Launching of a New Currency in a Simple Random Matching Model.*

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Abstract

This paper studies the launching of a new fiat currency within a search-theoretic framework. We show that legal tender laws may not be sufficient to guarantee the acceptability of the new currency, and that the withdrawal of a large fraction of the competing currency is essential to avoid the failure of such a launching. The possibility of converting the old currency into the new one can ease the transition to the new currency only if it is combined with strict legal tender laws. Finally, a network externality is identified that may generate inefficiencies in the conversion decision.

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1 Introduction

This paper studies the launching of a new fiat currency within a dual-currency search-theoretic model. Its aim is to assess the efficiency of three types of instruments: legal tender laws, a compulsory withdrawal of the old currency, and the possibility offered to individuals to convert their old units of money into new ones. Starting with a simple model, we draw some lessons for a successful currency reform when the strategy of acceptance of each currency and the decision to convert the old into the new one are endogenous.

A fiat currency is an object that no-one values for its intrinsic properties but one that is used as a means of payment by agents. The acceptability of such an object was difficult to achieve until the last century, and therefore, monies were essentially commodity monies (gold and silver coins), i.e. objects that can be valued independently of their medium of exchange function. Nowadays, as argued by Selgin (1994), the introduction of a new fiat money is something to take very seriously. People in general feel very hesitant to adopt a different currency than the one they usually accept: old habits indeed die hard.

Governments may want to introduce a new currency for different reasons. First, money is a symbol of national and political identity. The colonization of a country or the creation of an independent state is generally accompanied by the introduction of a new currency. Second, a new currency can be launched for technical purposes: to avoid counterfeiting (the introduction of a new bill), to save some production or issuing costs (the replacement of notes by coins), or to better serve the public need. Third, monetary integration can imply the replacement of national currencies by a common one (the euro, for instance).

1.1 Historical episodes

Several recent episodes teach us that some conditions must be fulfilled for a currency reform to be successful.

Shell currencies in Africa. In many East and West African countries, the shell currencies have proven to be "formidable rivals" for modern monies until the middle of the 20^{th} century, especially for the smallest purchases (Davies, 1994). A case in point is Nigeria, where the British government attempted to replace shell currencies (cowries and manillas) at the end of the 19^{th} Century (Ofonagoro, 1979).[1] It took fifty years for the pre-colonial currencies to stop being used by Nigerians as a means of payment, and for the British currency to be circulated effectively.

> ‹‹ *Coercive measures notwithstanding, the attitude of the local population to the use of British currency remained pragmatic. The gold*

[1] In fact, both cowries and British currencies could be put to other uses. Cowries had religious and ornemental uses and British currencies were made of gold, silver and copper. For surveys, see also Hawkins (1958), Jones (1958), Hopkins (1966), Johnson (1970) and Gregory (1996).

coins were never taken upcountry by the Delta middlemen. Rather, they kept them in circulation in the vicinity of the government stations on the coast, where they were required in the payment of revenue. (...) British currency at this time circulated within very narrow limits. The quantity in circulation among the African population was generally limited to what the government paid its clerks, soldiers and other employees, or what it spent in small-scale local purchases of imported goods.›› Ofonagoro (1979, p.634)

According to Gregory (1996), the fall in the value of shell currencies was not a consequence of an increase in their quantities, as often argued, but rather a fall in the demand for those currencies due to compulsory measures introduced by the new government.

‹‹*But the demonetization of the cowrie does not happen over night. Rival standards of value are at stake. This is a political struggle between the citizens of the old state, who have their wealth stored in the form of cowrie money, and the new rulers. The citizens who hold their wealth in the form of cowrie money have much to lose and fight it out. As the imperial state gradually assumes control, the demand for cowrie money falls because it is no longer legal tender.*›› Gregory (1996, p.208)

The wampum in North America A similar experience happened in North America. In the 17^{th} century, American Indians used a particular form of money, the *wampum*, which was composed of beads made of shells of the clam (Davies, 1994, pp.38-40).[2] Because of its popularity, the *wampum* was declared legal tender in several American colonies: in Massachusetts, it was legal tender at six beads a penny. The *wampum* ceased to be legal tender in the New England states in 1661, but continued in circulation in parts of North America for 200 years.

The coin/note substitutions A more recent illustration of the difficulty in launching a new currency is given by the unsuccessful attempt of the US government, in 1979, to introduce a new coin, the Susan B. Anthony dollar coin, to replace the existing one dollar bill (Caskey and St. Laurent, 1994). In April 1983, Britain experienced similar difficulty in introducing a pound coin to replace the pound note: one month after being introduced, the coin temporarily disappeared (Burgoyne *et al.*, 1999). In many respects, a parallel can be drawn between the coin/note substitution and the launching of a new currency *stricto sensu*: the new coin must be accepted as payment, its use comes up against agents' habits, and the transition to the new coin generates

[2] According to Davies (1994, p.40), the use of *wampum* as money cannot be dissociated from its ornemental qualities, but the *wampum* "was a durable bridge to more modern forms of money".

network externalities.[3] Canada, which benefited from the lessons offered by the American experience, successfully introduced the "loonie" dollar coin on July 1st, 1987.

New currencies in the Former Soviet Union The creation of new states (or independent states) is also an opportunity to institute new currencies. Since the dissolution of the Soviet Union in December 1991, many former republics decided to launch their own currencies (Estonia, Latvia, Lithuania, Kazakhstan, Kyrgyzstan).[4] A first step towards a separate currency was the introduction of coupon currencies for cash transactions in several republics, in response to a shortage of rouble notes. Even in the three Baltic states, where the currency reforms were successful overall, the launching of a new fiat currency required different steps. As an example, Latvia and Lithuania introduced interim currencies (the *Latvian rouble* and the *talonas*) that had to be replaced a year later by new ones (the *lats* and the *litas*). In Kyrgyztan, the *som* was introduced in May 1993. However, individuals continued to prefer the rouble, and the conversion to the som was not fully achieved. The launching of a national currency was even more chaotic in Ukraine. In January 1992, Ukraine introduced a coupon currency which was meant to coexist with the rouble, but without success. In November 1992, the Ukrainian government left the rouble area and launched its national currency, the *karbovanets*. However, this money became a victim of hyperinflation, lost its credibility and had to be replaced by the currency now in use, the *hryvnia*, launched on August 25th, 1996.

The euro The changeover to the euro is planned in countries joining the European Monetary Union. Since January 1st, 1999, the euro is a money of account and can be used in non-cash payments. The introduction of bills and coins will start on January 1st, 2002. The period of dual circulation of the old and new notes and coins should last between four weeks and two months. The abolition of the legal tender status of the national currencies is up to the member states for decision, but will not occur after the end of February 2002. For instance, the Deutschmark will cease to be legal tender from January 1st, 2002. The Dutch guilder will be abrogated in January 2002 at the latest. After February 2002, there will be a limited portion of national currencies still in circulation. The central banks will keep redeeming the national currencies after their legal tender status has been abolished, for either a certain period of time or for an indefinite time span.

1.2 General presentation

To study the launching of new fiat currency, we adopt the search-theoretic framework of Kiyotaki and Wright (1991, 1993). Trades are decentralized, and agents

[3] A positive network externality arises when a good is more valuable to a user the more users adopt the same good.

[4] These experiences are described by Lainela (1993), Abrams and Cortés-Douglas (1993), and Melliss and Cornelius (1994).

meet pairwise and at random. Barter is ruled out by restrictions on preferences and technology. As a consequence, people need a medium of exchange to carry out trades.

There are two currencies in this economy: a new currency and an old one. To promote the acceptability of the new unit of money, the government has at least three "launching vehicles" at hand: enact legal tender laws, implement a compulsory withdrawal of the old currency, or allow, at any time, the conversion of the old currency into the new one. This paper examines the role of these instruments in the launching of a new currency.

This article proceeds as follows. First, we study the consequences of a one-off partial withdrawal of the old currency on the acceptability of the two currencies when the conversion is not allowed. In the presence of legal tender laws, there is always a "natural" equilibrium where the legal tender currencies are accepted while the other currencies are rejected. Furthermore, if legal tender laws are very strict, this equilibrium is unique. Conversely, if a large fraction of trades are not subject to legal tender laws, then multiple equilibria are possible. If the two currencies are legal tender, and if only one currency is universally accepted, the likeliest one is the more abundant one. Even when the old currency becomes illegal, it may go on circulating if the supply of new currency is not sufficient. The main lesson of this first part is that, in any case, a massive withdrawal of the old currency induces agents to accept the new one in order to alleviate the absence of double coincidence of wants problem.

Second, there is no compulsory withdrawal of the old currency, but agents are allowed to initiate the conversion to the new currency. In the absence of coercive measures (for instance, legal tender laws), the possibility of converting the old currency into the new one is insufficient to guarantee the circulation of the new currency. When the two currencies are legal tender, it is very unlikely that the change occurs. However, when the old unit of money looses its legal tender status, the presence of strict laws makes the change-over to the new currency likelier. Still, there may appear a network externality that may block the switch. If no-one initiates the conversion to the new currency, then the quantity of money in the economy is only composed of money which is not legal tender; this reduces the incentive to convert the old currency into the new one, even though it would be better for everyone. Coordination failures are then possible.[5]

Third, we consider the explicit dynamics of the model in the absence of a compulsory withdrawal of the old currency. The government preannounces that the old currency will lose its legal tender status in the future. We describe the trajectory of the economy, that is the acceptability of the old currency and the date at which the conversion to the new currency occurs. Furthermore, we ask whether such an announcement may encourage agents to switch to the new currency earlier. Our answer is negative: there is no preannouncement effect.

[5] A similar effect has already been mentioned in the literature about innovation and network externalities: it is the so-called *bandwagon effect* (Farrell and Saloner, 1986). It means that if a set of individuals adopts one technology, then the same choice becomes more attractive to all other individuals.

Finally, we resort to simulations in order to describe the situation with both a continuous withdrawal of the old currency by the government and the possibility offered to agents to switch to the new currency at any time. We verify that the previous conclusions still hold.

As far as we know, there are a few theoretical papers devoted to the launching of a new fiat currency. Ritter (1995) describes the transition between a barter equilibrium and a monetary equilibrium. Contrary to ours, there is only one type of fiat money in his model. Aiyagari and Wallace (1997) and Li and Wright (1998) study the role of a government transaction policy in the establishment of a currency. In particular, Li and Wright describe an economy with two fiat currencies and show that the government is able to eliminate one type of money from circulation, or to affect the exchange rate. We will confirm some of their results but will also emphasize the role of a compulsory withdrawal of the old currency. Furthermore, we will allow agents to convert their old unit of money into the new one.

Most of the search models with two monies focus on the emergence of an international currency (Matsuyama *et al.*, 1993; Zhou, 1997; Trejos and Wright, 1996, 1999).[6] Few models introduce both a legal currency and an illegal one. Green and Weber (1996) consider the possibility of counterfeiting. Curtis and Waller (2000) introduce legal restrictions on the use of foreign currency for internal trade. Our context and our results differ in various aspects from theirs.

This paper is organized as follows. In the second section, we present the model and its main assumptions. Section 3 studies the case of a compulsory withdrawal of the old currency accompanied by legal tender laws. In the fourth section, we give some preliminary results about the dynamics of the model. In section 5 and 6, there is no compulsory withdrawal but agents can voluntarily convert their old currency into the new one at a certain cost. Steady-state equilibria, welfare considerations and the dynamics of the model are presented. Sections 7 and 8 (simulations) complement and exemplify the model. Finally, section 9 draws a parallel between the results of the model and historical episodes.

2 The model

The environment is similar to that of Kiyotaki and Wright (1991, 1993) and Matsuyama *et al.* (1993).

2.1 Main Assumptions

The economy is composed of a continuum of infinitely-lived agents of unit measure and $N \geq 3$ distinct goods. Agents are specialized in both production and consumption. There are N types of agents. An agent of type $i \in \{1, ..., N-1\}$ produces the good i and only consumes the good $i+1$. An agent of type N produces the good N but consumes the good 1. The distribution of agents between the different types is uniform.

[6] For a survey on dual-currency search models, see Craig and Waller (2000).

Meetings between agents are bilateral and occur at random with respect to agent types. Consider the situation where the individual i meets a partner j. The probability that j produces the good consumed by i is $1/N$; the single coincidence of wants probability is denoted by $x \equiv \frac{1}{N}$ in the sequel. Obviously, assumptions about tastes and technologies imply that double coincidence of wants meetings never occur.

For tractability, we add restrictions about agents' storage capacity. Agents cannot hold more than one unit of an object (money or goods) at a time.[7] Furthermore, the units of goods and money are indivisible. Thus, the determination of terms of trade is straightforward: agents always trade one unit of good for one unit of money.

When producing agents suffer no disutility.[8] Production is instantaneous and follows consumption: an agent who holds one unit of money cannot produce before this unit of money has been exchanged for a consumption good. This restriction can be interpreted as an implicit production cost. The consumption of one unit of good generates an instantaneous utility flow equal to $u > 0$.

There are two objects that no-one consumes or produces called fiat monies. The old money is labelled by 0 and the new one by 1. The quantity of old money is $M_0 \in (0, 1)$ and the quantity of new one is $M - M_0$, where $M < 1$ is the quantity of all the units of money in the economy. As previously mentioned, the units of money are indivisible; thus, the fraction of agents holding the old money is M_0 whereas the fraction of agents holding the new one is $M - M_0$.

Agents who do not hold money can either exit the market permanently and return to autarky, or become sellers (they are labelled by S).[9] A seller is someone who sells his good for one unit of any legal tender currency. In equilibrium, an agent without money will never choose to exit the market. Hence, the fraction of sellers among the population is $1 - M$.

Time is discrete and is indexed on $\mathbb{N}$. The discounted lifetime utility of an agent who consumes at the periods $\{t_n, n \in \mathbb{N}\}$ is:

$$\sum_{n \in \mathbb{N}} \beta^{t_n} u,$$

where $\beta \in (0, 1)$ is the discounting factor.

2.2 "Launching vehicles"

The government may use different instruments to promote the acceptability of a new currency. He can resort to coercive measures — legal tender laws,

[7] Craig and Waller (1999) develop a dual currency search model in which agents are allowed to hold multiple units of both currencies. In general, they must rely on numerical methods to solve the model.

[8] This assumption comes from Matsuyama *et alii* (1993) and Trejos and Wright (1996, 1999).

[9] We do not allow sellers to exit the market temporarily. This can be justified by a high cost to re-enter the market. This restriction has no role for the steady-state equilibria and only prevents a collapse of the economy in non-stationary environments.

an authoritarian replacement — to force agents to hold and accept the new currency, even when they do not want to do it. He can also use the established currency as a "launching vehicle" by guaranteeing the conversion between the old and the new currency. We indicate a distinction between the ability of the government to withdraw a given currency and its monitoring activity. For instance, in Nigeria, the British government declared shell monies illegal but did not choose to redeem them for British coins (Ofonagoro, 1979).

2.2.1 Legal tender laws

Legal tender laws specify which of the currency in the economy must be accepted as payment for goods. According to Simmel (1907, cited by Selgin, 1994), legal tender laws rely on "*the probability that every individual, in spite of his ability to refuse the money, will accept it*".

In our model, the government declares which currencies are legal tender and which are "illegal tender".[10] The coercive power of the state comes from its ability to monitor transactions. However, the monitoring technology is less than perfect and the government can only observe a fraction $g \in (0, 1)$ of all the meetings. As a consequence, the seller may renege his commitment to sell his good for legal tender currencies when he is not monitored.

When a seller and a buyer are matched, the buyer offers the currency he holds if a single coincidence of wants is realized. If this meeting is subject to government monitoring, the seller is forced to accept legal currencies and to refuse illegal ones.[11] In the absence of government control, the seller chooses freely to accept or to refuse the unit of money.

In short, the seller can choose rationally to accept or to refuse the units of money on a fraction $(1 - g)$ of all trades, namely the non-monitored trades. If g is close to one, the legal tender probability is highly constraining and individuals have rarely the choice to accept or refuse a currency. In particular, when $g = 1$ a currency which is legal tender is always accepted and a currency which is not legal tender is always refused.[12]

Our distinction between monitored and non-monitored trades can also reflect the opposition between the "official economy" and the "black economy". In the official economy, trades must be conducted with legal tender currencies. However, in the black economy, any medium of exchange may be used. For instance, in Germany, during the post-war period (1945-1948), the legal tender

[10] Alternatively, it could have been assumed that monies which are not legal tender are not necessarily illegal. Indeed, legal tender laws specify which monies cannot be refused in payments but have nothing to say about other means of payment. However, by requiring the payment of taxes in the legal tender money, the use of other monies is prohibited for this purpose.

[11] In Wright (1999), when people are monitored, their trades become common knowledge. Monitoring can then improve social welfare by making a larger number of trades implementable. In our model, even if the government can enforce the trade that period, it does not know the agents names; so it cannot use trigger strategies to enforce credit.

[12] An alternative formalization would consist in introducing agents, called *government agents*, whose transaction strategies are specified exogenously. See, for instance, Green and Weber (1996) or Li and Wright (1998).

currency (namely, the Reichsmark) was refused in the black market whereas other fiat and commodity monies were accepted (such as cigarettes).

It will be assumed in the sequel that the new currency is always legal tender. On the contrary, the old currency is legal tender before time $T \in \mathbb{N}$ and becomes illegal at time T.

2.2.2 Withdrawal and conversion of the old currency

The quantity of old money decreases over time because a compulsory withdrawal is initiated by the government and because old currency holders can decide, at any period, to switch to the new currency. When confiscated, the old currency units are redeemed for new ones. At period t, the probability for an old currency holder to become a new currency holder, at no cost, is equal to λ_t.

Furthermore, every old currency holder can voluntarily go to the Central Bank and ask for the new currency. In this case, agents bear a positive cost γ. Let Ω_t denote the average conversion decision: Ω_t is one if agents switch to the new currency and zero otherwise. The law of motion of the quantity of old currency units in the economy is indicated by the following equation:

$$M_{0,t+1} = (1 - \Omega_t)(1 - \lambda_t)M_{0,t} \tag{1}$$

If all agents in the economy switch to the new currency at time t $(\Omega_t = 1)$, then the quantity of old money in $t + 1$ is zero. Otherwise, the quantity of old money in $t + 1$ is equal to its quantity in t minus the amount withdrawn by the government in t $(\lambda_t M_{0,t})$.

The chronology of events within each period is the following. At the beginning of period t, an old currency holder becomes a new currency holder with probability λ_t (phase 1). With probability $1-\lambda_t$, he keeps his old currency unit, but can exchange it at cost γ (phase 2). Then, the random matching process occurs (phase 3).

Phase 1	Every old currency holder becomes a new currency holder with probability λ_t
Phase 2	Every individual still in possession of an old currency unit can exchange it at cost γ
Phase 3	Individuals meet at random and pairwise

2.3 Individual strategies

Basically, the game described in this paper is an anonymous sequential game. The main feature of this type of game is that a player's payoff depends on his opponents' actions only through their distributions.[13] Each individual is in one of the three following states: seller (S), holder of the old currency (0) or holder of the new currency (1).

Let us denote by $\mathcal{V}_{S,t}$ (resp. $\mathcal{V}_{0,t}$ and $\mathcal{V}_{1,t}$) the expected lifetime utility of an agent in the state S (resp. 0 and 1) at the beginning of phase 3 of period t.

[13] See Jovanovic and Rosenthal (1988).

Furthermore, let $\widetilde{\mathcal{V}}_{0,t}$ denote the expected lifetime utility of an agent who holds one unit of the old currency at phase 1 of period t. The different expressions for the value function of an old currency holder are recapitulated in the following table:

Time	t	*end of phase 2*	$t+1$
Value of an old currency holder	$\widetilde{\mathcal{V}}_{0,t}$	$\mathcal{V}_{0,t}$	$\widetilde{\mathcal{V}}_{0,t+1}$

At the beginning of each period, every old currency holder must decide whether to keep his old currency unit or to convert it into the new one. The trip to the Central Bank costs $\gamma > 0$ in terms of utility. Let ω_t denote the conversion strategy at time t: ω_t equals one if the agent switches to the new currency and zero otherwise. This decision is expressed by the comparison of two quantities: the difference between the value of being a new currency holder and the value of being an old currency holder $(\mathcal{V}_{1,t} - \mathcal{V}_{0,t})$, and the conversion cost (γ). More precisely,

$$\mathcal{V}_{1,t} - \mathcal{V}_{0,t} \begin{matrix} > \\ < \end{matrix} \gamma \Rightarrow \omega_t = \begin{matrix} 1 \\ 0 \end{matrix} \tag{2}$$

As a consequence,

$$\widetilde{\mathcal{V}}_{0,t} = (1 - \lambda_t) \max(\mathcal{V}_{0,t}, \mathcal{V}_{1,t} - \gamma) + \lambda_t \mathcal{V}_{1,t}. \tag{3}$$

Trade strategies specify which objects are accepted by individuals when matched with a partner. They depend on time and on the object agents hold. Let us denote by $\pi_{1,t}$ (resp. $\pi_{0,t}$) the probability that a seller accepts the new currency (resp. the old currency) if he is not monitored by the government. The strategy of acceptance of the new currency is given by the following decision rule:

$$\mathcal{V}_{1,t+1} - \mathcal{V}_{S,t+1} \begin{matrix} > \\ < \end{matrix} 0 \Rightarrow \pi_{1,t} = \begin{matrix} 1 \\ 0 \end{matrix} \tag{4}$$

A seller rationally chooses to accept the new currency at period t if the value of being a new currency holder in $t+1$ is superior to the value of being a seller in $t+1$. According to a similar reasoning, the strategy of acceptance of the old currency is given by the following rule:

$$\widetilde{\mathcal{V}}_{0,t+1} - \mathcal{V}_{S,t+1} \begin{matrix} > \\ < \end{matrix} 0 \Rightarrow \pi_{0,t} = \begin{matrix} 1 \\ 0 \end{matrix} \tag{5}$$

A seller rationally chooses to accept the old currency at period t if the value of being an old currency holder at phase 1 of period $t+1$ is superior to the value of being a seller in $t+1$.

Let us denote by $\Pi_{0,t} \in (0,1)$ (resp. $\Pi_{1,t}$) the probability that a seller chosen at random accepts the old currency (resp. the new currency) at period t if he is not monitored by the government. At the symmetric Nash equilibrium, $\pi_{i,t} = \Pi_{i,t}$ and $\omega_t = \Omega_t$.

2.4 The value functions

To specify the value function of agents according to their state, we introduce Bellman equations. We distinguish two periods, before and after the old currency loses its legal tender status.

For sake of simplicity, we restrict our attention to equilibria such that money holders never delay their purchases.[14] This implies the following conditions:

$$u > \beta\left(\mathcal{V}_{i,t} - \mathcal{V}_{S,t}\right), \qquad \forall t, \qquad i = 0, 1$$

This inequality is always satisfied in steady-state equilibria. It will also be satisfied in non stationary environments if agents are sufficiently impatient.

2.4.1 The old currency is legal tender ($t < T$)

The value of being a seller at period t is:

$$\mathcal{V}_{S,t} = \left(M - M_{0,t}\right) xg\beta\mathcal{V}_{1,t+1} + \left(M - M_{0,t}\right) x(1-g)\beta \max\left(\mathcal{V}_{1,t+1}, \mathcal{V}_{S,t+1}\right) +$$

$$+ M_{0,t}x(1-g)\beta \max\left(\widetilde{\mathcal{V}}_{0,t+1}, \mathcal{V}_{S,t+1}\right) + M_{0,t}xg\beta\widetilde{\mathcal{V}}_{0,t+1} + (1 - Mx)\beta\mathcal{V}_{S,t+1}. \tag{6}$$

Equation (6) has the following interpretation. Because meetings occur at random, a seller of type j is matched with a new currency holder with probability $M - M_{0,t}$, and is matched with an old currency holder with probability $M_{0,t}$; furthermore, the seller's partner consumes good j with probability x. According to the first term of RHS (6), if the seller's partner holds one unit of new currency (with probability $M - M_{0,t}$) and values good j (with probability x), the seller is forced to trade with probability g (the legal tender probability). According to the second term of RHS (6), the seller can rationally choose to accept or to refuse the new currency if he is not monitored by the government (with probability $1 - g$). The third and fourth terms of RHS (6) describe a similar case where the seller meets an old currency holder who wishes to consume good j.

The value of being a new currency holder is:

$$\mathcal{V}_{1,t} = (1-M)x\left[g + \Pi_{1,t}(1-g)\right]\left[u + \beta\mathcal{V}_{S,t+1}\right] +$$

$$+ \left\{1 - (1-M)x\left[g + \Pi_{1,t}(1-g)\right]\right\}\beta\mathcal{V}_{1,t+1}. \tag{7}$$

According to (7), the new currency holder meets a seller with probability $(1-M)$ and this seller produces the consumption good of the buyer with probability x. There is a trade between the buyer and the seller if the meeting is subjected to

[14] It may happen that new currency holders may prefer to delay their purchase until the fraction of new currency holders has increased. Indeed, the value of being a seller is higher when the quantity of new currency in the economy is also higher.

government monitoring (with probability g) or if, in the absence of government monitoring, the new currency is accepted (with probability $\Pi_{1,t}$).

Finally, the value of being an old currency holder is:

$$\mathcal{V}_{0,t} = (1-M)x\left[g + \Pi_{0,t}(1-g)\right]\left[u + \beta\mathcal{V}_{S,t+1}\right] +$$

$$+\left\{1-(1-M)x\left[g + \Pi_{0,t}(1-g)\right]\right\}\beta\widetilde{\mathcal{V}}_{0,t+1}. \quad (8)$$

2.4.2 The old currency is not legal tender ($t \geq T$)

The value of being a seller is:

$$\mathcal{V}_{S,t} = (M - M_{0,t})\,x(1-g)\beta\max\left(\mathcal{V}_{1,t+1},\mathcal{V}_{S,t+1}\right) + (M - M_{0,t})\,xg\beta\mathcal{V}_{1,t+1} +$$

$$+M_{0,t}x(1-g)\beta\max\left(\widetilde{\mathcal{V}}_{0,t+1},\mathcal{V}_{S,t+1}\right) + \left\{1 - Mx + M_{0,t}xg\right\}\beta\mathcal{V}_{S,t+1}. \quad (9)$$

The only difference with (6) is that a seller who meets an old currency holder will be free to trade only with probability $(1-g)$.

The value of being a new currency holder is given by (7). The value of being an old currency holder is:

$$\mathcal{V}_{0,t} = (1-M)x\Pi_{0,t}(1-g)\left[u + \beta\mathcal{V}_{S,t+1}\right] +$$

$$+\left[1-(1-M)x\Pi_{0,t}(1-g)\right]\beta\widetilde{\mathcal{V}}_{0,t+1}. \quad (10)$$

Equation (10) has the following interpretation. After period T, an old currency holder becomes a seller if he meets a seller who accepts the old currency and if there is no government monitoring (an event which occurs with probability $(1-M)x\Pi_{0,t}(1-g)$); otherwise, the old currency holder can benefit from a compulsory replacement at period $t+1$ and then become a new currency holder.

To end this section, we give a definition of an equilibrium in our model.

Definition 1 *An equilibrium is a 7-uplet* ($\{\mathcal{V}_{0,t}\}$, $\{\mathcal{V}_{1,t}\}$, $\{\mathcal{V}_{S,t}\}$, $\{M_{0,t}\}$, $\{\Pi_{0,t}\}$, $\{\Pi_{1,t}\}$, $\{\Omega_t\}$) *where:*

- ($\{\mathcal{V}_{0,t}\}$, $\{\mathcal{V}_{1,t}\}$, $\{\mathcal{V}_{S,t}\}$) *obey the Bellman equations (6)-(8) and (9)-(10),*
- $\{M_{0,t}\}$ *satisfies the law of motion (1),*
- $\{\Pi_{0,t}\}$, $\{\Pi_{1,t}\}$ *and* $\{\Omega_t\}$ *are consistent with the individual decision rules (2) and (4)-(5).*

3 Legal tender laws

In this section, it is assumed that old currency holders cannot initiate the conversion to the new currency: the conversion cost is too high or the Central Bank refuses to redeem the old currency. However, there is a one-off partial withdrawal of the old currency at the initial period. At $t = 0$, the government withdraws a quantity $M - M_0$ of old money and replaces it with the new one. In the following periods, the stocks of old and new currencies are unchanged:

$$\lambda_0 = \frac{M - M_0}{M}, \qquad \lambda_t = 0 \quad \forall t \geq 1$$

Thus, according to (3), the value function of an old currency holder at the beginning of period t is:

$$\widetilde{\mathcal{V}}_0(t) = \mathcal{V}_0(t), \qquad \forall t \geq 1$$

We restrict our attention to symmetric Nash equilibria in pure strategies where everyone adopts the same strategy of accepting or refusing the new and the old currency. As in most of the search-theoretic models of fiat money, the multiplicity of equilibria is a generic feature. We will study the influence of the legal tender status of the old currency on the typlogy of steady-state equilibria in the presence of a residual quantity of old currency.

3.1 The two currencies are legal tender

Proposition 2 *Assume that there is a residual quantity of old currency M_0 and that agents are not allowed to convert their old units of money into new ones. The two currencies are legal tender. Then, there are three types of equilibria: $(\Pi_0, \Pi_1) \in \{(1,1), (1,0), (0,1)\}$. The equilibrium where the two currencies are accepted always exists. An equilibrium where only one currency is accepted exists if the quantity of this currency is above the following threshold:*

$$\frac{g(1 - \beta + (1 - M)\beta x)}{(1 - g)\beta x}$$

Proof. See appendix. ■

The equilibrium where the two currencies are accepted is the "natural" equilibrium of the model. Indeed, if someone anticipates that everyone will accept a currency which is legal tender, then it is also rational that he accepts it. This equilibrium is such that each agent is indifferent between holding the new currency or the old one. However, proposition 2 indicates that other equilibria may occur. The typology of these equilibria is represented in figure 1 (a).

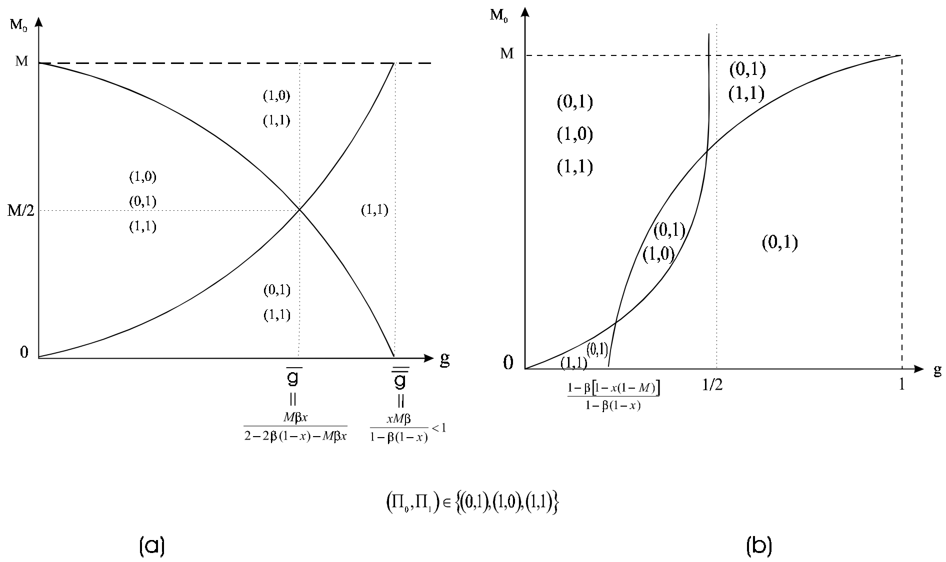


Figure 1. Compulsory withdrawal and legal tender.

If government monitoring is not too severe ($g < \overline{g}$) and the composition of the quantity of money is almost symmetrical (M_0 close to $M/2$), then all equilibria exist and the choice of the media of exchange is *a priori* indeterminate. If g is between $\overline{g}$ and $\overline{\overline{g}}$ then, if there is only one currency which is universally accepted, this will be the more abundant one. Finally, if the fraction of exchanges that are monitored is high ($g > \overline{\overline{g}}$), the unique equilibrium is such that the two currencies are accepted.

A first lesson of this model is that the compulsory withdrawal of the old currency initiated by the government plays an important role in the acceptability of the new currency. If there is only one currency which is accepted in non-monitored trades, the likelier one is the more abundant one.

3.2 Only the new currency is legal tender[15]

Proposition 3 *Assume that there is a residual quantity of old currency M_0 and that agents are not allowed to convert their old units of money into new ones. Only the new currency is legal tender. Then there are three types of equilibria: $(\Pi_0, \Pi_1) \in \{(1,1)\,,\ (1,0)\,,\ (0,1)\}$. The equilibrium where only the new currency is accepted always exists. The equilibrium where the two currencies are accepted exists when*

$$g < \frac{1-\beta+\beta x\,(1-M)}{1-\beta(1-x)-\beta x M_0}$$

[15] Curtis and Waller (2000) also describe a dual currency economy with a legal and an illegal currency. Contrary to us, they restrict their attention to the case where the legal currency is always accepted. Furthermore, they ignore the impact of the residual quantity of illegal currency on the acceptability of the two currencies.

The equilibrium where only the old currency is accepted exists when

$$M_0 > \frac{g\left[1-\beta+(1-M)x(1-g)\beta\right]}{\beta x\left(1-g\right)\left(1-2g\right)} \quad \textit{and} \quad g < \frac{1}{2}$$

Proof. See appendix. ■

The equilibrium where only the new currency is accepted is the natural equilibrium. If everyone accepts the currency which is legal tender, then it is individually rational to do so. In the same way, if everyone refuses the illegal currency, then it is individually rational to refuse it.

For the old currency to be accepted, the quantity of old currency (M_0) must be large and the legal tender probability low. Indeed, if the new medium of exchange is scarce and if it is difficult to find a trading partner, then it can be rational from an individual point of view to continue to accept the old currency, even if it is illegal. The equilibrium where only the old currency is accepted can occur if the quantity of old currency left is sufficient, and it must be higher when government monitoring becomes more severe. Furthermore, if more than half of the trades are monitored by the government ($g > 1/2$), then this equilibrium is impossible.

The typology of equilibria when only the new currency is legal tender is represented in figure 1 (b).[16] A lesson from the last proposition is the following one. If the government did not introduce enough new currency, then it may be advantageous to continue to accept the old one even if its circulation is hampered by government supervision. By withdrawing a large quantity of old money, the government can make the new money indispensable.[17]

4 Compulsory withdrawal

We now consider the acceptability of the two currencies along the trajectory of the economy when the quantity of old money varies from M to 0. First, it will be assumed that the old currency withdrawal is very slow so that the state of the economy at any point in time can be approximated by a steady-state. Second, we will assume that, in the presence of a rapid currency reform, agents have perfect foresight expectations.

4.1 An evolutionary approach

We assume that agents behave as if their environment were stationary. Hence, results from the previous section apply. In the presence of multiple equilibria, a simple selection criterion is adopted. Agents follow behavioral patterns that have been used traditionally unless it is not rational to do so.[18] More specifically, at

[16] For some parameter values, the two upward-sloping curves may not intersect.

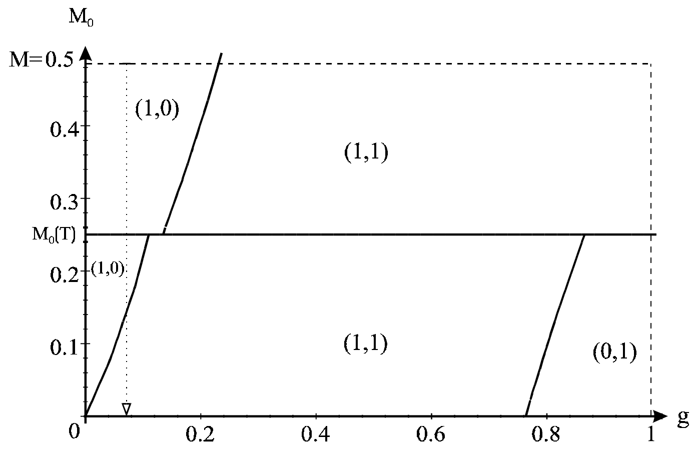
[17] This issue is not explicitly considered in Li and Wright (1998) because of their assumption about the pricing procedure. The seller has no bargaining power and gets no surplus from a trade with a buyer: so, he does not care about the composition of the quantity of money.

[18] A similar approach is used in Matsuyama *et al* (1993).

the beginning of each period, the inital belief of an agent is that a currency will be accepted if that was the case at the previous period.

At the beginning of time ($t = 0$ and $M_0 = M$), agents believe that only the old currency is accepted. Indeed, the sole currency is the old one and the unique equilibrium is such that this currency is accepted. The government gradually withdraws the old currency and replaces it by the new one. If, conditional on the belief that the old money is accepted and the new one is refused, it is individually rational to follow the same behavior, then agents remain coordinated on the Nash equilibrium where only the old currency is accepted, even if other equilibria exist.

Conditional on $\Pi_1 = 0$, if $\mathcal{V}_1 - \mathcal{V}_S$ becomes positive, then it is individually rational to accept the new currency although the initial belief was that the new currency is refused in non-monitored trades. The decision to accept the new currency generates a reciprocal externality which reinforces the incentive to use the new currency. Likewise, individuals will start refusing the old currency when the belief that this currency is accepted in non-monitored trades is not sufficient to guarantee a positive surplus $\mathcal{V}_0 - \mathcal{V}_S > 0$.



$$(\Pi_0, \Pi_1) \in \{(1,0),(1,1),(0,1)\}$$

Figure 2. Selection of the equilibrium and dynamics

The trajectory of the economy is described as a succession of steady states where beliefs are formed as indicated above and where the quantity of old currency varies from M to 0. At time T, the old currency ceases to be legal tender. The remaining quantity of old currency in circulation at that time is $M_0(T)$. In figure 2, a trajectory is drawn for a very loose government supervision.[19] Agents accept the old currency even after it has lost its legal tender status. They start

[19]The curves are drawn for the following parameter values: $M = 0.5$, $\beta = 0.9$, $x = 0.1$ and $M_0(T) = M/2$.

to accept the new one when the quantity of old currency is very small. Other scenarios are possible. Agents may accept the new currency at time T, where the old one is no longer legal tender, or even accept it before T. If the legal tender probability is high, individuals accept the two currencies from the beginning. In a similar way, agents may accept the old currency all the time, or start to refuse it at T, or even after T.

4.2 Withdrawal with perfect foresight expectations

Let us assume now that agents have perfect foresight expectations and anticipate that the old currency will have entirely disappeared at some future date T_{end}. At T_{end} the only currency in the economy (namely, the new one) is legal tender. As a consequence, the new currency will be accepted with probability one. Indeed, in so far as sellers are forced to accept this money on a fraction $g > 0$ of trades, and because double coincidence of wants meetings are ruled out, the non-monetary equilibrium cannot exist[20]. From (6) and (7) we have:

$$(1-\beta)\,\mathcal{V}_S = Mx\beta\,(\mathcal{V}_1 - \mathcal{V}_S)\,, \tag{11}$$

$$(1-\beta)\,\mathcal{V}_1 = (1-M)x\,[u + \beta\,(\mathcal{V}_S - \mathcal{V}_1)]\,. \tag{12}$$

According to (11) and (12), the surplus to be a buyer rather than a seller is:

$$\mathcal{V}_1 - \mathcal{V}_S = \frac{x(1-M)}{1-\beta(1-x)}u > 0 \tag{13}$$

It is then rational to accept the new currency for any parameter values.

The next proposition generalizes this result and shows that individuals, who anticipate that at some future time T_{end} no old currency units will be left in the economy, accept the new currency as soon as it is introduced.

Proposition 4 *If at some future date the old currency will be entirely withdrawn, then $\Pi_{1,t} = 1$ for all $t \in \mathbb{N}$.*

Proof. See appendix. ■

In a world with perfect foresight expectations, agents know that the new currency will be accepted in the final steady-state of the economy. Backward induction logic generalizes this result for the whole trajectory of the economy. Note that it does not matter for the acceptability of the new currency whether the government gradually withdraws the old currency until T_{end}, or withdraws it at once.

5 Conversion in steady-state

In the current section and the following one, old currency holders are allowed to convert their currency unit into a new one at cost γ. We study the incentives for

[20] In the case of a positive production cost, the monetary equilibrium is unique if and only if the legal tender probability is above a positive threshold. For a similar argument, see Li and Wright (1998).

agents to switch to the new currency. The conversion cost may be interpreted as the usual leather shoe cost generated by the travel to the Central Bank.[21]

For the sake of simplicity, it is assumed that there is no authoritarian replacement ($\lambda(t) = 0,\ \forall t$). Thus, according to (3), the value of being an old currency holder at the beginning of period t is simply:

$$\widetilde{\mathcal{V}}_0(t) = \max(\mathcal{V}_0(t), \mathcal{V}_1(t) - \gamma)$$

The decision to convert the old currency depends on the conversion cost and on agents' beliefs about the acceptability of the old and new currencies.

We restrict our attention to steady-state Nash equilibria in pure strategies. If old currency holders choose to initiate the conversion to the new currency ($\Omega = 1$) then $M_0 = 0$. Otherwise, $\Omega = 0$ and $M_0 = M$. In both cases the quantity of money is equal to M and is entirely composed of one type of money, the new or the old one, depending on the strategy of old currency holders.

5.1 The two currencies are legal tender

Proposition 5 *Assume that agents are allowed to convert their old units of money into new ones, and that there is no compulsory withdrawal by the government. The two currencies are legal tender. Then there are three types of equilibria:*

(i) An equilibrium without conversion where the two currencies are accepted: $(\Omega, \Pi_0, \Pi_1,) = (0, 1, 1)$ *always exists.*

(ii) An equilibrium without conversion where only the old currency is accepted: $(\Omega, \Pi_0, \Pi_1,) = (0, 1, 0)$ *exists if* $g < \overline{\overline{g}}$

(iii) An equilibrium with conversion where the new currency is accepted but the old one refused: $(\Omega, \Pi_0, \Pi_1,) = (1, 0, 1)$ *exists if* $\overline{\Gamma} < \frac{\gamma}{u} < \Gamma_g$.

The quantities $\overline{\overline{g}}$, $\overline{\Gamma}$ *and* Γ_g *are defined in appendix.*

Proof. See appendix. ■

As previously mentioned, the equilibrium where the two monies are accepted is the "natural" equilibrium. In that case, agents indifferently hold the old or the new currency; so, it is not rational to convert the old currency into the new one if the conversion is costly.

A necessary, but not sufficient, condition for the switch to occur is that the new currency is accepted and the old one refused. This condition can be fulfilled if the conversion cost is neither too low nor too high. Indeed, the belief that the old currency is not accepted, conditional on the fact that the new one is accepted, can be sustained by a Nash equilibrium if the convertion is hampered by a sufficiently high cost, namely $\frac{\gamma}{u} > \overline{\Gamma}$. If the conversion is inexpensive ($\frac{\gamma}{u} < \overline{\Gamma}$), individuals cannot rationally refuse the old currency: they will accept it even if they believe that others refuse it, just because they know that they

[21] We could also have imagined another story where the conversion cost would have been borne by sellers, to update their vending machines for instance. In the S. Anthony dollar experience, this updating cost ranged from \$25 to \$350 (Caskey and St Laurent, 1994).

can convert it easily into the new one. Finally, the more efficient government monitoring is, the lower the conversion cost must be to encourage agents to switch to the new currency.

These results are reported in figure 3 (a).

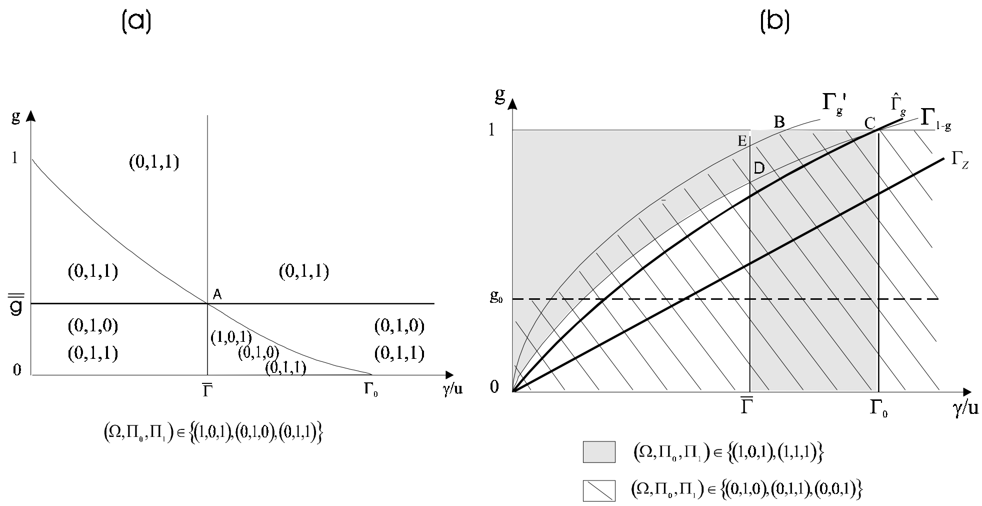


Figure 3. Conversion and legal tender.

The preceding figure shows that the conditions for the emergence of the new currency are very restrictive when the two currencies are legal tender. If government monitoring is strict ($g > \overline{\overline{g}}$), the unique equilibrium is such that the two currencies are accepted but only the old one circulates. For the new currency to emerge, the pair $\left(\frac{\gamma}{u}, g\right)$ must be situated in the region $(A, \Gamma_0, \overline{\Gamma})$ where the legal tender probability is low and the conversion cost lies between two intermediate values ($\overline{\Gamma}$ and Γ_0). Even when those conditions are fulfilled, there are other equilibria where the sole currency in the economy is the old one.[22]

5.2 Only the new currency is legal tender

Proposition 6 *Assume that agents are allowed to convert their old currency units into new ones, and that there is no compulsory withdrawal by the government. Only the new currency is legal tender. Then there are five types of equilibria:*

(i) Three types of equilibria without conversion: $(\Omega, \Pi_0, \Pi_1) \in \{(0,1,1), (0,1,0), (0,0,1)\}$. *At least one of these equilibria occurs if* $\frac{\gamma}{u} > \Gamma_g^{'}$ *or* $g < g_0$.

[22] An alternative assumption would be to consider that the government is able to subsidize agents who choose to convert their old currency into the new one: γ is then negative. Given that the natural equilibrium is such that the two currencies are accepted, individuals would gain by converting their old currency unit into the new one. In this case, the equilibrium $(\Omega, \Pi_0, \Pi_1,) = (1,1,1)$, where individuals switch to the new currency, would always exist. A related issue is the debasement problem in medieval economies: see Velde *et alii* (1998).

(ii) Two types of equilibria with conversion: $(\Omega, \Pi_0, \Pi_1) \in \{(1,1,1), (1,0,1)\}$. *At least one of these equilibria occurs if* $\frac{\gamma}{u} < \Gamma_{1-g}$ *or* $\overline{\Gamma} < \frac{\gamma}{u} < \Gamma_0$.

The quantities $\Gamma_g^{'}$, g_0, Γ_{1-g} *and* Γ_0 *are defined in appendix.*

Proof. See appendix. ■

According to proposition 6, agents may refuse to switch to the new currency $(\Omega = 0)$ if the conversion cost is too high $(\frac{\gamma}{u} > \Gamma_g^{'})$ or if the old currency is accepted and the new one is refused $(g < g_0)$. However, agents may initiate the conversion to the new currency $(\Omega = 1)$ if the conversion is inexpensive $(\frac{\gamma}{u} < \Gamma_{1-g})$ or if the old currency is refused and the new one accepted $(\overline{\Gamma} < \frac{\gamma}{u} < \Gamma_0)$.

According to us, the equilibrium $(\Omega, \Pi_0, \Pi_1) = (0, 0, 1)$ does not deserve too much attention. Indeed, it is unstable in the following sense: if a very small fraction of individuals decide to accept the old currency, then it is rational for all others to accept it.

Results of proposition 6 are illustrated in figure 3 (b). Contrary to proposition 5, when the old currency is illegal, there is a region in the space of parameters where the unique equilibrium is such that agents switch to the new currency. Indeed, a low conversion cost combined with strict monitoring from the government induces old currency holders to exchange their currency unit for a new one.

Paradoxically, there is also a region in the space of parameters, namely $\left(D, \overline{\Gamma}, 0\right)$, where the unique equilibrium is such that old currency holders keep their units of money even when the conversion cost is relatively low. Thus, a low conversion cost is not sufficient to guarantee the switch to the new currency. This can be explained by the following logic. Suppose that it is not costly to convert the old currency into the new one. A seller who is offered a unit of old currency may want to accept it just because it is easy to exchange for legal tender money. As a consequence, the old currency is accepted on the fraction $1-g$ of trades. Under such circumstances, old currency holders who can use their units of money in non-monitored trades will not want to bear the conversion cost to get the new currency.

5.3 Welfare considerations

Let us examine now in which circumstances it is efficient to adopt the new currency that is promoted by the government from the point of view of the whole community of private agents. Doing so, we ignore some gains related to the introduction of a new currency (the seigniorage revenue that can be collected by the government, the saving of some production or issuing costs...) but that are not described in this model. We only focus on the inefficiencies that are generated by the conversion decision.

5.3.1 Both currencies are legal tender

First, assume that the two currencies are legal tender and that agents vote for a coordinated change-over to the new currency. Then, there would be unanimous disagreement for such a switch. Indeed, individuals can be in two positions, seller or buyer (denoted by B). In general, their lifetime expected discounted utility depends on which type of money circulates in the economy. For sellers, according to (11) and (13) we have:

$$\mathcal{V}_S \mid_{\Omega=0} = \mathcal{V}_S \mid_{\Omega=1} = \frac{Mx^2\beta (1-M)}{1-\beta (1-x)} u$$

Sellers are indifferent to whether the old currency or the new one circulates. For buyers, we have:

$$\mathcal{V}_B \mid_{\Omega=0} = \mathcal{V}_0, \qquad \mathcal{V}_B \mid_{\Omega=1} = \mathcal{V}_1 - \gamma$$

If there is a coordinated switch to the new currency, buyers bear the conversion cost γ. The equality $\mathcal{V}_0 = \mathcal{V}_1$ implies that $\mathcal{V}_B \mid_{\Omega=0} > \mathcal{V}_B \mid_{\Omega=1}$. Equilibria such that $\Omega = 0$ Pareto-dominate the equilibrium with $\Omega = 1$. As a consequence, if the equilibrium $(\Omega, \Pi_0, \Pi_1,) = (1, 0, 1)$ occurs, the economy is subject to a coordination failure in strong form. This inefficiency is called an *excess momentum* in the literature about network externalities: people rush to the new currency just because they fear to have a money that is not universally accepted.

5.3.2 Only the new currency is legal tender

We rank the equilibria according to the Pareto criterion when it is possible, and according to an utilitarian criterion. In this last case, the social welfare $\mathcal{Z}$ is measured as follows:

$$\mathcal{Z} = M(1-\beta)\mathcal{V}_B + (1-M)(1-\beta)\mathcal{V}_S$$

It is the sum of the permanent income of buyers and sellers.

Proposition 7 *Assume that agents are allowed to convert their old units of money into new ones, and that there is no compulsory withdrawal by the government. Only the new currency is legal tender.*

(i) When $\frac{\gamma}{u} < \widehat{\Gamma}_g$, equilibria such that $\Omega = 1$ Pareto-dominate equilibria such that $\Omega = 0$. ($\widehat{\Gamma}_g$ is defined in appendix).

(ii) According to an utilitarian criterion, an equilibrium with conversion is socially preferable to an equilibrium without conversion when

$$\frac{\gamma}{u} < \frac{(1-M)\,xg}{1-\beta} \equiv \Gamma_Z$$

Proof. See appendix. ■

On the left side of curve $\widehat{\Gamma}_g$, equilibria where individuals switch to the new currency Pareto-dominate equilibria where old currency holders keep their units of money (see figure 3b). Indeed, sellers are better-off when a legal tender currency circulates because trades are easier. The same is true for buyers because the conversion cost is not too high. On the left of Γ_Z, equilibria where individuals switch to the new currency are socially prefered to equilibria where old currency holders do not initiate the conversion.

In the region $(E, D, 0)$ the two equilibria $(\Omega, \Pi_0, \Pi_1) = (0, 1, 1)$ and $(\Omega, \Pi_0, \Pi_1) = (1, 1, 1)$ exist. In both cases, the two currencies are accepted. The only difference is the decision to switch or not to the new currency. The equilibrium without conversion, namely $(\Omega, \Pi_0, \Pi_1) = (0, 1, 1)$, illustrates a coordination failure. If no-one initiates the conversion, then it is not individually rational to do so. If everyone goes to the Central Bank, then it is rational to follow the movement. The decision to switch to the new currency depends on agents' beliefs about the decision of others, and generates a spillover effect. The social gain of converting an old currency into a new one is superior to the private gain because the new currency will serve as a medium of exchange for the whole community. So, there is a positive network externality in the sense that the surplus an individual gets from converting his old currency into a new one depends on how many others have already opted for the conversion.[23] This effect is called a *bandwagon effect* in the literature about network externalities. Paraphrasing Tirole (1990, p. 407), each individual is happy to jump on the bandwagon, but neither one is sufficiently eager to set it rolling himself, as he might be adopting the new currency. This effect generates *excess inertia* in the sense that money holders stick to their old currency.[24]

In the region $(B, C, \Gamma_0, \overline{\Gamma}, E)$ in figure 3b, there is one equilibrium where the old currency is refused and where people switch to the new currency, and there is another equilibrium where the two currencies are accepted and people do not switch to the new currency. However, the equilibrium $(\Omega, \Pi_0, \Pi_1) = (1, 0, 1)$ may not Pareto-dominate the equilibrium $(\Omega, \Pi_0, \Pi_1) = (0, 1, 1)$. Indeed, sellers are better-off in a world where the only currency is legal tender, but old currency holders may suffer from the non-acceptability of the currency they hold and the conversion cost they bear. According to our utilitarian criterion, the equilibrium with conversion is more efficient then the one without conversion if the legal tender probability is sufficiently high and the conversion cost sufficiently low

[23] To see this point, we can express the difference between the value of being a new currency holder and the value of being an old currency holder as a fonction of the fraction of old currency holders that initiate the conversion. Assume, for simplicity, that the conversion is feasible only in some arbitrary period. The fraction of old currency holders that initiate the conversion is Ω and the quantity of old currency left in the economy is then equal to $(1 - \Omega) M$. From (17)-(19), it can be verified that:

$$\mathcal{V}_1 - \mathcal{V}_0 = \Psi(\Omega) u, \qquad \text{with } \Psi'(.) > 0$$

There is a positive threshold $\overline{\Omega}$ above which $\mathcal{V}_1 - \mathcal{V}_0 \geq \gamma$. As a consequence, if each individual anticipate that the fraction of individuals that adopt the new currency is lower than $\overline{\Omega}$, then no-one will choose to switch to the new currency.

[24] In fact, our model could be reinterpreted in order to consider the introduction of an innovation in the system of payments. This innovation would take the form of a new currency which speeds up trades and reduces transaction costs. This new currency could be digital money. Berentsen (1998) proposes a simple model to study the introduction of digital money in the presence of network externalities. He shows that individuals could still use their old money, even if the new digital money reduces transaction costs.

$(\frac{\gamma}{u} < \Gamma_Z)$.

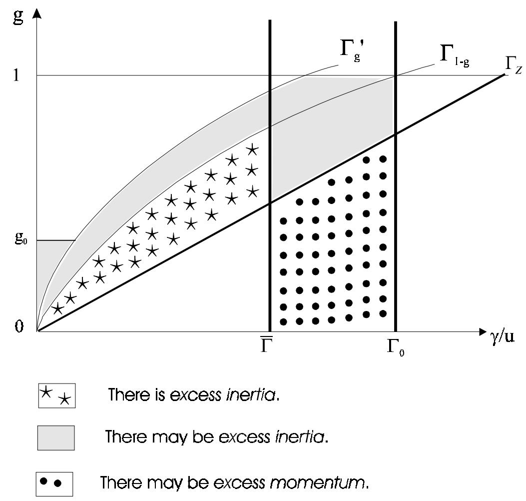


Figure 4. Inefficiencies of the conversion decision.

Results about efficiency can be summarized as follows (see figure 4):[25]
(i) There is a region in which conversion would be efficient but is not an equilibrium. There is *excess inertia*.
(ii) There is a region where equilibria with conversion coexist with equilibria without conversion, and the equilibrium without conversion is inefficient. There may be *excess inertia*.
(iii) There is a region where the unique equilibrium is an equilibrium with conversion and is efficient.
(iv) There is a region where equilibria with conversion coexist with equilibria without conversion, and the equilibrium without conversion is efficient. There may be *excess momentum*.
(v) There is a region where the unique equilibrium is an equilibrium without conversion and is efficient.

6 Conversion in dynamics

Let us now consider a currency reform that takes place over a very short period of time: for instance, the introduction of the Susan B. Anthony Dollar, or the launching of the euro. At time 0, the government announces that the old currency will become illegal at time T. Individuals are forward-looking and have perfect-foresight expectations. We characterize the trajectory of the economy and ask ourselves if such a preannouncement prompts agents to switch to the new currency earlier.

[25] These results are related to those of Farrell and Saloner (1986, p.947) in a model about innovation and network externalities.

For the sake of simplicity, we focus on the case where the new currency is always accepted, $\Pi_{1,t} = 1$ for all t.[26] To solve the dynamics of the model, we resort to a backward induction logic. From T, the environment is stationary and the typology of steady-state equilibria after T has been determined in the previous section. Bellman equations (6)-(8) allow us to deduce the whole trajectory of the economy. We rule out the case where the economy ends up in the fragile equilibrium $(\Omega, \Pi_0, \Pi_1) = (0, 0, 1)$.

Proposition 8 *Assume that agents are allowed to convert their old units of money into new ones, and that there is no compulsory withdrawal by the government. The old currency loses its legal tender status at time T. The different equilibria are:*

(i) Agents always accept the old currency but switch to the new one at time T, if $\frac{\gamma}{u} < \min\left(\Gamma_{1-g}, \overline{\Gamma}\right)$.

(ii) Agents always accept the old currency and never switch to the new one, if $\frac{\gamma}{u} > \Gamma'_g$.

(iii) Agents adopt the new currency from the beginning and always refuse the old one, if $\overline{\Gamma} < \frac{\gamma}{u} < \Gamma_g$.

(iv) Agents refuse the old currency from $t^* < T$. They switch to the new currency at time T. This equilibrium can occur if $\max\left(\Gamma_g, \overline{\Gamma}\right) < \frac{\gamma}{u} < \Gamma_0$.

The quantities Γ_g, Γ_{1-g}, Γ'_g, $\overline{\Gamma}$ and Γ_0 are defined in appendix.

Proof. See appendix. ■

To sum up, the possible outcomes of the dynamics are rather limited. If agents switch to the new currency, it happens either in time 0 or T. Furthermore, if the old currency is not always accepted, then it started to be refused in 0 or t^*.

The typology of equilibria is represented in figure 5.

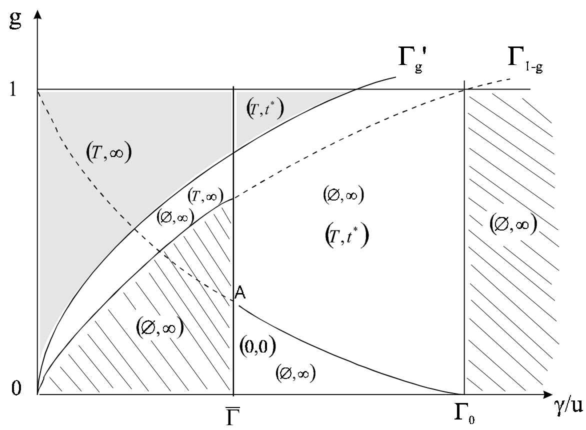


Figure 5 : Dynamics.

[26] There are two arguments to justify this simplification. First, if agents anticipate that the old currency will disappear at some future time, then it is the sole rational strategy. Second, the acceptability of the new currency is the "natural" strategy and it can always sustain a Nash equilibrium.

(T,∞): Switch to the new currency at T. The old currency is always accepted.
$(\emptyset,\infty)$: No switch to the new curreny. The old currency is always accepted.
(T,t^*): Switch to the new currency at T. The old currency is refused after t^*.
$(0,0)$: Switch to the new currency at 0. The old currency is never accepted.

There is a region in the space of parameters (namely, the grey-tainted region) where the unique equilibrium is such that agents switch to the new currency at time T. For this to happen, the legal tender probability must be high and the conversion not too expensive. The time when the old currency becomes illegal acts as a coordination device. The old currency is either always accepted or accepted until some date $t^* < T$. It is noteworthy that the length of the period of dual currency circulation has no role to play in the decision to switch.

There is another region where the unique equilibrium is such that agents keep using the old currency, even when it becomes illegal (namely, the region with oblique lines). There are two justifications for this type of equilibrium. The most obvious one is that a prohibitive cost (namely, $\frac{\gamma}{u} > \Gamma_0$) may prevent agents to initiate the conversion. The second explanation is the following one. If legal tender laws are not too strict and if the conversion is inexpensive ($\frac{\gamma}{u} < \overline{\Gamma}$), then the old currency will be accepted. Because the acceptabilities of the old and new currencies are not too different, old currency holders will prefer to keep their units of money rather than switch to the new one.

Finally, the announcement that the old currency will become illegal after T does not increase the chances of a switch to the new currency before T. Indeed, the region where the switch may occur at the initial period is $\left(A,\Gamma_0,\overline{\Gamma}\right)$. But in this region, a switch to the new currency can occur even if the government does not announce that the old currency will become illegal in the future. The absence of preannouncement effect comes from the fact that the decision not to adopt the new currency is not irreversible: individuals can change their mind when the old money becomes illegal.

The main lesson we can draw from this section is the following one. To induce agents to switch to the new currency, a low conversion cost is a necessary condition. However, this condition is not sufficient. In particular, if legal tender laws are very loose, the likeliest equilibrium is one where individuals keep using the old currency. As a consequence, the low conversion cost must be associated with strict legal tender laws.

7 Further results

A natural extension at this stage would be to consider both a partial withdrawal of the old currency, and the possibility offered to old currency holders to convert their units of money into new ones. We want to argue briefly that the previous results are not qualitatively modified by this extension.

First, let us consider the case where the two currencies are legal tender. The natural equilibrium is still $(\Omega,\Pi_0,\Pi_1) = (0,1,1)$ for any value of M_0.

Furthermore, withdrawing the old currency does not increase the chances of a switch to the new currency: that is, the equilibrium $(\Omega, \Pi_0, \Pi_1) = (1, 0, 1)$ exists for the same value of the parameters. However, a compulsory withdrawal of the old currency makes the equilibrium $(\Omega, \Pi_0, \Pi_1) = (0, 1, 0)$ less likely. Indeed, the new currency is refused and agents do not switch to the new one if:

$$g < \frac{\beta x M_0}{1 - \beta + (1 - M + M_0)\,\beta x}$$

Second, let us turn to the case where only the new currency is legal tender. A withdrawal of the old currency makes equilibria with no conversion less likely. As an example, the equilibrium $(\Omega, \Pi_0, \Pi_1) = (0, 1, 1)$ exists if it is rational to accept the old currency:

$$g < \frac{1 - \beta + \beta x(1 - M)}{1 - \beta\,(1 - x) - \beta x M_0},$$

and if old currency holders do not want to switch to the new currency, that is:

$$\frac{\gamma}{u} > \frac{(1 - M)\,xg\,(1 - \beta + x\beta\,(M - gM_0))}{(1 - \beta)\,(1 - \beta + \beta x\,(1 - gM_0)) + (1 - M)\,x\,(1 - g)\,\beta\,(1 - \beta + \beta x)}$$

The RHS is decreasing in M_0 and approaches Γ_{1-g} when M_0 is close to 0. As a consequence, a withdrawal of the old currency prevents the economy from a coordination failure where old currency holders keep their units of money even though it would be better for everyone to switch to the new currency. Such a measure is called a "pump-priming" in the literature about innovation.

Similarly, it can be shown that the equilibria $(\Omega, \Pi_0, \Pi_1) = (0, 1, 0)$ and $(\Omega, \Pi_0, \Pi_1) = (0, 0, 1)$ disappear if the government withdraws enough money of the old type. Indeed, when the quantity of old currency is small enough, individuals find it rational either to accept the new one or to switch to the new one, depending on the acceptability of the old currency.

8 Numerical exercises

This section aims to illustrate these last results. We assume that there is a withdrawal of the old currency at some constant rate λ and that old currency holders can switch to the new currency at any time.

The methodology to compute the dynamics is the following one. First, we choose some values for the parameters γ, g, M, β, x, T, and λ. At time $T_{end} = 1000$, $M_{0,T_{end}}$ is approximated by 0 and the value functions are set equal to their steady-state value. The lifetime expected utility of a buyer and a seller at the end of time, $\mathcal{V}_{1,T_{end}}$ and $\mathcal{V}_{S,T_{end}}$, are given by (11) and (12). To compute $\mathcal{V}_{0,T_{end}}$, we have to determine the acceptance strategy of the old currency and the conversion decision. In case of multiple equilibria, we arbitrarily select one of these equilibria.

We work by backward induction, using equations (6)-(10). The quantity of old currency at each period depends on the parameter λ and the decision of currency holders to switch to the new currency. Finally, agents' strategies are fully determined by (2), (4) and (5).

Simulations show that the dynamics of the model are not qualitatively altered by the introduction of a compulsory withdrawal of the old currency. The switch to the new currency occurs either in 0 or T, or never occurs. Furthermore, the old currency may cease to be accepted before it loses its legal tender status.

The first set of simulations illustrates the four cases of proposition 8. In the first case, the old currency is always accepted, but agents switch to the new one when it ceases to be legal tender. In the second case, the old currency is always accepted, and there is no switch to the new currency: the quantity of old money decreases at the rate λ over time. In the third case, the old currency is never accepted and agents immediatly convert their old currency units into new ones. In the fourth case, the old currency ceases to be accepted before T, but agents switch to the new currency in T.

The second set of simulations illustrates the multiplicity of equilibria. According to agents beliefs, the old currency may be accepted all the time, or may be refused after some time. Furthermore, agents beliefs about the acceptability of the old currency determine their decision to switch to the new currency. If they anticipate that the old currency will not be accepted, they immediatly switch to the new currency; otherwise, they never switch.

9 Back to historical episodes

In this section, we consider the historical episodes mentioned in the introduction in light of the results of the model, and see if they are consistent with the different propositions of the paper.

9.1 The coin/note substitution

Proposition 2 offers an explanation for the unsuccessful attempt of the US government to introduce the Susan B. Anthony dollar coin, to replace the existing one dollar bill, in 1979. Indeed, because the one dollar bills were still legal tender, people continued to accept them. Moreover, because the government did not withdraw the one dollar bills from circulation, individuals did not need another medium of exchange for small transactions. Furthermore, according to proposition 5, people who were allowed to convert their bills into coins, saw no good reasons to switch to the dollar coin. According to Caskey and St Laurent (1994, p.503), "*a well-marketed coin will generally fail to replace a circulating low-denomination bill, unless the government actively induces the public to switch to the new coin*".

Even in the presence of compulsory measures, legal tender laws for instance, propositions 6 and 7 indicate that a network externality may generate coordination failures and induce agents to stay with their old currency.

Caskey and St Laurent (1994) report the lessons learned by monetary authorities in six industrialized countries from their experiences in replacing low-denomination bills with coins. The lessons are the following ones: (i) Notes must be eliminated; (ii) An information campaign must be organized; (iii) The government must expect public resistance and be strong in its campaign to convert; (iv) Sufficient coins must be made available; (v) Coins must have a distinct appearance. The first and fourth lessons are consistent with proposition 2. The third lesson is taken into account in proposition 3. The second lesson may be related to the multiplicity of equilibria inherent in any model about currency acceptability. Indeed, an information campaign may be a way to coordinate agents to one equilibrium.[27]

Canada learned from the setbacks of the Susan B. Anthony dollar and, by withdrawing the competing form of money, successfully introduced the “loonie” dollar coin on July 1^{st}, 1987.

Since January, 2000, the United-States has introduced a new one dollar coin, the Golden dollar, to replace the existing one dollar bill again. However, it seems that this new currency will probably fail replacing the bill, as did the Susan B. Anthony dollar coin. Indeed, the causes of the failure of this coin (lack of information, absence of compulsory withdrawal of the one dollar bill, existence of network externalities) have not been seriously taken into account by the government to introduce the new Golden dollar, however the recommendations. Again, our model illustrates quite well the necessary measures that should be followed in order to finally garantee the success of the introduction of this Golden coin. In particular, the withdrawal of the one dollar bill and the abolition of its legal tender status should be implemented.

9.2 The long-lasting cowries and manillas

The Nigerian experience is a good example in support of our theory. Traditional monies (manillas and cowries) continued to circulate in Nigeria until 1950, although the British currency was introduced at the end of the 19^{th} century and the use of the old currencies were made illegal by the British government. This failure can be explained by proposition 3 : even if the old currency is made illegal by the government, individuals may go on accepting the old currency and refusing the new one if the withdrawal of the competing form of money is not large enough.

Until the beginning of the 20^{th} century, the British government considered cowries and manillas as barter items rather than full monies. Because of this misunderstanding, he did not want to redeem them. Later on, he understood the need to withdraw shell currencies in order to guarantee the acceptability of its own currency and the abandonment of the old one. That is the reason why

[27] Allison and Pianalto (1997) report the issuing strategy that has been followed by the Fed for the newly designed $100 note: no recall of notes with the old design, no deadline for exchanging them, but the withdrawal of pre-series-1996 notes when they are deposited at Federal Reserve Banks.

he first prohibited the import of shells currencies by a decree[28] in 1902, and then started, in 1948, a compulsory withdrawal of the manilla from circulation with payment of compensation in British currency.[29]

9.3 The rapid move in the FSU republics and in the Federation of Yougoslavia

According to Hansson (1993), there are two dinstinct ways to introduce a new currency. On the one hand, the new currency can be issued gradually without immediatly withdrawing the old money form (Latvia, Lithuania, Ukraine). In this case, the two currencies coexist during a limited period of time. On the other hand, the government can choose a rapid exchange, over a few days (Estonia, Kyrgyzstan, Slovenia, Macedonia).

A rapid currency reform requires a voluntary switch to the new currency. The Estonian outcome, that can be explained by proposition 6, highlights the importance of legal tender laws in such a switch. By declaring the Russian rouble not legal tender, the Estonian government triggered the switch to the kroon. Karmo (1995) documents the long lines at the exchange offices when the Estonian kroon became the only legal tender currency in Estonia. Between June 20$^{\text{th}}$ and 22$^{\text{nd}}$, 1992, about 2.2 billion cash rubles were converted into kroons (Lainela, 1993). Karmo also describes how the Estonian authorities resisted legalizing the rouble by fear of the "roubleisation" that could have been generated by such a decision.

In Kyrgyzstan, only a partial conversion to the new currency was achieved: one quarter of cash roubles in circulation were converted into *som* (Melliss and Cornelius, 1994). In the same way, in Slovenia, the amount of dinars that were converted into the Slovene tolar was far less than the amount of cash that had entered circulation (Mencinger,1993). According to our model, these scenarios could be interpreted as excess inertia generated by the network externality in the decision to switch (see propositions 6 and 7).

In Latvia, the Latvian rouble became the only legal tender currency from July 20$^{\text{th}}$, 1992, but the use of foreign currencies was not forbidden. As expected from proposition 2, companies continued to use the Russian rouble to a large extent in their payments: it is estimated that, at the end of 1992, between one third and a half of all transactions were still in foreign currencies, and approximately 15 per cent in Russian roubles (Lainela and Sutela, 1994). The announcement of the gradual introduction of the lats was made in March 1993. By June, two-thirds of the currency in circulation was in lats, and the last Latvian rouble notes were withdrawn from circulation in October 1993 (Lainela and Sutela, 1994).

[28] This decree is the Native Currency Proclamation No 14. See Ofonagoro (1979, p.643).

[29] It was the so-called "operation manilla". See Ofonagoro (1979, p.646).

10 Conclusion

This paper has studied the launching of new fiat currency within the search-theoretic framework developed by Kiyotaki and Wright. Although the model is simple, it has allowed us to highlight some of the salient features of a currency reform consisting of replacing an old fiat currency with a new one. We have first emphasized the role of a compulsory withdrawal of the old currency to promote the acceptability of the new one. Without a massive withdrawal, a currency reform is doomed to failure. There is evidence to confirm this result: Nigeria, where it took more than fifty years before the British currency were accepted, or the United States, where the Fed did not successfully introduce the Susan B. Anthony dollar coin at the end of the 70's.

Second, we have determined the circumstances under which individuals voluntarily switch to the new currency. It has been demonstrated that legal tender laws could favor a voluntary conversion of old currency units into new ones. Furthermore, a new externality in search models of money has been identified. Indeed, the decision to switch to the new currency generates spillover effects. For some values of the parameters, this network externality can generate excess inertia: old currency holders keep using their unit of money even though it would be mutually beneficial to adopt the new one. Again, there are examples in support of this prediction: in Kyrgyzstan, the introduction of a new currency following the collapse of the Soviet Union was a partial success: only one quarter of cash roubles in circulation were converted into the new currency.

Finally, it has been demonstrated that the length of the period of dual currency circulation does not play any role in the decision to switch. However, the time at which the old currency becomes illegal acts as a coordination device and triggers the transition to the new currency.

A natural extension of this model would be to introduce prices and to endogenize the terms of trade between old and new currencies. This part is left to future investigation.

Appendix

A1 : Proof of propositions 2 and 3

(i) The two currencies are legal tender.

In the steady-state where $(\Pi_0, \Pi_1) \in \{0,1\}^2$ sustains a Nash equilibrium, the Bellman equations (6)-(8) can be rewritten as follows:

$$(1-\beta)\,\mathcal{V}_S = x\,(M-M_0)\,[g+(1-g)\Pi_1]\,\beta\,(\mathcal{V}_1-\mathcal{V}_S) +$$

$$+xM_0\,[g+(1-g)\Pi_0]\,\beta\,(\mathcal{V}_0-\mathcal{V}_S)\,, \tag{14}$$

$$(1-\beta)\,\mathcal{V}_1 = (1-M)\,x\,[g+(1-g)\Pi_1]\,[u+\beta\,(\mathcal{V}_S-\mathcal{V}_1)]\,, \tag{15}$$

$$(1-\beta)\,\mathcal{V}_0 = (1-M)\,x\,[g+(1-g)\Pi_0]\,[u+\beta\,(\mathcal{V}_S-\mathcal{V}_0)]\,. \tag{16}$$

(ii) Only the new currency is legal tender.

The Bellman equations (9), (7) and (10) associated with any (Π_0, Π_1) equilibrium can be rewritten as follows:

$$(1-\beta)\,\mathcal{V}_S = x\,(M-M_0)\,[g+(1-g)\Pi_1]\,\beta\,(\mathcal{V}_1-\mathcal{V}_S) +$$

$$+xM_0(1-g)\Pi_0\beta\,(\mathcal{V}_0-\mathcal{V}_S)\,, \tag{17}$$

$$(1-\beta)\,\mathcal{V}_1 = (1-M)\,x\,[g+(1-g)\Pi_1]\,[u+\beta\,(\mathcal{V}_S-\mathcal{V}_1)]\,, \tag{18}$$

$$(1-\beta)\,\mathcal{V}_0 = (1-M)\,x(1-g)\Pi_0\,[u+\beta\,(\mathcal{V}_S-\mathcal{V}_0)]\,. \tag{19}$$

For a given M_0, a steady-state equilibrium is a 5-uplet $(\mathcal{V}_0,\ \mathcal{V}_1,\ \mathcal{V}_S,\ \Pi_0,\ \Pi_1)$ where $(\mathcal{V}_0,\ \mathcal{V}_1,\ \mathcal{V}_S)$ satisfy (14)-(16) or (17)-(19) and $(\Pi_0,\ \Pi_1)$ are consistent with the individual rationality criteria (4) and (5).

For each possible equilibrium, we replace Π_0 and Π_1 by their value into (14)-(16) or (17)-(19), and we determine the set of conditions under wich the individual rationality criteria (4) and (5) are fulfilled.

A 2: Proof of proposition 4

Let us first demonstrate the following lemma.

Lemma 9 *For all* t*,* $\mathcal{V}_{1,t} > \mathcal{V}_{0,t}$.

Proof. Assume that for a given $t < T_{end}$, $\mathcal{V}_{1,t+1} > \mathcal{V}_{0,t+1}$. It follows that:

$$\mathcal{V}_{1,t+1} > \max\left[(1-\lambda_{t+1})\,\mathcal{V}_{0,t+1} + \lambda_{t+1}\mathcal{V}_{1,t+1};\ \mathcal{V}_{1,t+1} - \gamma\,(1-\lambda_{t+1})\right],$$

with

$$\max\left[(1-\lambda_{t+1})\,\mathcal{V}_{0,t+1}+\lambda_{t+1}\mathcal{V}_{1,t+1};\ \mathcal{V}_{1,t+1}-\gamma\,(1-\lambda_{t+1})\right]=\widetilde{\mathcal{V}}_{0,t+1}$$

As a consequence:

$$u+\beta\mathcal{V}_{S,t+1}>\beta\mathcal{V}_{1,t+1}>\beta\widetilde{\mathcal{V}}_{0,t+1}$$

Furthermore,

$$\mathcal{V}_{1,t+1}-\mathcal{V}_{S,t+1}>\widetilde{\mathcal{V}}_{0,t+1}-\mathcal{V}_{S,t+1}.$$

Thus:

$$\Pi_{1,t}\geq\Pi_{0,t}$$

and

$$(1-M)x\left[g+\Pi_{1,t}(1-g)\right]\geqslant(1-M)x\left[g+\Pi_{0,t}(1-g)\right]>(1-M)x\Pi_{0,t}(1-g).$$

It can be deduced from (7), (8) and (10) that:

$$\mathcal{V}_{1,t}>\mathcal{V}_{0,t}$$

To conclude, it can be verified that $\mathcal{V}_{1,T_{end}}>\mathcal{V}_{0,T_{end}}$. ■

At time T_{end}, there is no old currency left in the economy: so, the new currency is accepted. Assume now that $\Pi_{1,t}=1$, the new currency is accepted at the period $t\leq T_{end}$. Then:

$$\mathcal{V}_{1,t+1}>\mathcal{V}_{S,t+1}.$$

Because $u+\beta\mathcal{V}_{S,t+1}>\beta\mathcal{V}_{1,t+1}$, and from (7), it can be shown that:

$$\mathcal{V}_{1,t}>\beta\mathcal{V}_{1,t+1}. \tag{20}$$

According to the previous lemma, $\mathcal{V}_{0,t+1}<\mathcal{V}_{1,t+1}$. Furthermore, by assumption $\mathcal{V}_{S,t+1}<\mathcal{V}_{1,t+1}$. Thus, from (6) and (9):

$$\mathcal{V}_{S,t}<\beta\mathcal{V}_{1,t+1}. \tag{21}$$

Then, (20) and (21) imply:

$$\mathcal{V}_{1,t}>\beta\mathcal{V}_{1,t+1}>\mathcal{V}_{S,t}\Rightarrow\Pi_{1,t-1}=1.$$

The new currency is accepted at period $t-1$. Backward reasoning permits to conclude.

A 3: Proof of proposition 5

From (8), if the old currency is legal tender, $\mathcal{V}_0$ is given by the following Bellman equation:

$$\mathcal{V}_0 = (1-M)x\left[g + \Pi_0(1-g)\right](u + \beta\mathcal{V}_S) +$$

$$+\left\{1-(1-M)x\left[g+\Pi_0(1-g)\right]\right\}\beta\left\{\Omega\left(\mathcal{V}_1-\gamma\right)+(1-\Omega)\mathcal{V}_0\right\} \qquad (22)$$

The novelty in (22) is the second term of RHS. If the old currency holder has not traded his currency unit for his consumption good at the end of the period, then he switches to the new currency if Ω is equal to one.

A steady-state equilibrium is a 6-uplet ($\mathcal{V}_0$, $\mathcal{V}_1$, $\mathcal{V}_S$, Π_0, Π_1, Ω) where ($\mathcal{V}_0$, $\mathcal{V}_1$, $\mathcal{V}_S$) satisfy Bellman equations (14), (15) and (22) and (Π_0, Π_1, Ω) are consistent with individual rationality criteria (2), (4) and (5).

There are three types of equilibria:

$$(\Omega, \Pi_0, \Pi_1) \in \left\{(0,1,1), (0,1,0), (1,0,1)\right\}$$

Other equilibria are impossible.

1. $(\Omega, \Pi_0, \Pi_1) = (0,1,1)$

   This equilibrium always exists.

2. $(\Omega, \Pi_0, \Pi_1) = (0,1,0)$

   The condition $\mathcal{V}_0 > \mathcal{V}_S$ is always verified. The new currency is refused if:

   $$g < \frac{xM\beta}{1-\beta(1-x)} = \overline{\overline{g}}$$

3. $(\Omega, \Pi_0, \Pi_1) = (1,0,1)$

   The old currency must be refused on the fraction $(1-g)$ of the meetings that are not supervised by the government. The old currency is not accepted if $\widetilde{\mathcal{V}}_0 - \mathcal{V}_S < 0$, which can be rewritten as $\mathcal{V}_1 - \mathcal{V}_S < \gamma$, or

   $$\frac{\gamma}{u} > \frac{(1-M)\,x}{1-\beta\,(1-x)} = \overline{\Gamma} \qquad (23)$$

   Furthermore, it is rational to convert the old currency into the new one if $\mathcal{V}_1 - \mathcal{V}_0 > \gamma$, that is:

   $$\frac{\gamma}{u} < \frac{\left\{1-\beta\,(1-xM)\right\}(1-M)x(1-g)}{\left\{1-\beta+(1-M)\,\beta xg\right\}\left\{1-\beta\,(1-x)\right\}} = \Gamma_g \qquad (24)$$

A 4: Proof of proposition 6

From (10), $\mathcal{V}_0$ obeys:

$$\mathcal{V}_0 = (1-M)x\Pi_0(1-g)\,(u+\beta\mathcal{V}_S) +$$

$$+\{1-(1-M)x\Pi_0(1-g)\}\,\beta\,\{\Omega\,(\mathcal{V}_1-\gamma)+(1-\Omega)\mathcal{V}_0\} \tag{25}$$

There are five types of equilibria:

$$(\Omega,\Pi_0,\Pi_1)\in\{(0,1,1)\,,(0,1,0)\,,(0,0,1)\,,(1,1,1)\,,(1,0,1)\}$$

It can be verified that other equilibria are impossible.

1. $(\Omega,\Pi_0,\Pi_1)=(0,1,1)$

   According to proposition 3, the old currency is accepted even if it is not legal tender. Old currency holders choose not to convert their currency if $\mathcal{V}_1-\mathcal{V}_0<\gamma$, which gives:

   $$\frac{\gamma}{u}>\frac{[1-\beta+\beta x(1-g)M]}{[1-\beta+\beta x(1-g)]}\frac{(1-M)xg}{[1-\beta+(1-M)\beta x]}=\Gamma_g' \tag{26}$$

2. $(\Omega,\Pi_0,\Pi_1)=(0,1,0)$

   The new currency is refused on the fraction of meetings that are not monitored if $\mathcal{V}_1-\mathcal{V}_S<0$, which gives:

   $$g<g_0$$

   where g_0 satisfies

   $$M=\frac{g_0\,[1-\beta+x\beta(1-g_0)]}{\beta x(1-g_0)^2}$$

   This condition automatically implies that $\mathcal{V}_1-\mathcal{V}_0<\gamma$: it is rational not to convert the old currency into the new one.

3. $(\Omega,\Pi_0,\Pi_1)=(0,0,1)$

   The condition $\mathcal{V}_1>\mathcal{V}_S=\mathcal{V}_0=0$ is verified for all the parameters. There is no switch to the new currency if $\mathcal{V}_1-\mathcal{V}_0<\gamma$, or equivalently,

   $$\frac{\gamma}{u}>\frac{(1-M)\,x}{1-\beta+\beta x\,(1-M)}=\Gamma_1'$$

4. $(\Omega,\Pi_0,\Pi_1)=(1,0,1)$

   The old currency is refused by sellers if $\widetilde{\mathcal{V}}_0-\mathcal{V}_S=\mathcal{V}_1-\mathcal{V}_S-\gamma<0$, that is:

   $$\frac{\gamma}{u}>\frac{x\,(1-M)}{1-\beta\,(1-x)}=\overline{\Gamma} \tag{27}$$

   Agents convert the old currency into the new one if $\mathcal{V}_1-\mathcal{V}_0>\gamma$, or:

   $$\frac{\gamma}{u}<\frac{(1-M)\,x\,(1-\beta+\beta xM)}{[1-\beta\,(1-x)]\,(1-\beta)}=\Gamma_0 \tag{28}$$

   The new currency may circulate but agents can still believe that the old currency is accepted if the conversion cost is neither too high nor too low.

5. $(\Omega, \Pi_0, \Pi_1) = (1, 1, 1)$

The old currency is accepted if (27) is not fulfilled. Finally, the condition for a conversion ($\mathcal{V}_1 - \mathcal{V}_0 > \gamma$) can be rewritten as:

$$\frac{\gamma}{u} < \frac{(1-M)\,xg}{[1-\beta+\beta\,(1-M)\,x(1-g)]}\frac{[1-\beta\,(1-xM)]}{[1-\beta\,(1-x)]} = \Gamma_{1-g} \qquad (29)$$

A 5: Proof of proposition 7
(i) Pareto criterion.
From (6) and (9), the value of being a seller satisfies:

$$(1-\beta)\,\mathcal{V}_S = M\widetilde{x}\beta\,(\mathcal{V}_\Omega - \mathcal{V}_S)$$

where $\widetilde{x} = x$ if $\Omega = 1$ and $\widetilde{x} = x(1-g)$ if $\Omega = 0$, and where $\mathcal{V}_\Omega$ is the value of holding one type of money conditionnal on the fact that this is the only type of money in circulation in the economy. From (8), (7) and (10), the value of being a currency holder satisfies:

$$(1-\beta)\,\mathcal{V}_\Omega = (1-M)\,\widetilde{x}\beta\,(\mathcal{V}_S - \mathcal{V}_\Omega)$$

From the two previous equations, we obtain:

$$\begin{aligned} \mathcal{V}_S &= M\widetilde{x}\beta\frac{(1-M)\,\widetilde{x}}{[1-\beta\,(1-\widetilde{x})]\,(1-\beta)}u, \\ \mathcal{V}_\Omega &= (1-M)\,\widetilde{x}\frac{(1-\beta+\beta\widetilde{x}M)}{[1-\beta\,(1-\widetilde{x})]\,(1-\beta)}u \end{aligned}$$

It can be verified that $\mathcal{V}_S$ is increasing with $\widetilde{x}$. As a consequence:

$$\mathcal{V}_S\,|_{\Omega=1} > \mathcal{V}_S\,|_{\Omega=0}$$

The buyer is better-off in an equilibrium with conversion if $\mathcal{V}_1\,|_{\Omega=1} - \gamma > \mathcal{V}_0\,|_{\Omega=0}$, which can be rewritten as:

$$\frac{\gamma}{u} < \widehat{\Gamma}_g$$

with

$$\widehat{\Gamma}_g = \frac{(1-M)\,x}{1-\beta}\left[\frac{(1-\beta+\beta xM)}{(1-\beta+\beta x)} - \frac{(1-g)\,[1-\beta+M\beta x\,(1-g)]}{(1-\beta+\beta x\,(1-g))}\right]$$

The curve $\widehat{\Gamma}_g$ is increasing with g, is equal to 0 in $g = 0$ and equal to Γ_0 in $g = 1$. Furthermore, $\frac{\widehat{\Gamma}_g}{\Gamma_{1-g}} > 1$ can be written as

$$\frac{(\beta x - \beta xM + 1 - \beta)}{[1-\beta+\beta\,(1-M)\,x\,(1-g)]} > \frac{[1-\beta\,(1-x)]\,[1-\beta+\beta Mx\,(1-g)]}{[1-\beta\,(1-xM)]\,(1-\beta+\beta x\,(1-g))}$$

and then

$$M\beta^2x^2g\,(1-M)\,(\beta x\,(1-g)+(1-\beta)\,(2-g))>0$$

As a consequence, $\widehat{\Gamma}_g > \Gamma_{1-g}$.
(ii) Utilitarian criterion.
After some calculation, we obtain:

$$\mathcal{Z}=M(1-M)x(1-g\,(1-\Omega))u-M(1-\beta)\Omega\gamma$$

The social welfare is equal to the number of trades per period, multiplied by the utility of consumption, minus a term proportional to the conversion cost.

A 6: Proof of proposition 8

The perfect foresight trajectory of the economy is determined by a backward induction logic. After T, there are four types of steady-state equilibria such that the new currency is accepted. Let $(\Omega_\infty, \Pi_{0,\infty})$ denote a Nash equilibrium where a fraction Ω_∞ of old curreny holders switch to the new curreny at each period, and where the old currency is accepted with probability $\Pi_{0,\infty}$:

$$(\Omega_\infty, \Pi_{0,\infty}) \in \{(1,1)\,,(0,1)\,,(1,0)\,,(0,0)\}$$

By assumption, the fragile equilibrium $(\Omega_\infty, \Pi_{0,\infty}) = (0,0)$ is ruled out. The value functions after T are denoted $\mathcal{V}_{S,\infty}$, $\mathcal{V}_{0,\infty}$ and $\mathcal{V}_{1,\infty}$.

$$\mathcal{V}_{S,t}=\mathcal{V}_{S,\infty},\quad \mathcal{V}_{0,t}=\mathcal{V}_{0,\infty},\quad \mathcal{V}_{1,t}=\mathcal{V}_{1,\infty}\qquad \forall t\geq T$$

1. $(\Omega_\infty, \Pi_{0,\infty}) = (1,1)$

   Assume that the equilibrium is such that agents do not switch to the new currency before T and that $\Pi_{0,t}=1$ for all $t<T$. According to (6) and (8)

   $$\mathcal{V}_{S,t}=Mx\beta\widetilde{\mathcal{V}}_{0,t+1}+(1-Mx)\beta\mathcal{V}_{S,t+1},$$

   $$\widetilde{\mathcal{V}}_{0,t}=(1-M)x\,[u+\beta\mathcal{V}_{S,t+1}]+\{1-(1-M)x\}\,\beta\widetilde{\mathcal{V}}_{0,t+1},$$

   where $\widetilde{\mathcal{V}}_{0,t}=\mathcal{V}_{0,t}$ for all $t<T$. By difference,

   $$\widetilde{\mathcal{V}}_{0,t}-\mathcal{V}_{S,t}=(1-M)xu+(1-x)\,\beta\left(\widetilde{\mathcal{V}}_{0,t+1}-\mathcal{V}_{S,t+1}\right)$$

   As a consequence, $\widetilde{\mathcal{V}}_{0,T}-\mathcal{V}_{S,T}>0$ implies $\widetilde{\mathcal{V}}_{0,t}-\mathcal{V}_{S,t}>0$ for all $t<T$. Therefore, it is always individually rational to accept the old currency.

   According to (7), the value of being a new currency holder in t satisfies:

   $$\mathcal{V}_{1,t}=(1-M)x\,[u+\beta\mathcal{V}_{S,t+1}]+[1-(1-M)x]\,\beta\mathcal{V}_{1,t+1},$$

By difference, we have:

$$\mathcal{V}_{1,t} - \widetilde{\mathcal{V}}_{0,t} = [1-(1-M)x]\,\beta\left(\mathcal{V}_{1,t+1} - \widetilde{\mathcal{V}}_{0,t+1}\right) \qquad \forall t < T$$

Furthermore, $\mathcal{V}_{1,T} - \widetilde{\mathcal{V}}_{0,T} = \gamma$. Therefore, for all $t < T$, $\mathcal{V}_{1,t} - \widetilde{\mathcal{V}}_{0,t} = \mathcal{V}_{1,t} - \mathcal{V}_{0,t} < \gamma$. Thus, it is not rational to convert the old currency in $t < T$. As a consequence, the profile of strategies $\{(\Omega_t, \Pi_{0,t}) = (0,1), \forall t < T; (\Omega_t, \Pi_{0,t}) = (1,1), \forall t \geq T\}$ sustains a subgame perfect equilibrium.

2. $(\Omega_\infty, \Pi_{0,\infty}) = (0,1)$
   A similar reasoning shows that $(\Omega_t, \Pi_{0,t}) = (0,1)$ for all $t \in \mathbb{N}$.

3. $(\Omega_\infty, \Pi_{0,\infty}) = (1,0)$
   $\widetilde{\mathcal{V}}_{0,T} - \mathcal{V}_{S,T} < 0 \Rightarrow \Pi_{0,T-1} = 0$. At time $T-1$, the old currency is not accepted. From (7) and (8), $\mathcal{V}_{1,T-1}$ and $\mathcal{V}_{0,T-1}$ obey the following equations:

$$\mathcal{V}_{1,T-1} = (1-M)x\,(u + \beta\mathcal{V}_{S,T}) + [1-(1-M)x]\,\beta\mathcal{V}_{1,T},$$

and

$$\mathcal{V}_{0,T-1} = (1-M)gx\,(u + \beta\mathcal{V}_{S,T}) + [1-(1-M)gx]\,\beta\,(\mathcal{V}_{1,T} - \gamma)$$

By difference,

$$\mathcal{V}_{1,T-1} - \mathcal{V}_{0,T-1} = (1-M)\,x(1-g)\,\{u + \beta\,(\mathcal{V}_{S,T} - \mathcal{V}_{1,T})\} + [1-(1-M)xg]\,\beta\gamma$$

From (13),

$$\mathcal{V}_{1,T} - \mathcal{V}_{S,T} = \frac{x(1-M)}{1-\beta\,(1-x)}u$$

It is rational to switch to the new currency at time $T-1$ if $\mathcal{V}_{1,T-1} - \mathcal{V}_{0,T-1} > \gamma$, or:

$$\frac{\gamma}{u} < \Gamma_g = \frac{(1-\beta+\beta xM)\,(1-M)\,x\,(1-g)}{[1-\beta\,(1-x)]\,\{1-[1-(1-M)\,xg]\,\beta\}}$$

So, we distinguish two cases.

- $\frac{\gamma}{u} < \Gamma_g$
  Assume that $(\Omega_t, \Pi_{0,t}) = (1,0)$ for all t. From (6), (7) and (9) we have:

$$\begin{aligned} \mathcal{V}_{S,t} &= Mx\beta\mathcal{V}_{1,t+1} + (1-Mx)\beta\mathcal{V}_{S,t+1}, \\ \mathcal{V}_{1,t} &= (1-M)x\,(u + \beta\mathcal{V}_{S,t+1}) + (1-(1-M)\,x)\,\beta\mathcal{V}_{1,t+1} \end{aligned}$$

The stationary trajectories $\{\mathcal{V}_{S,t} = \mathcal{V}_{S,\infty}\}$ and $\{\mathcal{V}_{1,t} = \mathcal{V}_{1,\infty}\}$ satisfy the two previous equations. Furthermore, for all t, $\widetilde{\mathcal{V}}_{0,t} = \mathcal{V}_{1,\infty} - \gamma$. Agents always refuse the old currency if $\widetilde{\mathcal{V}}_{0,t} < \mathcal{V}_{S,t}$ for all t, or:

$$\mathcal{V}_{1,\infty} - \gamma < \mathcal{V}_{S,\infty} \Longleftrightarrow \frac{\gamma}{u} > \overline{\Gamma}$$

According to case 4 of proof of proposition 6, this condition is fulfilled. Old currency holders always initiate the conversion to the new currency if $\mathcal{V}_{1,t} - \mathcal{V}_{0,t} > \gamma$. It can be verified that, for all $t \in \{0, ..., T-1\}$, $\mathcal{V}_{0,t} = \mathcal{V}_{0,T-1}$. Thus, the previous inequality can be rewritten as follows:

$$\mathcal{V}_{1,T-1} - \mathcal{V}_{0,T-1} > \gamma \Longleftrightarrow \frac{\gamma}{u} < \Gamma_g.$$

To conclude, the profile of strategy $\{(\Omega_t, \Pi_{0,t}) = (1,0), \forall t\}$ sustains a subgame perfect equilibrium.

- $\frac{\gamma}{u} > \Gamma_g$
  Assume that there is $t^* \in \{0, ..., T-1\}$ such that $(\Omega_t, \Pi_{0,t}) = (0,0)$ for all $t \in \{t^*, ..., T-1\}$. From (3),

$$\widetilde{\mathcal{V}}_0(t) = \mathcal{V}_0(t), \qquad \forall t \in \{t^*, ..., T-1\}$$

Let us focus first on the strategy of acceptance of the old currency. From (6) and (8) we have:

$$\left(\mathcal{V}_{S,t} - \widetilde{\mathcal{V}}_{0,t}\right) = \beta(1-xg)\left(\mathcal{V}_{S,t+1} - \widetilde{\mathcal{V}}_{0,t+1}\right) - (1-M)\,xgu, \quad \forall t \in \{t^*, ..., T-1\}$$

where $\widetilde{\mathcal{V}}_{0,t} = \mathcal{V}_{0,t}$ for all $t < T$. Furthermore, we have the following terminal condition:

$$\mathcal{V}_{S,T} - \widetilde{\mathcal{V}}_{0,T} = \mathcal{V}_{S,T} - \mathcal{V}_{1,T} + \gamma = -\frac{x(1-M)u}{1-\beta(1-x)} + \gamma$$

After some calculation we get:

$$\mathcal{V}_{S,t} - \widetilde{\mathcal{V}}_{0,t} = [\beta(1-xg)]^{T-t}\left\{-\frac{x(1-M)(1-\beta)(1-g)}{(1-\beta+\beta xg)(1-\beta+\beta x)}u + \gamma\right\}$$

$$-\frac{(1-M)\,xgu}{1-\beta(1-xg)}$$

It can be verified that

$$\frac{\gamma}{u} > \Gamma_g > \frac{x(1-M)(1-\beta)(1-g)}{(1-\beta+\beta xg)(1-\beta+\beta x)}$$

Hence, the term between brackets is positive and $\mathcal{V}_{S,t} - \widetilde{\mathcal{V}}_{0,t}$ is increasing with t. Let define $\widetilde{t}^*$ such as:

$$\widetilde{t}^* = \max \left\{ t \in \mathbb{Z} \left| \left[\beta\left(1 - xg\right)\right]^{T-t} \left(-\frac{x\left(1-M\right)\left(1-\beta\right)\left(1-g\right)}{\left(1-\beta+\beta xg\right)\left(1-\beta+\beta x\right)} u + \gamma \right) \right. \right.$$

$$\left. - \frac{\left(1-M\right) xgu}{1-\beta\left(1-xg\right)} < 0 \right\}$$

Then,

$$t^* = \max\left(0, \widetilde{t}^*\right)$$

Given that $\mathcal{V}_{S,t^*+1} \geq \mathcal{V}_{0,t^*+1}$, we have $\Pi_{0,t^*} = 0$. Thus, $\Pi_{0,t} = 0$ for all $t \in \{t^*, ..., T-1\}$.

Let us turn on the strategy of conversion of the old currency into the new one. From (7) and (8) we have:

$$\mathcal{V}_{1,t} - \widetilde{\mathcal{V}}_{0,t} = \left(1-M\right) x(1-g) \left[u + \beta \left(\mathcal{V}_{S,t+1} - \widetilde{\mathcal{V}}_{0,t+1} \right) \right] +$$

$$+ \left[1 - (1-M)x\right] \beta \left(\mathcal{V}_{1,t+1} - \widetilde{\mathcal{V}}_{0,t+1} \right), \qquad \forall t \in \{t^*, ..., T-1\}$$

From the fact that $\mathcal{V}_{S,t} - \widetilde{\mathcal{V}}_{0,t}$ is increasing with t for all $t \in \{t^*, ..., T-1\}$, we deduce that:

$$\mathcal{V}_{1,t+1} - \widetilde{\mathcal{V}}_{0,t+1} > \mathcal{V}_{1,t} - \widetilde{\mathcal{V}}_{0,t} \Rightarrow \mathcal{V}_{1,t} - \widetilde{\mathcal{V}}_{0,t} > \mathcal{V}_{1,t-1} - \widetilde{\mathcal{V}}_{0,t-1}$$

Furthermore, for all $t \in \{t^*, ..., T-1\}$, $\mathcal{V}_{0,t} = \widetilde{\mathcal{V}}_{0,t}$ and $\widetilde{\mathcal{V}}_{0,T} = \mathcal{V}_{1,T} - \gamma$. Thus,

$$\mathcal{V}_{1,T} - \widetilde{\mathcal{V}}_{0,T} = \gamma > \mathcal{V}_{1,T-1} - \mathcal{V}_{0,T-1} = \mathcal{V}_{1,T-1} - \widetilde{\mathcal{V}}_{0,T-1}$$

Given that for all $t \in \{t^*, ..., T-1\}$ $\mathcal{V}_{0,t} = \widetilde{\mathcal{V}}_{0,t}$, it has been demonstrated that $\mathcal{V}_{1,t} - \mathcal{V}_{0,t}$ is increasing in time. Thus, $\Omega_t = 0$ for all $t \in \{t^*, ..., T-1\}$.

In conclusion, it has been shown that the profile of strategies $\{(\Omega_t, \Pi_{0,t}) = (0,0);$ $\forall t \in \{t^*, ..., T-1\}\}$ sustains a subgame perfect equilibrium.

Finally, an analogous reasoning would permit to demonstrate that if $t^* \geq 1$, then for all $t \in \{0, t^* - 1\}$, $(\Omega_t, \Pi_{0,t}) = (0,1)$.

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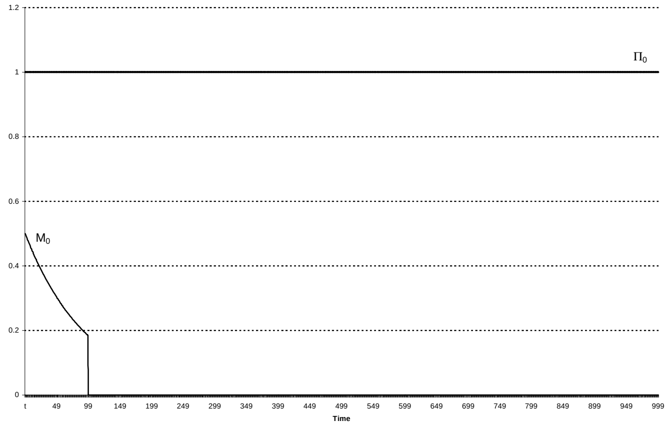
SIMULATIONS 1



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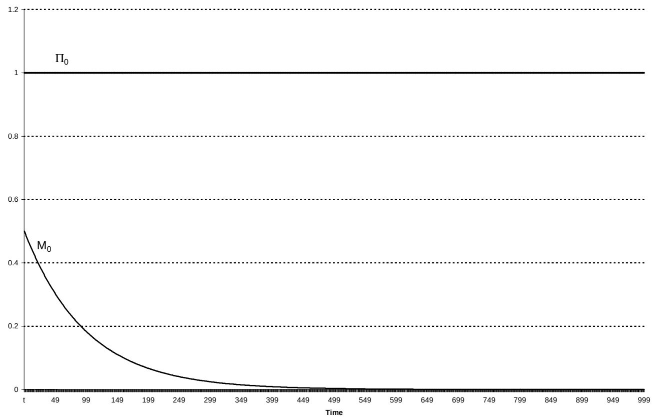

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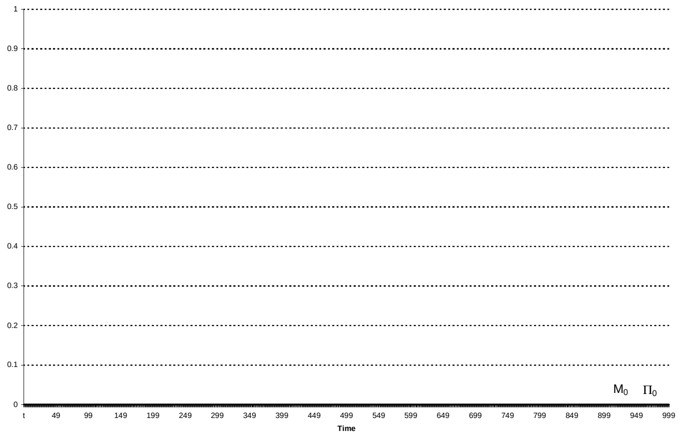

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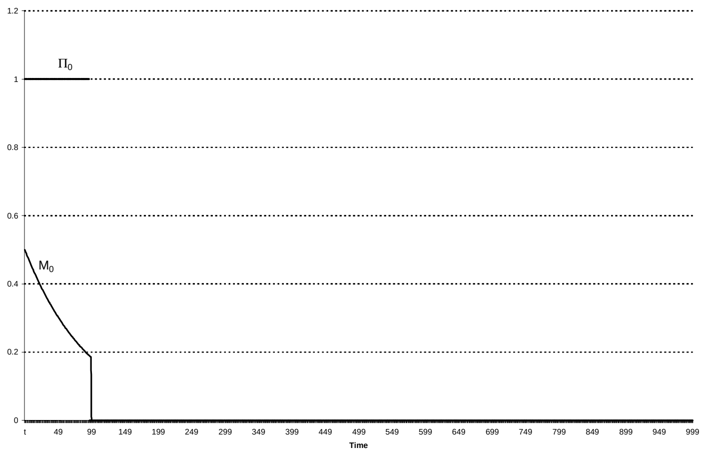

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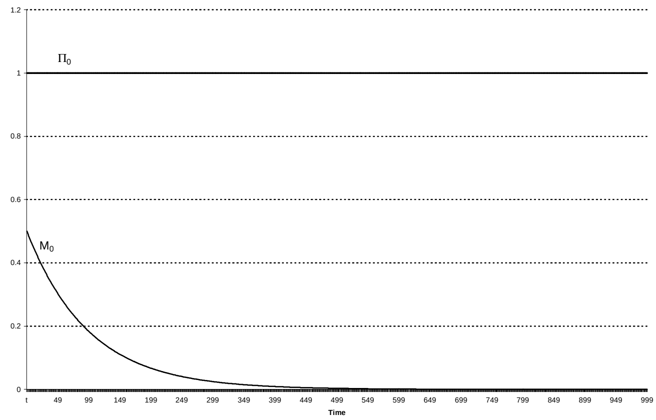

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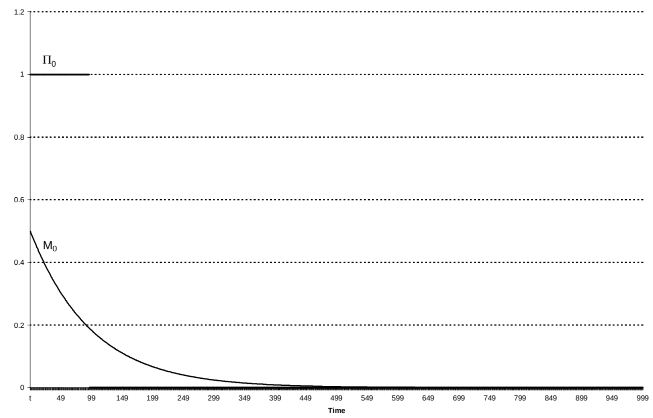

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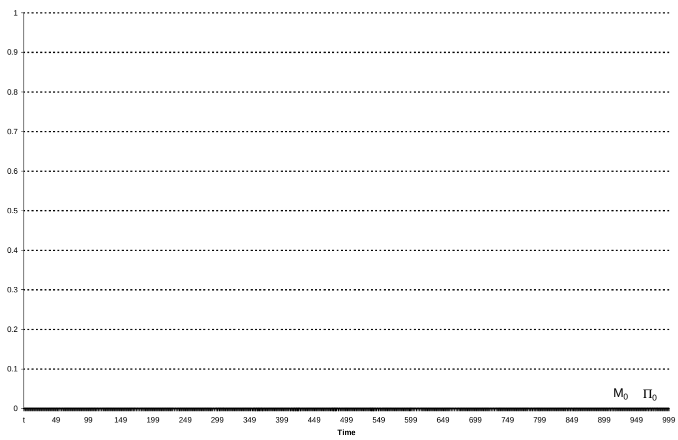

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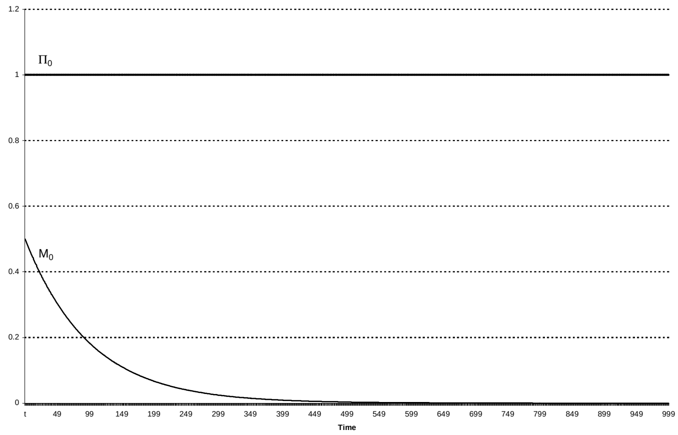

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