotone evaluation rule $?_{min}(y)$ and a scalar $z_{min}(G) < \infty$ such that if $s \in E_V(G)$ then $x(s) - ?_{min}(y(s)) \geq z_{min}(G)$.

This assumption states that for an outcome to be a victory, the player's earnings must exceed the evaluation of his opponents' earnings by at least z_{min}. So, for example, if the evaluation rule $?_{min}$ is the average of the opponents' payoffs, for an outcome to be a victory the player's payoff must be at least as great as the average of his opponents' payoffs. Note that z_{min} could be negative, so that the player does not necessarily need a higher monetary payoff than his opponents to feel victorious. Also note that S1 implicitly assumes that the evaluation rule is independent of the game, so that if the player uses the average of his opponents' payoffs as the point of comparison in one game, he uses it in all games.

The second assumption governs uneven distributions of payoffs when the distribution favors the player under consideration. Assumption S1 states that to be considered a victory, the individual's payoff must be high relative to his opponents' payoffs. The next assumption states that if the his payoff is too high relative to his opponents' average payoff, he does not feel victorious.

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bounded by the lowest and highest components of the vector being evaluated.

S2 – Maximum victory threshold: There exists a monotone evaluation rule $?_{max}(y)$ and a scalar $z_{max}(G)$ such that if $x(s)$! $?_{max}(y(s)) > z_{max}(G)$ then s 0 $E_N(G)$.

The assumption posits the existence of a maximal payoff difference above which the individual no longer feels victorious. The idea is that a win that is too lopsided is not a victory, and it could reflect either inequity aversion or feelings of guilt. Of course, if $z_{max}(G)$ is sufficiently large, this effect has no consequence for behavior.

The third assumption deals with defeat.

S3 – Minimum neutral threshold: There exists a monotone evaluation rule $?_D(y)$ and a scalar $z_D(G)$ such that if $x(s)$! $?_D(y(s))$ # $z_D(G)$ then s 0 $E_D(G)$.

The idea behind this assumption is straightforward – if the player's payoff is too low compared to his opponents' payoffs, he considers it a defeat.

Assumptions S1 – S3 identify three, possibly different monotone evaluation rules for determining events. This allows for quite a bit of flexibility in modeling behavior, and the identification of the evaluation rules is an interesting question for further study. Most of the experimental evidence in the next section concerns two-person games, in which case there is a unique evaluation rule, $?(y) = y$. For games with more than two players, other evaluation rules are possible. For example, a player might use the minimum for $?_D$, the average for $?_{min}$, and the maximum for $?_{max}$. Under these conditions the player feels defeated if his own payoff is too far below the lowest of his opponents' payoffs, and he feels victorious if his own payoff compares favorably to the average of his opponents' payoffs but is not too high compared to the highest of his opponents' payoffs.

There are good reasons for assuming that all three evaluation rules are the same. First, as noted above, when there is only one opponent there is only one evaluation rule. Second, other studies of players' concern for others use only one evaluation rule. Notably, Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) use a comparison of the player's own payoff to the average of his opponents' payoffs in their models of inequity aversion.[8] The first lemma shows how the setting is simplified when the same evaluation rule is used for all three thresholds.

Lemma 1. If $?_D = ?_{min} = ?_{max} = ?$ and if $z_D(G) \# z_{min}(G) \# z_{max}(G)$ then

$$E_D(G) = \{s|\ x(s)\ !\ ?(y(s)) \# z_D(G)\},$$

$$E_V(G) = \{s|\ z_{min}(G) \# x(s)\ !\ ?(y(s)) \# z_{max}(G)\}, \text{ and}$$

$$E_N(G) = \{s|\ z_D(G) < x(s)\ !\ ?(y(s)) < z_{min}(G)\}\ \text{c}\ \{s|\ x(s)\ !\ ?(y(s)) > z_{max}(G)\}.$$

According to Lemma 1, if the difference between a player's own payoff $x(s)$ and his evaluation of the vector of his opponents' payoffs $?(y(s))$ is low, the player feels defeated, and when it is high but not too high he fells victorious. There are two reasons why an outcome might be considered neutral. First, it could be that $z_D < x(s)\ !\ ?(y(s)) < z_{min}$, so that the individual's payoff is too high to be considered a defeat but too low to be considered a victory. Second, it could be that $x(s)\ !\ ?(y(s)) > z_{max}$, so that the individual's payoff is too high compared to his opponents' payoffs. Note that these too-high payoffs can result in a neutral state, but not a defeat.

One notion of concern for others that has garnered increasing attention over the years is fairness or inequity aversion (see Fehr and Schmidt (1999) and Bolton and Ockenfels (2000)). Basing their arguments on experimental evidence from ultimatum and other games, researchers have posited that players' preferences exhibit inequity aversion in the sense that they sometimes choose strategies that reduce their own monetary payoffs but result in more equitable monetary allocations across players. It can be shown that if VD preferences satisfy assumptions S1 - S3 they are consistent with inequity aversion.

Not all games provide an opportunity for players to exhibit inequity Let G be a game, let A_i denote player i's strategy set, let $x(a_i, a_{!i})$ denote the monetary payoff of player i when he plays strategy a_i and his opponents play the strategy vector $a_{!i}$, and let $y(a_i, a_{!i})$ denote the vector of monetary payoffs received by his opponents under the same

[8] In Fehr and Schmidt's (1999) model players care about the difference between their payoffs and the average of their opponents' payoffs, while in Bolton and Ockenfels' (2000) model players care about the ratio.

strategy combination. Given the evaluation rule $?(y)$, define $z(a_i, a_{-i}) = x(a_i, a_{-i}) - ?(y(a_i, a_{-i}))$. Let $(G, ?, a_{-i})$ be a triple consisting of a game, a monotone evaluation rule, and a strategy vector for player *i*'s opponents.

Definition. $(G, ?, a_{-i})$ *potentially reveals inequity aversion for player i* if there exist strategies a_i^{**} and a_i^{*} such that

(i) $x(a_i^{**}, a_{-i}) \geq x(a_i, a_{-i})$ for all $a_i \in A_i$, and
(ii) $|z(a_i^{**}, a_{-i})| < |z(a_i^{*}, a_{-i})|$.

The idea behind this definition is that given the game and his opponents' strategies, the player under consideration has two strategies of interest. Strategy a_i^{**} is the best response in purely monetary terms, while a_i^{*} pays less in monetary terms but results in less inequity as measured by the payoff difference z. In such a setting, it is possible for player i to choose a_i^{*} over a_i^{**}, thereby trading some amount of money for decreased inequity. If so, he *exhibits inequity aversion*.

Definition. A class of preferences *accommodates inequity aversion* if for any triple $(G, ?, a_{-i})$ that potentially reveals inequity aversion for player *i* and any pair of strategies a_i^{**} and a_i^{*} satisfying (i) and (ii) above, there exists a member of that class of preferences for which a_i^{*} is chosen over a_i^{**}.

Note that the class of standard, selfish preferences cannot accommodate inequity aversion, because purely selfish players always choose the selfish best-response strategy a_i^{**}. As the next proposition shows, the class of VD preferences can accommodate inequity aversion.

Proposition 1. The class of VD preferences that satisfies S1 - S3 accommodates inequity aversion.

Proof. Suppose that $(G, ?, a_{-i})$ potentially reveals inequity aversion, and let a_i^{**} and a_i^{*} be as in the definition. It suffices to find the threshold levels $z_D(G)$, $z_{min}(G)$ and $z_{max}(G)$, as in Lemma 1, that lead to a preference for a_i^{*} over a_i^{**}. Assume first that $z(a_i^{**},a_{-i}) < z(a_i^{*},a_{-i})$. If $z(a_i^{**},a_{-i}) \leq z_D(G) < z(a_i^{*},a_{-i}) < z_{min}(G)$, then playing a_i^{**} results in a defeat but playing a_i^{*} results in a neutral outcome. If $u_D(x(a_i^{**},a_{-i})) \leq u_N(x(a_i^{*},a_{-i}))$, player i chooses a_i^* in response to a_{-i}.

Now assume instead that $z(a_i^{**},a_{-i}) > z(a_i^*,a_{-i})$. Set $z_{min}(G) \leq z(a_i^*,a_{-i}) \leq z_{mzx}(G) < z(a_i^{**},a_{-i})$. Playing a_i^* results in a victory while playing a_i^{**} results in a neutral outcome. If $u_V(x(a_i^*,a_{-i})) \leq u_N(x(a_i^{**},a_{-i}))$, player i chooses a_i^* in response to a_{-i}. **9**

The VD model accommodates inequity aversion through a different mechanism than most models, such as those proposed by Fehr and Schmidt (1999) and Bolton and Ockenfels (2000). In those models opponents' payoffs enter directly into the player's utility function, and a small increase in the opponents' payoffs has a small effect on the player's utility. Here, in contrast, opponents' payoffs only matter for determining the events. A small increase in the opponents' payoffs can either lead to a different event, thereby causing a large change in utility, or it can lead to the same effect, in which case there is no change in utility. Thus, The VD model treats opponents' payoffs much more discretely than existing models.

4. Behavior in some common games

The model of behavior in games constructed in the preceding sections is able to explain patterns of play observed in a variety of settings, including the behavior of the proposer in dictator games, the responder in ultimatum games,[9] players in a one-shot prisoner's dilemma, and players in a public good contribution game. Of course, the first two of these games potentially reveal inequity aversion, so the results follow from Proposition 1. Nevertheless, the analysis illustrates the usefulness of the VD model in games.

Dictator game

In the dictator game, proposer is assigned the task of splitting a prize of size k. Let x denote the amount he keeps for himself, so that he gives $k - x$ to his opponent. While standard game-theoretic analysis predicts that $x = k$, experimental studies show that proposer tends to give away about 20% of the pot, consistent with $x = 0.8k$.

Proposer gives away money if two things happen. First, keeping the entire pot of

[9] Analyzing the behavior of the proposer in ultimatum games involves analyzing responses to beliefs about the behavior of the responder, which moves beyond the main point of this paper.

k must be inconsistent with victory. This occurs if $z_{max} < k$.[10] If, in addition, $z_{max} > 0$, it is possible for proposer to feel victorious and the highest monetary payoff consistent with the victory is $(z_{max} + k)/2$. When this offer is made, the other player gets $(k - z_{max})/2$ and the payoff difference is z_{max}. For proposer to be willing to give away part of the pot, $u_N(k)$, the neutral utility from keeping k, must be below $u_V((z_{max} + k)/2)$, the victory utility of keeping the smaller amount $(z_{max} + k)/2$. Thus, two conditions are required for proposer to give away a positive amount of money in the dictator game, as seen in the following proposition, whose proof is obvious.

Proposition 2. In the dictator game, proposer keeps an amount $x < k$ if $0 < z_{max} < k$ and $u_V((z_{max} + k)/2) \geq u_N(k)$.

The basic idea behind this result is that the proposer considers it a victory if he earns a higher payoff than the responder, as long as it is not too much higher. If his payoff is too high, he no longer considers it a victory, and so he keeps the largest amount consistent with victory.

Note that proposer's generosity is governed entirely by the parameter z_{max}, and that experimental evidence that proposer gives away 20% of the pot is consistent with $z_{max} = 0.6k$.

Ultimatum game

In the ultimatum game, proposer is assigned the task of splitting a prize of size k. He offers to give x to responder and keep $k - x$ for himself. Responder can either accept the offer or reject it. If she accepts, the payoffs are as proposed. If she rejects, both players get zero. From responder's point of view, accepting leads to $z_1 = 2x - k$, while rejecting leads to $z_2 = 0$. Experiments show that responders often reject offers of less than about 20% of the pot.

Unlike the proposer's decision in the dictator game, the behavior of the responder in the ultimatum game is driven by the parameter z_D, which reflects the (negative) payoff difference below which the individual feels defeated. The next proposition shows that in the VD model responders reject offers that are too low.

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[10] Since there are only two players, the only evaluation rule is $?(y) = y$, where y is the opponent's monetary payoff.

Proposition 3. In the ultimatum game, if $-k < z_D < 0 < z_{min}$ responder rejects an offer of x if and only if $x \leq (k + z_D)/2$ and $u_D(x) < u_N(0)$.

Proof. By hypothesis, rejecting an offer leads to a neutral outcome for responder. So, utility from rejecting is $u_N(0)$. First suppose that $x \leq (k + z_D)/2$ and $u_D(x) < u_N(0)$. The first inequality guarantees that $z = x - 2k \leq z_D$, so that responder feels defeated if she accepts. The second inequality states that accepting the offer generates less utility than rejecting it does, so she rejects the offer.

Now suppose that one of the two inequalities fails. If $x > (k + z_D)/2$, responder does not experience defeat when she rejects the offer, and $u_N(x) > u_N(0)$.If, instead, x is considered a defeat but $u_D(x) \geq u_N(0)$, she does not reject the offer of x, regardless of whether it is considered neutral or a defeat, because accepting leads to higher utility. ∎

The basic idea behind this result is that responder considers it a defeat if her earnings are too far below proposer's earnings, while she considers the rejection outcome to be neutral. If she finds the zero monetary payoff in a neutral setting more attractive than the offered payoff in a defeat setting, she rejects the offer.

Proposition 3 implies that if $z_D < 0$, responder accepts any offer of $k/2$ or more. Also, if $z_D \leq -k$, responder accepts any positive offer. Experimental evidence that responder rejects offers of about $0.2k$ or less is consistent with $z_D = -0.6k$.

Prisoner's dilemma

Consider the following version of the prisoner's dilemma.

	C	D
C	3, 3	0, 4
D	4, 0	2, 2

Standard game-theoretic analysis prescribes that both players choose the dominant strategy *D*, and this is the unique equilibrium. The VD model allows other equilibria as well. In particular, under appropriate conditions there exists a Nash equilibrium in which both players play *C*.

Proposition 4. If, for both players, $z_{min} \leq 0 < z_{max} < 4$ and $u_V(3) \geq u_N(4)$, then the strategy combination (C,C) is a Nash equilibrium.

Proof. Since $z_{min} \leq 0 < z_{max}$, both on-diagonal outcomes are considered victories by both players. Since $z_{max} < 4$, the off-diagonal outcomes are considered neutral by the player with the higher monetary payoff. If his opponent plays C, a player receives utility $u_V(3)$ from playing C and utility $u_N(4)$ from playing D, and, by the hypothesis, prefers to play C. $\blacksquare$

The basic idea behind this result is that the players consider it a victory if they both get the same payoff, but not if one player gets a much larger payoff than the other. While defecting has a higher monetary payoff than cooperating when the opponent cooperates, the extra utility from feeling victorious is enough to compensate and remove any incentive for defecting.

The requirements for cooperation in the prisoner's dilemma as presented above and the requirements for contributions in the dictator game are remarkably consistent. In the dictator game the maximal payoff difference is k, and behavior is consistent with $z_{max} = 0.6k$. In the prisoner's dilemma the maximal payoff difference is $k = 5$, and when $z_{max} = 0.6k$ there is an equilibrium in which both players cooperate.

Public good provision

Suppose that n individuals are endowed with some amount $e > 0$, and each player i can split his endowment between consumption a_i and contributions b_i to a public good. Letting $b = \Sigma b_i$, each individual receives benefit gb from the public good, regardless of his own contribution, with $g < 1 < ng$. Thus, his total "monetary" payoff is $a_i + gb$. Since $g < 1$, in the unique Nash equilibrium each individual contributes $b_i = 0$. But, if every player contributed some positive amount, they would all be better off. This is the standard free-riding problem.

The VD model provides a way around this free-riding problem. In particular, under appropriate parameter conditions there exist other Nash equilibria besides the free-riding equilibrium.

Proposition 5. If all players are identical with $z_{max} = 0$, then for every $0 \leq \beta \leq e$ such that $u_V(e - \beta + ng\beta) \geq u_N(e + (n-1)g\beta)$ there exists a Nash equilibrium in which every player contributes β.

Proof. Suppose the other $n - 1$ players each contribute β. This means that all of the other players receive identical payoffs, so that all monotone evaluation rules for the player under consideration yield the same evaluation of his opponents' payoffs. If the player under consideration contributes ß, all *n* players have the same monetary payoff of *e* ! ß + *ng*ß. If, instead, he contributes *b* < ß, his monetary payoff is *e* ! *b* + (*n* ! 1)*g*ß + *gb* and the other *n* ! 1 players each get *e* ! ß + (*n* ! 1)*g*ß + *gb*. Thus, when he contributes ß, $z = 0$ and he considers it a victory, but when he contributes less than ß, $z >$ 0 and he considers the outcome neutral. His highest possible neutral payoff, given that all of the other players contribute ß, is *e* + (*n* ! 1)*g*ß, which corresponds to a contribution of zero. By the hypothesis, he prefers to contribute ß. **9**

As with the dictator game, all of the action in the public good provision game is determined by the parameter z_{max}, which demarks a payoff-difference threshold below which the player considers himself victorious but above which he considers the outcome neutral. In the public good game with identical players, contributions occur in Nash equilibrium when players have an extremely strong notion of fairness so that they only consider themselves victorious if they have contributed at least as much to the public good as the average player.

Discussion

A number of other models of preferences in games can explain this same evidence. Bolton (1991), Rabin (1993), Fehr and Schmidt (1999), and Bolton and Ockenfels (1999), all construct models in which individuals dislike inequitable allocations.[11] Here these notions are captured by the parameters z_D and z_{max}. When a player earns less than z_D below his opponents' average earnings, he feels defeated. This effect makes strategies which can yield neutral outcomes more appealing, and these neutral outcomes must have more equitable allocations. When a player earns z_{max} above his opponents' average earnings, he considers himself victorious, but if he earns more than z_{max} above his opponents' average earnings, he considers the outcome neutral. This effect makes strategies that generate more equitable allocations more appealing. The major difference between the VD model and the existing models of fairness or inequity aversion is that in the

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[11] Rabin (1993), Levine (1998), and Bolton and Ockenfels (1999) all assume that inequity aversion depends not only on the payoff allocation, but also on the intentions of the other players. This added consideration could be incorporated into the VD model, but is not here.

existing models payoff differences enter directly into the player's utility function, with players willing to trade small decreases in their own payoffs for small decreases in payoff differences. In the VD model, in contrast, payoff differences enter the utility function more discretely.

Sugden (1984), Croson (1999), and Falk and Fischbacher (1999) consider notions of reciprocity. Positive reciprocity entails rewarding kindness, while negative reciprocity means punishing unkindness.[12] In the VD model, negative reciprocity is governed by the parameter z_D. If a player's opponent plays a strategy which helps the opponent at his expense, causing the payoff difference to fall below z_D, strategies that hurt the opponent but make the payoffs more equal become more attractive. A good example is the propensity of responders to reject low offers in the ultimatum game, as discussed above. Positive reciprocity can be discussed using the prisoner's dilemma. If his opponent cooperates, putting himself at risk, if z_{max} is too high the player under consideration does not find it attractive to defect, since this would punish his opponent's kind action. Thus, the VD model builds in tastes for reciprocal behavior, but only through tastes for inequity aversion.

5. Victory and defeat in risky choice

In this section the event-dependent preferences discussed in Section 2 are used to incorporate notions of victory and defeat into expected utility theory. Since the VD model was originally developed in the context of games against other players, it is necessary to transform the model to allow for games against nature. In these settings, only the payoff the individual receives and the set of payoffs he might have received determine the events. The remainder of this section is devoted to placing restrictions on the events and the utility functions, and these restrictions are then used in the next section to discuss evidence on risky decisions.

Let S denote a state space with typical element s, as before, and let $x(s)$ be the payoff the individual receives from his choice when the state of the world is s. Let F_S be a probability distribution over states, and let A be the individual's *choice set*, that is, the set of probability distributions among which the individual chooses. For considerations of risky decisions, the choice set A replaces the game G in specifications of the events and the event-dependent utility functions.

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[12] Spite is closely related to negative reciprocity. See ...

The first restriction on preferences states simply that if a given state is considered a victory, any state that yields a higher payoff is also considered a victory. Similarly, if a state is considered a defeat, any state that yields a lower payoff is also a defeat.

R1 — Monotonicity across events: If $s \in E_V(A)$ and $x(s') \geq x(s)$ then $s' \in E_V(A)$, and if $s \in E_D(A)$ and $x(s') \leq x(s)$ then $s' \in E_D(A)$.

Besides being plausible, assumption R1 allows events to be characterized as payoff intervals.

Proposition 6. If condition R1 holds, there exist payoffs $x_D(A)$ and $x_V(A)$ such that $E_D(A) = \{s| \, x(s) \leq x_D(A)\}$, $E_N(A) = \{s| \, x_D(A) < x(s) < x_V(A)\}$, and $E_V(A) = \{s| \, x_V(A) \leq x(s)\}$.

Proof. Let $x_D(A) = \sup\{x(s)| \, s \in E_D(A)\}$. Then by R1, $E_D(A) = \{s| \, x(s) \leq x_D(A)\}$. Similarly, let $x_V(A) = \inf\{x(s)| \, s \in E_V(A)\}$. Then by R1, $E_V(A) = \{s| \, x(s) \geq x_D(A)\}$. Since $x_D(A) \in x(E_D(A))$, $x_V(A) \in x(E_V(A))$, and $E_D(A)$ and $E_V(A)$ are disjoint, it must be the case that $x_D(A) < x_V(A)$. Then $E_N(A) = \{s| \, x_D(A) < x(s) < x_V(A)\}$.

Let $F(x)$ be the probability distribution over payoffs induced by the distribution function $F_s(s)$ defined over states. Since both events and probability distributions can be defined in terms of payoffs instead of states, the preference function V_s can be, as well. Let

$$V(F;A) = \int_{x_0}^{x_D(A)} u_D(x;A)dF(x) + \int_{x_D(A)}^{x_V(A)} u_N(x)dF(x) + \int_{x_V(A)}^{x_n} u_V(x;A)dF(x), \qquad (3)$$

where x_0 and x_m are the lowest and highest possible payoffs, respectively, and the events and utility functions are assumed to depend on the choice set, as in (2). By construction, $V(F^*) \geq V(F)$ if and only if $V_s(F_s^*) \geq V_s(F_s)$ where F^* and F are the probability distributions over payoffs induced by the probability distributions over states, F_s^* and F_s, respectively. Thus, V represents the individual's preferences. For the remainder of the discussion of preferences toward risk, states will be ignored and events will be treated as payoff intervals.

Now define $u(x;A) = u_i(x;A)$ on $E_i(A)$. Then $V(F;A) = \int u(x;A)dF(x)$, which is a standard expected utility representation, albeit with a complicated utility function, and all of the standard results from expected utility theory can be extended to this setting. Most importantly, results about first-order-stochastic dominance (FOSD) preferences can be established under conditions A1 and A2 in Section 2, which state that $u_V(x;A) \geq u_N(x) \geq u_D(x;A)$ for all x and A and that $u_i(x;A)$ is increasing in x for all A, respectively.

Proposition 7. Under conditions R1, A1, and A2, preferences exhibit FOSD preference.

Proof. Under R1, it is enough to show that the function $u(x)$ is increasing. Suppose that $x^* > x$. There are two cases. First, if both x^* and x are in the same event E_i, then A2 implies that $u(x^*;A) = u_i(x^*;A) \geq u_i(x;A) = u(x;A)$. Alternatively, if $x \in E_i(A)$ and $x^* \in E_j(A)$ with $i \neq j$, then A1 and A2 together imply that $u(x^*;A) = u_j(x^*;A) \geq u_i(x;A) = u(x;A)$. ∎

This proposition states that if the conditions A1, A2, and R1 hold, the decision-maker will only choose from among the *undominated* alternatives in A, where $F \in A$ is undominated in A if there is no $F' \in A$ such that F' FOSD F.

Now turn attention to the issue of determining when a payoff constitutes either a victory or a defeat. First, for an outcome to be considered either a victory or a defeat, it must have resulted from a purposeful choice; an inevitable outcome does not lead to the extra utility boost. This leads to the next assumption.

> **R2 — Avoidability:** Payoffs in $E_V(A)$ and $E_D(A)$ must be *avoidable*; that is, given A, there exists an undominated distribution F in A such that $F(x_D(A)) = 0$, and there exists an undominated distribution F^* in A such that $F(x_V(A)) = 1$.

Take the defeat case first. Defeats occur when the decision-maker suffers a low outcome. But, if there was nothing he could have done to avoid this outcome, the outcome is not considered a defeat. Put another way, defeats cannot be caused by bad luck; instead, they must result, at least in part, from a conscious decision. So, the defeat occurs when the decision-maker suffers a low outcome after choosing not to take a "safe" option which would have made the low outcome impossible.

The reasoning for victories is similar. A victory occurs when the decision-maker obtains a high payoff, but a high-payoff alone is not sufficient. To feel victorious, the decision-maker must have passed up a safe option which would have precluded the high outcome.

The other restriction on preferences is that victories and defeats must be unlikely:

R3 — Unlikeliness: Payoffs in $E_V(A)$ and $E_D(A)$ must be *unlikely*; that is, there exist $p_D(A), p_V(A) \in (0,1)$ such that for all undominated $F \in A$, $F(x_D(A)) \leq p_D(A)$ and $F(x_V(A)) \geq 1 - p_V(A)$, with $p_D(A) + p_V(A) < 1$.

If someone plays a lottery in which he can win \$1000 with probability 0.99 and win \$0 otherwise, winning \$1000 is unlikely to make him feel victorious. But, winning \$0 is likely to make him feel defeated. Condition R4 places bounds on how likely victories and defeats can be, with the provision that some payoffs must be considered neutral.

Conditions R2 and R3 combined state that if a payoff x is considered a victory by the decision maker, there must be some undominated probability distribution in the choice set for which the highest payoff is less than x, so that x is avoidable, and there must be no undominated probability distribution in the choice set for which the probability of receiving at least x is greater than p_V, so that x is unlikely. If the payoff y is considered a defeat, there must be some undominated probability distribution for which the lowest payoff is above y, so that y is avoidable, and there must be no undominated probability distribution for which the probability of receiving y or less is greater than p_D, so that y is unlikely.

The final assumption concerns the shapes of the utility functions. The neutral utility function u_N, which is independent of the choice set, is assumed to be S-shaped as in Kahneman and Tversky (1979), so that it is risk averse over gains and risk seeking over losses.

R4 — Diminishing sensitivity. $u_N(x)$ is concave when $x > 0$ and convex when $x < 0$, and $u_N(0) = 0$.

The resulting utility function is shown in Figure 1. Assumption R4 holds particular importance for the issue of gambling and insurance in the next section. In particular, without victory and defeat considerations, an expected utility maximizer satisfying R4 would never take a fair gamble over gains and would never purchase fair insurance against losses.

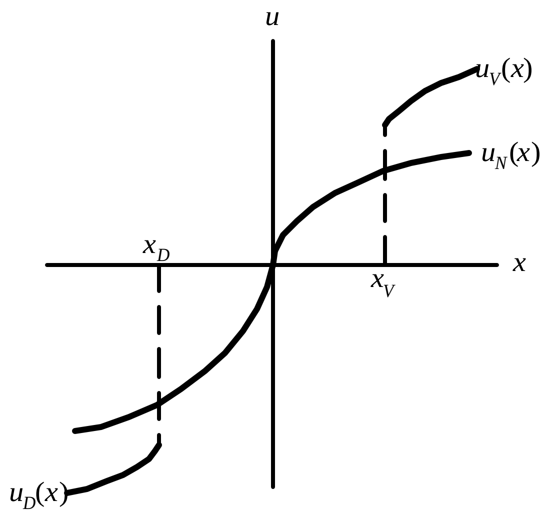

Figure 1

6. Evidence on risky choice

In this section it is demonstrated that the VD model is able to accommodate several important choice patterns that have been discussed in the literature. In particular, the discussion includes the Allais paradox, Conlisk's (1989) variants of the Allais paradox including boundary effects, and simultaneous gambling and insurance.

Table 1

	Probabilities									
	Choice set=A_1		A_2		A_3		A_4		A_5	
Payoff	$\boldsymbol{F_1^*}$	$\boldsymbol{F_2}$	$\boldsymbol{F_3}$	$\boldsymbol{F_4^*}$	$\boldsymbol{G_1}$	$\boldsymbol{G_2^*}$	$\boldsymbol{H_1}$	$\boldsymbol{H_2^*}$	$\boldsymbol{H_3}$	$\boldsymbol{H_4^*}$
$5M	0	.10	0	.10	.88	.98	.10	.20	.10	.20
$1M	1	.89	.11	0	.11	0	.89	.78	.19	.08
$0	0	.01	.89	.90	.01	.02	.01	.02	.71	.72

Asterisks denote the modal choices in the pairs.

The Allais paradox

The Allais paradox involves the two pairs of choices A_1 and A_2 in Table 1. It arises because individuals typically prefer F_1 to F_2 in A_1, but prefer F_4 to F_3 in A_2, and this choice pattern is inconsistent with standard expected utility specifications. The VD model can accommodate the Allais pattern. First, note that when the choice set is $A_1 = \{F_1, F_2\}$, the outcome \$0 can be considered a defeat, because it is both avoidable and unlikely. The payoff \$5M may or may not be considered a victory. To make F_2 as attractive as possible, \$5M will be treated as a victory when the choice set is A_1. Thus, the individual chooses F_1 over F_2 iff

$$u_N(\$1\text{M}) \geq .01u_D(\$0; A_1) + .89u_N(\$1\text{M}) + .10u_V(\$5\text{M}; A_1), \tag{4}$$

assuming that \$5M is considered a victory. When the choice set is $A_2 = \{F_3, F_4\}$, \$0 is no longer avoidable, so it cannot be considered a defeat. The outcome \$5M is both avoidable and unlikely, while the outcome \$1M is not avoidable. Thus, \$1M is neutral, while \$5M may or may not be a victory. To make F_3 as attractive as possible, \$5M is treated as neutral when the choice set is A_2. The decision maker chooses F_4 over F_3 iff

$$.90u_N(\$0) + .10u_N(\$5\text{M}) \geq .89u_N(\$0) + .11u_N(\$1\text{M}). \tag{5}$$

Simplifying and combining (5) and (6), the individual chooses F_1 and F_4 if[13]

$$.01u_D(\$0; A_1) + .10u_V(\$5\text{M}; A_1) \leq .11u_N(\$1\text{M}) \leq .01u_N(\$0) + .10u_N(\$5\text{M}). \tag{6}$$

The intuition behind (6) fits exactly that originally given by Allais (1953) to explain his paradox. In the choice between F_1 and F_2, the former is chosen because receiving \$0 would be very bad. This is captured by \$0 being considered a defeat in the left side of the expression. In the choice between F_3 and F_4, the decision maker will probably get \$0 anyway, so he might as well choose F_4 and go for the \$5M. This is captured by \$0 being considered neutral in the right side of the expression.

Conlisk's displaced Allais paradox

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[13] Similar conditions could be found if \$5M is considered neutral in one or both of the choice pairs. The key to the result lies in treating \$0 as a defeat in A_1 but as neutral in A_2.

Conlisk (1989) considers two variants of the Allais questions. The first variant consists of having subjects choose between F_1 and F_2 in choice set A_1 in Table 1, and also between G_1 and G_2 in choice set A_3. The first choice is governed by expression (4) above. In the second choice neither of the extreme outcomes is avoidable, so the neutral utility function is used for all of the outcomes. Conlisk finds that a majority of subjects prefer G_2 to G_1, which is implied by

$$0.98u_N(\$5M) + 0.02u_N(\$0) \geq 0.88u_N(\$5M) + 0.11u_N(\$1M) + 0.01u_N(\$0). \quad (7)$$

This reduces to $0.10u_N(\$5M) + 0.01u_N(\$0) \geq 0.11u_N(\$1M)$, which also implies the choice of F_4 over F_3 in (5). So, condition (6) above implies Conlisk's displaced Allais behavior as well as the original Allais behavior.

Boundary effects

The distributions in A_1, A_2, and A_3 in Table 1 all lie along the boundary of the probability triangle.[14] Conlisk (1989) also gave subjects choices that moved the original Allais distributions off of the boundary. Notice that the movement from H_1 to H_2 is the same as the movement from F_1 to F_2 in the Allais paradox, removing mass 0.11 from the intermediate outcome and adding mass 0.10 to the high outcome and mass 0.01 to the low outcome. Similarly, the movement from H_3 to H_4 is the same as the movement from F_3 to F_4 in the Allais paradox.

The choices in this problem differ from the Allais choices in an important dimension, though. In none of the alternatives is the individual able to avoid any of the outcomes; all of the probabilities are strictly positive. Since none of the outcomes are avoidable, the neutral utility function u_N is used for all of the outcomes, and the individual must choose either H_1 and H_3 or he must choose H_2 and H_4. This is what Conlisk (1989) finds, with 66 of the 215 subjects choosing H_1 and H_3, and 81 choosing H_2 and H_4. So, 68% of the subjects' choices were consistent with the VD model. Once again assuming that u_N is independent of the choice set, the modal choice, H_2 and H_4, is implied by the

[14] When there are three fixed outcomes, the set of lotteries with those three outcomes is the two-dimensional simplex given by $\{(p_1,p_2,p_3) \in \mathbb{R}_+^3 | \, p_1 + p_2 + p_3 = 1\}$. This is the probability triangle. The boundary is the set where at least one of the three probabilities is zero. Consequently, the distributions F_1, F_3, F_4, G_1, and G_2 in Table 1 all lie on the boundary of the probability triangle.

condition $0.10u_N(\$5\text{M}) + 0.01u_N(\$0) \geq 0.11u_N(\$1\text{M})$, which is identical to the condition (5) for choosing F_4 over F_3 and condition (7) for choosing G_2 over G_1.

An ability to predict the choices of F_1 over F_2 but H_2 over H_1 is important. Typically this choice pattern has been labeled a "certainty" or "boundary" effect, and in their meta-study Harless and Camerer (1994) conclude from the evidence of boundary effects that expected utility works well when the number of outcomes with positive probability is constant, but not when the number of probable outcomes changes. While the VD model does not make exactly this prediction, it comes close. As the next proposition shows, the VD model coincides with the standard expected utility model when all distributions under consideration have the same support, but that Allais-type violations can occur when distributions have different supports, as with the Allais paradox.

Proposition 8. Let $A_1 = \{F_1, F_1^*\}$ and $A_2 = \{F_2, F_2^*\}$, with $F_1 - F_1^* = F_2 - F_2^*$ and $\text{supp } F_1 \subseteq \text{supp } F_1^*$ and $\text{supp } F_2 \subseteq \text{supp } F_2^*$. Then all VD preference maximizers choose either F_1 and F_2 or F_1^* and F_2^* if and only if $\text{supp } F_1 = \text{supp } F_1^*$ and $\text{supp } F_2 = \text{supp } F_2^*$.

Proof. Suppose that $\text{supp } F_1 = \text{supp } F_1^*$ and $\text{supp } F_2 = \text{supp } F_2^*$. Then no outcomes are avoidable, and the utility function u_N is used to evaluate all monetary payoffs. Then $F_1 \succsim F_1^*$ if and only if $0 \leq \int u_N(x)[dF_1(x) - dF_1^*(x)] = \int u_N(x)[dF_2(x) - dF_2^*(x)]$ if and only if $F_2 \succsim F_2^*$.

Now suppose that $\text{supp } F_1 \neq \text{supp } F_1^*$. There are two cases. First suppose that $\inf \text{supp } F_1 > \inf \text{supp } F_1^*$. Then choose $\inf \text{supp } F_1^* < x_D(A_1) \leq \inf \text{supp } F_1$ and $p_D(A_1) = F_1^*(x_D(A_1))$. Also let $x_D(A_2) < \inf \text{supp } F_2^*$, $x_V(A_1) > \sup \text{supp } F_1^*$, and $x_V(A_2) > \sup \text{supp } F_2^*$. Then the individual uses u_N to evaluate all monetary payoffs in A_2, and uses u_N for all monetary payoffs above $x_D(A_1)$ in A_1. He uses $u_D(\cdot;A_1)$ for all monetary payoffs below $x_D(A_1)$ in A_1, and these payoffs only occur if he chooses F_1^*. If $u_D(x;A_1)$ is sufficiently below $u_N(x)$ for $x \leq x_D(A_1)$, it is possible to have $F_1 \succ F_1^*$ but $F_2^* \succ F_2$.

The other case has $\inf \text{supp } F_1 = \inf \text{supp } F_1^*$ but $\sup \text{supp } F_1 < \sup \text{supp } F_1^*$. A similar argument shows that it is possible for the individual to experience victory from F_1^* but not from the other three lotteries, and have the preferences $F_1^* \succ F_1$ but $F_2 \succ F_2^*$. ■

This proposition shows that Allais-type violations occur when one of the two lotteries in a pair has a different support than its alternative. It also shows that Allais-type violations are not caused by any sort of certainty effect – if the \$1M for sure gamble in the original Allais lottery were replaced by a uniform distribution over [\$0.95M, \$1.05M], the VD model predicts the same choice pattern.

Insurance and gambling

Begin with a simple insurance problem in which the individual faces the prospect of losing $L > 0$ with probability p. Fair insurance against the loss costs pL. The issue is under what conditions does the individual prefer paying pL for sure over facing the potential loss. Let (x,p) denote the probability distribution which pays x with probability p and zero otherwise. The choice set $A = \{(-L,p), (-pL, 1)\}$, so there are only two choices. Note that the loss of L is avoidable, and so is the payoff of zero. If $p \leq p_D$, then using (3), the individual purchases insurance if and only if

$$u_N(-pL) \geq pu_D(-L;A). \tag{8}$$

The standard reference dependence condition R4 tells us that $u_N(-pL) < pu_N(-L)$, so that without the effect of defeat the individual would not buy insurance. But, since $u_D(-L;A) < u_N(-L)$, it is possible for (8) to hold if $u_N(-L) - u_D(-L;A)$ is sufficiently large.

Proposition 9. *If* $-L \leq x_D(A)$ and if $u_N(-p_D(A)L) \geq p_D(A)u_D(-L;A)$, there exists $q \leq p_D(A)$ such that the individual insures when $p \in [q, p_D(A)]$.

Proof. If $p \leq p_D(A)$, then the expected utility from facing the risk is $pu_D(-L;A)$ and the expected utility from insuring is $u_N(-pL)$. The hypothesis states that the individual insures when $p = p_D(A)$. If $p < p_D(A)$ and $u_N(-pL) \geq pu_D(-L;A)$, then note that $d/dp[u_N(-pL) - pu_D(-L;A)] = -Lu_N'(-pL) - u_D(-L;A) \geq -Lu_N'(-pL) - u_N(-pL)/p = -L[u_N'(-pL) - u_N(-pL)/(-pL)]$, and the term in brackets is negative by the convexity of u_N over losses. So, if the individual insures when the probability of loss is q, he insures whenever $q \leq p \leq p_D(A)$. ∎

This result shows that it is possible for the model to accommodate insurance against losses if the loss is large enough and unlikely enough to constitute a defeat, and if the effect of a defeat on utility is sufficiently large. Note that individuals only insure when losses are unlikely.

A similar analysis allows for gambling over large, unlikely gains. Here the condition is that the probability of the gain G must be at most $p_V(A)$ and the other condition is that $p_V(A)u_V(G;A) \geq u_N(p_V(A)G)$.

Discussion

There are, of course, many models of behavior toward risk that can accommodate the choice patterns discussed above, and there are models that share many of the elements used in the construction of the VD model. However, none of the models look explicitly at notions of victory and defeat, and none of the models have counterparts which can be used to address the game theory evidence.

The VD model handles expected utility violations through event-dependent preferences. Currently, the most prominent existing models are rank-dependent expected utility theory (e.g. Quiggin (1993)) and the closely-related cumulative prospect theory (Tversky and Kahneman (1992)). Both of these models rely on probability transformation schemes to explain expected utility violations, while the VD model leaves probability distributions untransformed. There are similarities, though. Most of the action from probability transformations comes in the tails of the distributions, and all of the action from the victory and defeat events comes in the tails of the distributions. Still, in the existing models the probability transformations are based only on the distribution under consideration, while in the VD model the events are based on the choice set. Rank-dependent expected utility theory and cumulative prospect theory are both able to accommodate the Allais paradox as well as simultaneous gambling and insurance, but unless the probability transformation function is discontinuous at its endpoints, they cannot accommodate boundary effects.

Several other models do not use probability transformations, and they are closer to the VD model.[15] Landsberger and Meilijson (1990) propose a model in which the utility function is segmented, as here, and it predicts that people will gamble on large, unlikely gains and insure against large, unlikely losses. To accomplish this, they propose the condition that the utility function is star-shaped, i.e. it exhibits nonincreasing average utility with respect to some anchor point. The event-dependent utility function of the VD model is not star-shaped, because it is convex over losses.

Conlisk (1993) constructs a model in which the decision-maker has a preference for gambling. In his model, the riskier of two choices generates additional utility, and he shows that it can explain the Allais paradox and simultaneous gambling and insurance. His

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[15]Luce et al. (1993) develop a model that uses threshold levels for gains and losses with these reference levels dependent on the choice set. The model can accommodate the Allais paradox and a number of other choice anomalies. But, the model also relies on probability weighting schemes.

model is close to the VD model, in that both augment utility according to something other than the monetary payoff, and this augmentation depends on characteristics of the choice set.[16] But, this preference for gambling has no obvious application to the games discussed here.

The two categories of models that are closest to the VD model are those with dissapointment[17] and elation (Loomes and Sugden (1986) and Bell (1988)) and those constructed to accommodate boundary effects (Cohen (1992), Neilson (1992)). Using the terminology from this paper, one can think of the former class of models, which are similar in nature to regret theory (Loomes and Sugden (1982), Bell (1982)), as beginning with a neutral utility function, and adding utility from elation when the monetary payoff is above the certainty equivalent computed using the neutral utility function, and subtracting disutility from disappointment when the monetary payoff is below the certainty equivalent. This leads to two events, with the neutral event missing, and the overall utility function is continuous, unlike the one depicted in Figure 1. While this model can accommodate many behavioral patterns, it cannot explain boundary effects.

A variety of models have been proposed to accommodate boundary effects. The original version of prospect theory (Kahneman and Tversky (1979) could accommodate boundary effects through the discontinuity of the probability weighting function. Neilson (1992) proposes a model with discontinuous utility functions where different utility functions are used for different probability distributions depending on how many outcomes are assigned positive probability.[18] Neither of these models exhibits stochastic dominance preference, while the VD model does.

The closest model to the VD model for analyzing behavior toward risk is the three-criteria model of Cohen (1992). There the individual compares the expected utility of different distributions, but different utility functions are used for different distributions based

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[16] There are also models in which the parameters of the utility function depend on the lottery being considered, rather than the choice set. See, for example, Becker and Sarin (1987).

[17] The notion of disappointment discussed here differs from Gul's (1991) notion of disappointment aversion, which can be described as a preference for avoiding the risky gamble in situations like the first Allais choice pair.

[18] See Humphrey (1998) and Schmidt (1998) for extensions.

on the highest and lowest payoffs available from the distribution.[19] Proposition 8 above shows that in the VD model, expected utility with a single, event-independent utility function works well only when all lotteries under consideration have the same support, which matches exactly the prediction of Cohen's model. However, the likelihood of the outcomes never enters into Cohen's model.

7. Conclusions

This paper presents a single model that can explain “anomalous” behavior in games and toward risk. In particular, it can accommodate evidence of inequity aversion, cooperation in the one-shot prisoner’s dilemma, and contributions in a free-riding setting. It can also accommodate the Allais paradox and its variations, boundary effects, and simultaneous gambling and insurance. The success of a single, unified model for fitting this wide variety of patterns suggests that it may not be necessary to use separate approaches for games and for risk, and that it is fruitful to explore models that can do both. The success also validates the interpretation of the evidence lent by the model.

According to the VD model proposed here, the propensity of responders to reject low offers in ultimatum games and the Allais paradox behavior both arise from the same underlying motivation – both arise because the individual wishes to avoid states in which they feel defeated. For the ultimatum game, this means avoiding states in which his opponent’s payoff is too high compared to his own. For the Allais behavior, this means avoiding states in which an avoidable, unlikely, low payoff occurs. If, as the model proposes, individuals do lose utility in states in which they feel defeated, then one should expect them to take actions to avoid this utility loss, and these actions manifest themselves in predictable patterns in a variety of settings, including both risk and games.

Inequity aversion is captured through two effects. First, when the player’s payoff is low relative to his opponents’, he can avoid defeat by reducing his own payoff but reducing his opponents’ payoffs by more. Second, the player feels victorious when his payoff is high relative to his opponents’ but not too high, and if his payoff is too high he can retain victory by reducing his own payoff and raising his opponents’. Thus, inequity aversion comes not from a natural sense of fairness, but instead from a combination of a

[19] Gilboa (1988) and Jaffray (1988) propose similar models but using only the lowest possible payoff (the security level) and not the highest possible payoff (the potential level).

dislike of losing and a sense of guilt for winning by too much.

Gambling and insurance arise through two effects as well. Individuals have the standard reference-dependent neutral utility function introduced by Kahneman and Tversky (1979), so that it is concave over gains and convex over losses. Without some other consideration, then, individuals would never insure against large, unlikely losses and they would never gamble on large, unlikely gains. But they do in the VD model. Insurance against large, unlikely losses occurs because people consider themselves defeated when they suffer a large, unlikely loss, and insure against this sense of defeat. Gambling on large, unlikely gains occurs because people consider themselves victorious when they win a large, unlikely prize, and they gamble on this sense of victory. This willingness to gamble on victory suggests that players in games might choose riskier strategies in an effort to achieve a victory, although no games in which this effect can occur are explored in this paper.

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