RATIONALITY IN ECONOMETRICS

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The idea of rationality enters an econometrician's work in many ways; e.g., in his presuppositions about sample populations, in his model selections and data analyses, and in his choice of projects. I shall consider some of these ways and their ramifications for the econometrician's own life and for the development of econometrics.

I begin in the first two sections of the paper with a discussion of rationality that I have found in the writings of Aristotle and other leading philosophers. My aim here is to establish the characteristics that we in good faith can expect rational members of a sample population to possess. The characteristics with which I end up have no definite meaning. Instead they are like undefined terms in mathematics that an econometrician can interpret in ways that suit the purposes of his research and seem appropriate for the population he is studying. When interpreted, the pertinent characteristics of the rational members of a given population become hypotheses whose empirical relevance must be tested.

To emphasize the undefined aspect of the characteristics that constitute my idea of rationality, I designate a rational individual by the term 'rational animal.' As such my 'rational animal' shares many of the characteristics of Thomas Paine's 'common man' and of John Stuart Mill's 'economic man.' My 'rational animal' also looks like Donald Davidson's 'rational animal' whose rationality consists in it having all sorts of propositional attitudes (cf. Davidson, 1982). Whether my 'rational animal,' like Davidson's, does have the use of language is a question that I do not raise. However, all the populations that I have in mind have the use of some kind of language. Finally, the fact that the empirical relevance of a given interpreted version of my 'rational animal' cannot be taken for granted, accords with Hempel's insistance that the assumption that man is rational is an empirical hypothesis (cf. Hempel, 1962, p.5).

Donald McCloskey has written an interesting and enjoyable book about **The Rhetoric of Economics**. On pp. 83-86 in his book he discusses a paragraph from one of Robert Solow's many seminal articles, (Solow 1957), and chides Solow for making use of the four master tropes of literary form, *metaphor*, *metanymy*, *synecdoche*, and *irony*. 'Irony,' the most sophisticated of these master tropes, is at work already in the first sentence of the paragraph: "In this day of rationally designed econometric studies and super input-output tables, it takes something more than the usual 'willing suspension of disbelief' to talk seriously of the aggreagate production function…" Supposedly, Solow is bowing ironically to rationally designed econometric studies. He as well as part of his audience knew well that their rationality was in doubt.

I am not sophisticated enough to recognize the absurdity of Solow's reference to 'rationally designed econometric studies.' Instead of being ironic I believe that Solow had in mind the kind of econometrics that he himself so ingeniously displayed in the remainder of his article. So, for better or worse, in sections three and four of my paper I walk along the path that Solow was walking in his article and give an interpretation of 'rationally designed econometric studies' that I think Solow in 1957 would have liked.

In rationally designed econometric studies the interpretation of a 'rational animal' that seems appropriate for a given study is usually an interpretation that the pertinent econometrician extracts from various economic theories. I take a close look at some of these interpretations in section three of the paper and discuss their empirical relevance. The interpretations of particular interest concern consumer choice under certainty, choice under risky and uncertain conditions, and choice in game-theoretic situations. These interpretations appear in various representations in the ways econometricians model rationality. I cannot discuss all such models. So in section four I limit the discussion to microeconometric models of consumer choice and macroeconometric rational expectations models. All the models I consider appear in good examples of rationally designed econometric studies.

Thomas Kuhn and Imre Lakatos have proposed two thought provoking models of the historical development of scientific endeavors. Kuhn envisions science as a field of various 'normal sciences' in which researchers are solving puzzles that one or more past scientific achievements have supplied (Kuhn, 1970, pp. 10-11 and 35-36). Lakatos views science as a field of all sorts of scientific research programmes in which researchers are seeking ways to extend the positive heuristic of the programme (Lakatos, 1978, pp. 47-52). In a given 'normal science' researchers "are committed to the same rules and standards for scientific practice." Also they do not "normally aim to invent new theories, and they are often intolerant of those invented by others" (Kuhn, 1970, pp. 11 and 24). In a scientific research program the scientist builds his models in accordance with prescriptions that are articulated in the positive heuristic of his programme. "He ignores *actual* counter examples"…"in the hope that they will turn, in due course, into corroborations of the programme" (Lakatos, 1978, pp.50 and 52). Neither Kuhn nor Lakatos deliberates about the motives that guide a scientist in his choice of puzzles and positive heuristics. Also, they have little to say about the social aspects involved in the writing of scientific reports.

In the last two sections of the paper I consider the two lacunas in Kuhn's and Lakatos' theories, and see how econometricians go about solving puzzles and extending positive heuristics. Specifically, I discuss, first, the considerations that guide an econometrician in his choice of research projects. Then, I argue about the determinants of rational choice in model selection. Since Kuhn and Lakatos are primarily concerned with the choices of scientists in the hard sciences, the ideas I present here differ from theirs. These ideas might also differ from those of the average econometrician, since he might not agree with the econometric methodology in which I believe (cf.. Stigum, 1995). Finally, I consider the politics of writing research reports. The contents of these sections concern aspects of an econometrician's rational choice that are relevant for the orderly development of econometrics. I hope, therefore, that the reader will find my views interesting enough to warrant discussion.

I. *A Philosopher's Concept of Rationality*

One of the many socially constructed facts that I have learned to accept, insists that *a human being is a rational animal*. Philosophers, usually without proper reference, attribute this assertion to Aristotle. I shall begin my search for a suitable concept of rationality for econometrics by delineating Aristotle's idea of a rational animal. My main sources are translations of and philosophical commentaries to Aristotle's treatises, De Anima and the Nicomachean Ethics.[1]

I.1 **Rational Animals**

Aristotle had a simple scheme for classifying elements in the physical world. There were two realms, the inorganic world and the organic world. All living things and no others belonged to the organic world. Aristotle arranged the living things in turn into three life classes according to their possession of various basic faculties. Vegetable life consisted of all living things that had just two faculties, the powers of nutrition and reproduction. Animal life consisted of all living things that had the powers of nutrition and reproduction and the sensation of touch. Finally, human life consisted of all living things that had the powers of nutrition and reproduction, the sensation of touch, and the faculty of deliberative imagination.

For our purposes it is important to keep in mind the role of touch and deliberative imagination in Aristotle's characterization of animal life and human life. Aristotle allowed that animals may have many sensations in addition to touch. However, touch was to him "the only sense, the deprivation of which necessitates the death of animals. For neither [was] it possible for anything that is not an animal to have this sense, nor [was] it necessary for anything that is an animal to have any sense beyond it" (Aristotle, de Anima, p. 143). Consequently, an animal was a living thing that had the sensation of touch and the faculties of a member of vegetable life.

The case of deliberative imagination is a bit more involved. According to Aristotle, imagination was the faculty of mind by which an animal forms images of things that are not present to the senses or within the actual experience of the animal involved. As such imagination was not sensation although an animal could not have imagination without having the faculty of sensation. Aristotle conceived of two kinds of imagination, imagination derived from sensation and deliberative imagination. Sensitive imagination, he claimed, "[was] found in the lower animals but deliberative imagination [was] found only in those animals which [were] endowed with reason" (de Anima, Book III, Chapter XI). Also an animal could not have the power of reason without having imagination. Since none of the lower animals had reason, it follows that the higher animals; i.e., the human beings had to be animals with the power of deliberative imagination.

A rational animal is more than an animal with deliberative imagination. Here is why. It was the case that an animal with deliberative imagination had the faculty of reason, and an animal with reason had the faculty of deliberative imagination. To deliberate means to weigh alternatives, and that is an act of reason. Therefore, it was the case that an animal with the faculty of deliberative imagination and the power of reason would also have the ability to form opinions. Finally, it was the case that an animal could not have the ability to form opinions without having the faculty of deliberative imagination (de Anima, Book III, Chapter III). But if that is so, we may claim that a human being is an animal with the faculties of deliberative imagination, opinion, and reason. Consequently, if a human being is a rational animal, we may adopt the following characterization of rational animals:

An animal is *rational* if an only if it has deliberative imagination and is able to opine and reason.

I.2 **Deliberative Imagination, Opinion, and Reason**

I believe that the preceding assertion gives an adequate rendition of Aristotle's idea of *rational* animals. However, to get a good idea of what it means for an animal to be rational, we must take a closer look at the meanings of deliberative imagination, opinion, and reason.

To me the three terms, deliberative imagination, opinion, and reason are like undefined terms in mathematics. They have no definite agreed-upon meaning. Instead their meanings are culturally determined and vary over individuals as well as over groups of individuals. Here is what I have in mind.

The wealth of images that an individual can create with his deliberative imagination depends on many things. It depends on the physical and mental sensations that he has experienced: e.g., the places he has visited and the persons he has met.. It also depends on his schooling and his abilities and inclinations. Finally, it depends on the cultural traditions with which he grew up. A Californian might dream of UFOs traversing the sky, and a Norwegian youngster might fantasize about trolls and seductive maidens with long tails roaming a nearby forest at night.

Individuals have all sorts of opinions. Some opinions pertain to personal lives. They determine likes and dislikes of things and ideas and proper attitudes to fellow men.. Other opinions pertain to the society in which the individuals in question live. They may concern the appropriateness of customs; e.g.., the circumcision of women, and the usefulness of various structural aspects; e.g., the political independence of a central bank. Still other opinions pertain to the validity of theories and socially constructed facts. They may concern the power of ghosts and the right of biologists to produce clones of human beings. Opinions may vary in surprising ways over individuals as witnessed in the next example.

E.1 In her book, **In a Different Voice**, Carol Gilligan theorizes about differences in the character development of men and women. One of the experiments she describes is relevant for us.

A psychiatrist tells two eleven-year-old children, a boy and a girl, a sad story. Al and Liz are happily married. Liz is sick and will die unless she gets a medicament, C. Al has no money to buy C and the nearest druggist refuses to give it to him. That presents Al with two options, steal C to save his wife's life or watch his wife die. What should he do?

The boy argues that Al ought to steal C since the druggist's loss of C would be minor relative his gain from C saving Liz's life. He also suggests that, if Al were caught, he would receive a light sentence, since the judge would agree that Al did the right thing.

The girl insists that it is not right that a person should die if her life could be saved. Still Al ought not to steal C. Instead he should try to persuade the druggist to give him C and promise to pay him back later.

Gilligan recants that the boy in the experiment actually preferred English to math and that the girl aspired to become a scientist. Still, the boy relied on conventions of logic to resolve the dilemma, assuming that most people would agree to these conventions. The girl relied on a process of communication to find a solution to the dilemma, assuming that most people will respond to a reasonable plea for help (cf. Gilligan, pp. 24-32).

It is interesting here that Aristotle insisted that a person could not have an opinion without believing in what he opined. Hence, opinion was followed by belief. In fact, "every opinion [was] followed by belief, as belief [was] followed by persuasion, and persuasion by reason" (de Anima, p. 108). Persuasion concerns all matters on which an individual might have an opinion. It occurs in familiar places; e.g., at home, in class rooms and lecture halls, and in day-to-day interactions with friends and acquaintances. We experience it in different forms; e.g., as gentle parental coaxing, as socially constructed facts in books and newspapers, and as results of heated discussions. The beliefs we acquire in this way may be well founded or just fixed ideas as evidenced in E.2, where I paraphrase an observation of E.E. Evans-Pritchard.[2]

E.2 I have no particular knowledge of meteorology. Still I will insist that rain is due to natural causes. This belief of mine is not based on observation and inference. It is part of my cultural heritage. A savage might believe that, under suitable natural and ritual conditions, the appropriate magic can induce rain. His belief will also not be based on observation and inference. It forms part of his cultural heritage, and he has adopted it simply by being borne into it. Both he and I are thinking in patterns of thought that the societies in which we live have provided for us.

To Aristotle reason was the instrument by which a person thinks and forms conceptions. Reason could be passive or active. The passive reason in an infant was pure potentiality. In a learned person the passive reason became the capacity of thinking itself (de Anima, Book III, Chapter IV). The active reason was reason activated by desire. Depending on the pertinent object of desire the active reason would result in choice of action or judgment concerning truth and falsehood or what is right or wrong. The reasoning involved was *true* if it was logical and based on premises that either were true by necessity or accepted as true by the wise. Also, the desire was *right* if it reflected an appetition for a good end. The choices and the judgments concerning right or wrong were *good* if they resulted from true reasoning and a right desire. The judgments concerning truth and falsehood were *good* if they were logical consequences of premises that were true by necessity.

The validity of necessary truths and the well-foundedness of premises that the wise have accepted are often questionable. Besides, some of the premises of the wise may reflect attitudes that we cannot condone. Here are a few examples to illustrate what I have in mind.

Consider the law of the excluded middle. Aristotle believed that the law was true by necessity, and most mathematicians today agree with him. The Dutch intuitionists (DI), however, think differently. They insist that a declarative sentence denotes truth or falsehood according as it, or its negation, can be verified. If neither the sentence nor its negation is verifiable, the sentence is neither true nor false. Here are two cases in point:

(1) *There are three consecutive 7 in the decimal expansion of* p, and

(2) *There are integers of which nobody ever will have thought*.

At present the truth value of the first sentence is unknown, and it is conceivable that no human being will ever determine it. Satisfying the predicated relation in the second sentence involves a contradiction, and determination of its falsehood is inconceivable. According to the DI these sentences are at present neither true nor false.

Similarly, the well-foundedness of premises that the wise have accepted is often uncertain. We see that in the way scientific knowledge changes over time. For example, Aristotle believed that water was one of five basic substances that could combine with other substances to form compounds but could not be broken down to simpler substances. His idea was dispelled when H. Cavendish in 1775 succeeded in showing that hydrogen and oxygen combined to form water. We also see it in historical records describing dire consequences of government policies that were based on defunct economic theories. One recent example is from Peru.. There the changes in land ownership that the military junta carried out between 1968 and 1980 had a disastrous effect on farm output. The military and their US advisers based their policies on the economic theory of labor managed firms, a theory that had next to no empirical relevance under Peruvian conditions. The Peruvian farm laborers did not have the knowledge nor the ability to acquire the knowledge of how to manage large farms.

Finally, some of the premises that the wise accept, may reflect attitudes that we cannot condone. One example is the maxim that *the end always justifies the means*. When other arguments fail, governments use this maxim to justify all sorts of interventions. One example is from the nineteen nineties' 'Bank Crisis' in Norway. To solve the crisis the labor government assumed ownership of the three largest Norwegian banks. As far as I can tell, there was no good economic reason why the take-overs were necessary. Besides, the government's handling of the case displayed a shocking disregard for the individual citizen in its refusing to compensate the stockholders. The Russian 1956- and 1968- massacres of innocent people in Hungary and Czechoslovakia provide us with a different example. In private conversations, an internationally known scientist: i.e., a very wise man, told me that the Russians' vision of Eastern Europeans living in bliss under communism justified the atrocities

Now the upshot as far Aristotle's active reason goes: If necessary truths might be invalid, and if the judgments of the wise may be questioned, then true reason and right desire cannot be well determined. They must vary with the culture in which an individual has been brought up, and they are likely to vary with individuals in one and the same community. But if that is so, then Aristotle's active reason is like an undefined term with no definite meaning. This indefiniteness faces us everywhere and at all times. We become acutely aware of it when we meet people and experience events in real life or in books and newspapers that we cannot understand. The associated problems may concern family relations or the increasing violence in the streets. They may also concern aspects of human rights or just the senseless wars on the Balkans. Finally, they may concern the disparate premises of the religions that provide people with standards of right and wrong. The next example describes an important case in point.

E.3 The question is: *Should a doctor be allowed to help a mortally sick patient die prematurely?* In Norway the Church and the Law say no. However, many influential people say yes and argue strongly for changing the Law accordingly.

In 1996 a forty-five-year old person, mortally sick with multiple sclerosis, asked her doctor to help her die. The doctor agreed and gave her an overdose of morphine. Later, the doctor asked the Courts to try him for murder. He was hoping that the trial would start the process of changing the Law so that active death help under strict provisions would be allowed

As the first of its kind, the case is currently going through the court system in Norway. The lower court found the doctor guilty of premeditated manslaughter, but refused to punish him. Both the doctor and the prosecutor appealed the verdict to a higher court. In the higher court the jury found him not guilty of manslaughter. The judges, however, insisted that the doctor was guilty and overruled the jury. That meant that the same court must convene with different judges and try the case anew. Knowledgeable people believe that the case will end in the Supreme Court and be decided for good there.

I.3 **Universality, Necessity, and Rules**

The preceding discussion left us with a vague idea of a rational animal. We found out that a rational animal was an animal with deliberative imagination, beliefs, opinions, and reason. We also noticed that the four characteristic features of rational animals were to be likened to undefined terms in mathematics that have no definite meaning. The same must be the case for Aristotle's idea of rational animals. Hence, if we accept Aristotle's ideas, we should expect to meet all sorts of rational animals. That is not disconcerting since the people we meet in life are all so different.

There are three aspects of Aristotle's idea of rational animals that are striking. His notion of a rational animal is universal in the sense that it characterizes all human beings, be they atheists or priests, ignoramuses or scholars, or just infants or grown-ups. Also, he insists that the choices and judgements of rational animals are good choices and judgements only if they, in accordance with the rules of logic, follow by necessity from proper premises and right desires. In our search for a concept of rationality for econometrics, we shall look for a concept that share these two characteristics of Aristotle's idea of rationality. Hence we shall insist that *a rational person is an animal with deliberative imagination, beliefs, opinions, and reason.* Also *rational choices and judgements are good choices and judgements that, in accordance with the rules of logic, follow by necessity from proper premises and right desires.*

Since rational choices and judgements are to be good choices and judgements, a few remarks concerning Aristotle's idea of the 'good' are called for. To Aristotle the 'good' was that at which every art and inquiry and every action and pursuit aim. Also the 'good' was something that people search for its own sake. Finally, the 'good' was an activity of the soul in accordance with moral and intellectual virtue. This extraordinary good Aristotle identified with *happiness* (Nicomachean Ethics, pp. 1-24).

Most people associate living well and faring well with being happy. Yet, different persons are likely to differ considerably in their ideas as to what happiness actually is. A sick person may identify happiness with health and a poor person may associate it with wealth. Similarly, a youngster may associate happiness with pleasure, and a politician may identify it with honor. They are all wrong. In Aristotle's vocabulary health, wealth, pleasure and honor were not different aspects of the 'good.' Instead they were means in the pursuit of the 'good' which Aristotle designated by happiness.

According to Aristotle, happiness was an activity of the soul in accordance with perfect virtue. Also, a virtue was a state of character. Virtues came in two forms. One was moral. The other was intellectual. Individuals acquired moral virtues by habit. For example, they became just by carrying out just acts and brave by performing brave acts. Similarly, persons developped intellectual virtues in schools and through individual studies. For example, they acquired philosophical wisdom from intuitive reasoning and scientific research. They acquired practical wisdom by developing a true and reasoned capacity to act with regard to the things that are good or bad for man. Thus a person acquired virtues by habit and practice. In the process the person also learned to appreciate the virtuous and to enjoy the happiness it brought him.

What is virtuous in one society need not be virtuous in another. Also what one wise person considers virtuous another may dispute. In the Nichomachean Ethics Aristotle insisted that truly virtuous persons would, in all situations that involve choice and moral judgements, adhere to the so-called Doctrine of the Golden Mean. In such situations "excess [was] a form of failure, and so [was] defect, while the intermediate [was] praised and [was] a form of success; and being praised and being successful [were] both characteristics of virtue." Thus virtue was kind of a mean. For example, courage was a mean between fear and confidence and pride was a mean between dishonor and honor (Nichomachean Ethics, pp. 38-41). Inter-esting aspects of this doctrine have a good hold on the minds of many of my fellow citizens in Norway. There children grow up with the idea that it is not virtuous to show off. The school system is designed so that poor students may survive, average students may do well, and bright students have little or no opportunity to perform according to their abilities. And the solidarity principle, which underlies the income-distribution policy of the government, ensures that disposable income does not vary too much over a huge majority of the population.

Aristotle's Doctrine of the Golden Mean will not play a role in my search for a useful concept of rationality in econometrics. However, as the examples from Norway indicate, I cannot ignore the fact that rules and regulations that emanate from lawmakers and other authorities have a determining influence on what the average citizen considers virtuous. To the extent that these rules and regulations reflect aspects of Aristotle's doctrine, the doctrine will form a part of my idea of what constitutes rational conduct and judgement

II *Universality, Proper Premises, and Right Desires*

Rational animals live and function in the social reality we described in Chapter 2. We know that they are alike in having deliberative imagination, beliefs, opinions, and reason. In this section we shall see if they are alike in other respects as well. To find out, we look for answers to three questions: Is it true that two different rational animals in a given choice situation necessarily will make the same choices? If they happen to make different choices, what are the chances that we may persuade one of them to change his or her choice? Finally, is it likely that two individuals from one and the same society will act and make judgments on the basis of premises that all the members of their society accept?

II.1 **Rational Animals in Choice Situations**

We go through life experiencing all sorts of situations in which we are called upon to make a choice. Some times the number of alternatives we face is small. It is for Norwegian parents when their children come of age and *must* attend primary school. There are many public schools and just a few private schools. Most parents send their children to a public school in their neighborhood. Other times the number of alternatives we face is considerable. It is for the unfortunate person who must acquire a new car. There are many different car makes. Each make comes in many different models, and every model has any number of representatives from which the person may choose. Finally, there are times when the number of alternatives is hard to fathom. It is for the young man who must choose an education for access to a life-long stream of income that will enable him to satisfy his various future needs. He can decide to start his education now or wait a year. If he waits a year, he can take a parttime job and devote lots of time to his favorite hobbies and leisure activities. He can also take a full-time job and save money to pay for his education.

Two rational animals in one and the same choice situation are unlikely to be alike in all pertinent matters. A second look at the choice situations I described above will bear witness to that. In the choice of school for a child the abilities and character of the child are pertinent matters. So are also the upbringing of the parents, their visions for the future of the child, and the roles they play in the neighborhood in which they live. In the choice of a new car the buyer's budget constraint, his knowledge of cars, and the purposes for which he needs a car are pertinent matters. So are also the time the buyer has allotted to searching for a new car, the supply of cars in his neighborhood, and his ideas as to the kind of car he ought to be driving. In the choice of career the number of pertinent matters are as numerous as the alternatives the young man faces. A few of them are his character and upbringing, his abilities and good health, and the financial constraints he is confronting. So also are the availability and costs of schooling, the current employment situation, and his visions of opportunities and the kind of life he wants to live.

We are looking for a concept of rationality according to which rational choices follow by necessity from proper premises and the use of rules of logic. The preceding observations suggest that there are only two reasonable ways to test whether the choices of rational animals have this property: We can observe the actions of a given person in similar choice situations, or we can construct simple choice experiments in which a subject's impertinent attributes are stripped of influence. I shall describe two such tests below.

If a given rational animal's choices follow by necessity from proper premises and the use of rules of logic, we should expect that he in similar choice situations always will make the same choice. In the following example we question such consistency of a single rational animal's choices. Specifically, the example describes a way of testing the consistency of consumer choice of commodity vectors. The test is one that Paul Samuelson suggested many years ago (cf. Samuelson 1950, pp.   ).

E.4 Consider a consumer, A, and suppose that in the price-income situation, (p^0, I^0), A chooses the commodity vector, x^0. Suppose also that x^1 is an other commodity vector and that x^1 satisfies the inequality, $p^0x^1 \leq p^0x^0$. Then A has revealed that he prefers x^0 to x^1. Consequently, if A in some price-income situation, (p^1, I^1), buys x^1, it must be the case that $p^1x^1 < p^1x^0$.

In this case the premises are threefold. One insists that A has a consistent ordering of commodity vectors. The other claims that A, in a given price-income situation, always chooses the vector in his budget set that he ranks the highest. The third prescribes that A's ordering of commodity vectors is such that there always is just one vector in a pertinent budget set that he will rank as the highest.

If A is rational in accordance with the concept of rationality that we are seeking, the conclusion: i.e., the claim that the inequality, $p^1x^1 < p^1x^0$ must hold, is valid with one very strong proviso: A's ordering of commodity vectors when he chooses x^1 is the same as when he chose x^0.

The proviso in E.4 is hard to accept. A consumer's ordering of commodity vectors is likely to change over time for many reasons. It may change with the ages of family members or because some family member develops a liking for a special component of the commodity vector. The ordering may also change with the price-income expectations of the consumer. If the expectations at (p^1, I^1) differ from those at (p^0, I^0), Samuelson's test would be invalid.

Two-person non-cooperative game theory is full of ideas for laboratory experiments in which we can check whether two persons in one and the same choice situation will make the same choices. I describe one of them in E.5.

E.5 Consider two persons, A and B, who dislike each other enough so that they under no circumstances would be willing to communicate. A third person, C, has persuaded them to participate in a game of the following sort: A and B are to choose one of two letters, α and β. If they both choose α (β), C will pay them both \$ 100 (\$10). If A chooses α (β) and B chooses β (α), C will charge A \$ 200 for the game and pay B \$ 300 (pay A \$ 300 and charge B \$ 200 for the game). A (B) must make his choice without knowing which letter B (A) is choosing. Both A and B know all the consequences of the game and have agreed to the rules of the game.

The game A and B will be playing is a model of the game called "A Prisoner's Dilemma" with payoff matrix as shown below. All the entries in the matrix denote so many dollars.

		B's Strategies	
		α	β
A's Strategies	α	100,100	-200,300
	β	300,-200	10,10

An equilibrium in game theory is a Nash Equilibrium. It has the property that each player, once his opponent's strategy has been revealed, is satisfied with his own choice of strategy. In the present game there is one and only one equilibrium. It prescribes that both A and B choose the letter β.

In E.5 A and B start out with the same premises and the same information. Also the payoff matrix is symmetric in the sense that neither A nor B would have anything against exchanging names. It seems, therefore, that if A and B are rational, according to any concept of rationality that we may seek, they must end up choosing the same letter. Game theorists insist that the rational thing for A and B to do is to choose β. This is so because β is the only choice which A and B afterwards will not regret having made. So far the idea of regret has played no part in our discussion of rationality. Hence a philosopher is still free to reject the game theorists' claim by arguing as follows. A and B are both rational. Therefore, in the given choice situation they will choose the same letter. Being rational, A and B will figure out that they will choose the same letter. The best letter for them both is α. Hence the rational thing for A and B to do is to choose α (cf. Brown 1990, pp. 3-6).

It is one thing to argue what rational people ought to do when facing a prisoner's dilemma. Another thing is to find out what people actually do in such a choice situation. The evidence from experimental economics is mixed. In some cases the pertinent pair chooses an α. In other cases the pair chooses a β. In still other cases one member of the pair chooses an α while the other member chooses a β. On the whole, there seems to be more βs among the chosen letters than αs. That is not strange since choosing β is not just a Nash equilibrium strategy for the two players. It is a dominant strategy for both players. And, if that is not enough, it is also a *maxi-min* strategy for them. To wit: If A (or B) chooses α he may **loose** $ 200 ($ 200). If he chooses β, he might **receive** $ 10 rather than $ 100.

The experiment in E.6 bears out the import of the preceding remarks. The experiment was conceived of and carried out by Russel W. Cooper, Douglas V. DeJong, Robert Forsythe, and Thomas W. Ross(cf. Cooper et al, 1991). Douglas DeJong provided me with the numbers I have recorded in the table.

E.6 Consider a prisoner's-dilemma experiment in which we pair 42 subjects with each other in a sequence of *matchings.* By letting each subject encounter each other subject only once we can generate a sequence of forty different matchings in which each matching contains twenty single-stage games. In 1991|Cooper et al. carried out such an experiment. They recorded the following results:

Matchings	1-5	6-10	11-15	16-20
Percentage of (α,α) pairs	17%	10%	7%	6%
Percentage of (β,β) pairs	32%	45%	57%	67%
Percentage of (α,β) and (β,α) pairs	51%	45%	36%	27%

II.2 **Regrets and Rational Choice**

We have seen that it is next to impossible to divine circumstances in real life in which we can check whether rational animals in one and the same choice situation make the same choice. We have also seen that in experimental settings in which we would expect rational animals to make the same choice, they need not do so. The last example suggests that when two individuals in a prisoner's dilemma situation make different choices, one of them will have regrets and choose differently the next time he is called upon to choose. That fact raises an interersting question: Suppose that we have observed that a supposedly rational person makes a choice that we consider to be nonrational. Will it be possible for us to persuade him to change his choice? We shall try to answer that question next.

Philosophers believe that it is an essential characteristic of a rational person that he will be willing to mend his ways if he finds out that he has erred in choice or conduct. I am not so sure. If two rational animals in the same choice situation make different choices, one of three things must be the case. Both individuals consider the consequences of their choices equivalent. One of them have made a logical error. Or the two have made use of different logics. In the first case it makes no sense to ask one of the two individuals to mend his ways. In the other two cases the success of persuasion may depend on many factors some of which I shall indicate below.

First logical errors. Logical errors in our reasoning crop up in many situations. Since a logical error is like a false arithmetical calculation, we would expect that an erring person who discovers his error will recalculate and change his choice. Whether he actually will recalculate, however, may depend on the situation. For example, at home a given person may miscalculate the remaining balance in his checking account and write a check too many. Then the bank will make sure that he mends his ways. At work his search for solutions to a given problem may be based in part on unfounded beliefs and flimsy evidence. What is worse, the arguments he employs may suffer from ill specified a priori assumptions that are either inapplicable in the circumstances he envisages or lead to circular reasoning. If he is a reasonable person , he probably would redo his reasoning if he became aware of such fallacies. Finally, as a consultant on the side, he may base his forecasts of future business conditions on assumptions that others might consider utterly unrealistic. Even if these others manage to convince him of that, he might not change his ways as long as his forecasts are good.

Next different logics. It might at first sound impossible that in a given situation two persons may choose differently because they make use of different logics. However, E.5 provides us with a good illustration of such a possibility There A might argue as a game theorist and choose β while B argues as a philosopher and chooses α. We might convince B that his logics was fallacious. However, it would be to no avail since the rules of the game do not allow him to change his strategy.

There are many reasons why two supposedly rational persons in a given choice situation might make use of different logics. For example, their attitudes toward pertinent logical and nonlogical premises may differ. So also may their conceptions of the choice alternatives they face. The possibility that decision makers make use of different logics is important to econometrics. Therefore, I shall give several examples below that illustrate how it can happen.

When I compare the choice of two rational animals in a given choice situation, I presume that the two start out with the same premises and face the same consequences of their actions. The premises may insist that the two obey the laws of their society and adhere to a certain moral code. For example, you shall not steal or cheat on taxes, and you shall not inflict wounds on other persons willfully. The consequences may include a record of possible monetary gains as well as a list of various penalties for breaking the law. If two persons' premises and consequences are the same, only different logics cause them to make different choices. Different logics may reflect different attitudes toward the strictures of the law as well as different assignment of probabilities to possible gains and losses. E.7 bears witness to that.

E.7 In Norway many persons find it to their advantage to sneak out of paying fares on trolleys and busses. The fares are costly. Also, the chance of being caught sneaking is small, and the penalty when caught is low. Finally, since there are no public records of who has been caught sneaking, the punishment for sneaking amounts to only a smidgen more than the fine and a day of bad mood.

The sneaking on trolleys and busses is extensive enough to make the companies that run them concerned. A few years back the companies carried out a campaign in which they appealed to the moral attitudes of their customers and asked them to mend their ways. The campaign had no appreciable effect. It seemed that the only way to induce a sneaking person to stop sneaking was to raise the penalty and to increase the frequency of controls. The companies ended up doubling the penalty for being caught while leaving the frequency of controls more or less as it had always been.

In the next example I describe a choice situation in which a very wise person and an ordinary citizen apply different logics because of their radically different attitudes toward uncertainty.

E.8 In a classical article on exchangeable events and processes Bruno de Finetti insisted that subjective probabilities had to be additive. If a person's assignment of probabilities to mutually exclusive uncertain events were not additive, de Finetti could make a fool of the person by inducing him to partake in an unfavorable bet (cf. De Finetti 1964,pp. 102-104). To me this was long an incontestable fact that I now will show is no fact at all.

Consider an ordinary grown-up rational animal, A, who faces two events, $E_1$and its complement E_2. We may think of E_1 as the event that *Clinton will be impeached*. Also, suppose that A is a person who deals with uncertainty by shading his subjective probabilities. He assigns probability 1/3 to the occurrence of E_1 and probability 1/3 to the occurrence of E_2. Finally, suppose that A's utility function on the set of consequences is linear in all situations in which gains and losses can be measured in dollars. Let us see how de Finetti would make a Dutch Book against A

To present deFinetti's arguments I let $S=(S_1,S_2)$ designate a security that will pay the owner $\$S_1$ if E_1 occurs and $\$S_2$ if E_2 occurs. De Finetti takes for granted that S is worth $1/3(\$S_1) + 1/3(\$S_2)$ to A and that A should be willing to issue S and sell S for a smidgen more than what S is worth to him. With that assumption in hand, de Finetti shows that, for any positive pair of numbers, (g,h), he can find a pair, (S_1,S_2), that satisfies the equations,

$$g = S_1 - (1/3)S_1 - (1/3)S_2 \quad \text{and} \quad h = S_2 - (1/3)S_1 - (1/3)S_2.$$

From these equations it follows that, by buying the solution, $S = (S_1,S_2)$, from A, de Finetti can ensure himself a gain of about $\$g$ if E_1 were to occur and $\$h$ if E_2 were to occur.

In the present case de Finetti is wrong about the value of S to A. In choice under uncertainty A orders uncertain options in accordance with the axioms that we discuss on pp. 34-36 below. If $S = (S_1,S_2)$ is the solution to the two equations, then to him $(S_1,0)$ is worth $1/3(\$S_1)$, $(0,S_2)$ is worth $1/3(\$S_2)$, and S is worth $1/3(\$S_1) + 2/3(\$S_2)$ if $S_2 \leq S_1$, and $2/3(\$S_1) + 1/3(\$S_2)$ if $S_1 \leq S_2$. There is no way in which de Finetti can make a Dutch Book against A.

In E.8 I followed de Finetti in assuming that A's utility function is linear in money. Also, for simplicity, I considered a case with just two events. Neither of these assumptions is critical for the conclusion: It need not be irrational to shade one's probabilities in the face of uncertainty. If it is not, it makes no sense to ask A in E.8 to mend his ways.

There are many situations in which it makes no sense to ask somebody to mend his ways if his ways appear irrational to us. In some such cases the attempt would be futile. For example, persuasion will be to no avail in the case of an alcoholic. He must find out by himself that life would improve if he were to stop drinking. In other cases sound arguments will have next to no effect. Certainly, a lecture on the characteristics of purely random processes is unlikely to find an open mind in somebody who is suffering from an acute case of the *gambler's fallacy*. In still other cases any arguments that we may dream up might be incomprehensible to the supposedly irrational person and hence come to naught. Here is an example that illustrates what I have in mind.

E.9 The African Azande entertain beliefs that appear strange to us. For example, they believe that certain fellow members are witches who exercise malignant occult influence on the lives of other members. Also, they engage in practices that are incomprehensible to us. For example, they perform rituals to counteract witchcraft, and they consult oracles to protect themselves against harm.

Since oracles often contradict themselves and come up with fallacious prophecies, it seems that a Zande's life must be filled with disturbing contradictions. Contradictions once brought down George Cantor's beautiful 'house' of sets. However, they do not seem to have much effect on the lives of the Azande. According to E. E. Evans-Pritchard, using contradictions in which oracles are involved to demolish the oracles' claim to power, would be to no avail. If such arguments were translated into Zande modes of thought, they would serve to support the Zande's entire structure of belief. The Zande's mystical notions are eminently coherent They are interrelated by a network of logical ties, and are so ordered that they never too crudely contradict sensory experience. Instead, experience seems to justify them (cf. Winch 1964, p. 89)

It is interesting to me that the Zande's mystical notions are ordered so that they **never too crudely** contradict sensory experience. In that way the Zande are able to function in a world filled with contradictions. I believe that the Zande may not be too different from other rational animals in this respect. A rational animal has all sorts of beliefs. Also, if he believes in the propositions, p, q, r, s, and t, logic demands that he must believe in all the logical consequences of these propositions as well. The latter requirement imposes a structure on individual beliefs whose validity it is beyond the intellectual capacity of most people to check. But if that is so, rational animals are likely to entertain systems of beliefs that harbor oodles of contradictory propositions. Most individuals probably cope with such contradictions in two ways. They do not push their search for logical consequences too far. Also, when they run across contradictions, they are content to make local changes in their belief systems.

II.3 **Proper Premises for Choice and Judgements**

We have seen that two rational animals in the same choice situation need not make the same choices. There are all sorts of reasons for that; e.g., differences in attitudes toward premises and the use of different logics. We have also seen that only when a person's logical arguments are fallacious, do we have a good chance of persuading him to mend his ways. In this section we shall see if it is reasonable to believe that persons in the same society will make choices and pass judgements on the basis of the same scientific and ethical principles.

Aristotle insisted that a rational animal's reasoning was *true* if it was logical and based on premises that either were true by necessity or accepted as true by the wise. Also, true reasoning would result in *good* choices and judgements if it was activated by an appetition for the *good*. In Aristotle's days, a person's reasoning was logical only if it adhered to the rules and regulations of Aristotle's own syllogistic logic. Today most people would insist that to be logical a person's arguments would have to satisfy the strictures of the first-order predicate calculus (FPC). However, there are dissenters. For example, in the FPC the Law of the Excluded Middle (LEM) is a tautology. In the formal logic of the Dutch intuitionists LEM is true in some cases and not in others. Also, the FPC is monotonic in the sense that if C follows from A necessarily, then C follows by necessity from A and B as well. The formal logic that artificial-intelligence people have developed for choice with incomplete information is nonmonotonic.

For us it is useful to distinguish between logical premises and nonlogical premises. Different logics make use of different axioms and different rules of inference. The mixture of axioms and rules of inference in a given formal logic may vary from one presentation to another. Therefore, I shall think of the rules of inference of a formal logic as premises on a par with the axioms of the same logic. Logical premises are premises that belong to some formal logic. Two individuals who make use of different formal logics base their arguments on different families of logical premises. It is possible that one and the same person may make use of one formal logic in one situation and another in a different situation. In that case the family of logical premises that the person employs varies over the situations he faces. As long as the person keeps the different families of logical premises apart, this variation need not involve him in contradictions.

I shall distinguish between two families of nonlogical premises. One concerns scientific matters. The other deals with ethical matters. The scientific principles are of two kinds, those that are true by necessity and those that wise men have surmised from theory and from observations by inductive and appropriate analogical reasoning. The ethical premises are also of two kinds, those that concern moral virtues and those that pertain to political science proper.

The scientific premises that are true by necessity comprise a varied lot of assertions. Some of them are true by definition: e.g., "all widows have had husbands." Others can be established by analysis: e.g., "for all integers n, 1+2+…+n = n(n+1)/2." Still others are intuitively obvious: e.g., "agricultural skill remaining the same, additional labour employed on the land within a given district produces in general a less-than-proportionate return." Finally, there are some assertions for the truth of which wise men provide both inductive and theoretical reasons: e.g., "any living organism has or has had a parent."

The second class of scientific premises contains propositions that the wise, without good theoretical reasons, believe to be true. Some are laws that scientists have established by induction; e.g., "all ruminants have cloven hooves." Others are laws that can be inferred by analogy from introspection or other pertinent observations; e.g., "any man is either in selfless persuit of some spiritual goal or desires to obtain additional wealth with as little sacrifice as possible." Still others are laws of nature that knowledgeable wise men, on rather flimsy evidence, take to be valid; e.g., John M. Keynes' *Principle of the Limited Variability of Nature*.

It is possible that in a given society there is a family of scientific premises that all the members, if prodded, would accept as valid. Still, *the scientific premises on which one person in the society bases his reasoning will vary over the situations he faces. Also the scientific* *premises that different persons employ in similar situations are likely to differ for various reasons. Their educational background and their stock of tacit knowledge may differ. Besides, their access to information retrieval systems may be very different*. These facts of life have interesting analogues in mathematics. In developing one and the same theory different mathematicians may make use of different axioms and rules of inference. Also what are axioms in one theory may appear as theorems in another. For example, the axioms of the theory of real numbers are derived theorems in set theory and universal theorems in Euclidean geometry.

The ethical premises that concern moral virtues are prescriptions for good behavior. Some describe what it is morally right to do; e.g., "you shall honor your mother and father." Others list actions that are morally wrong; e.g., "you shall not kill." Still others formulate general principles for right behavior; e.g., "do to others only what you would like them to do to you." Prescriptions for good behavior as well as sanctions for bad behavior vary both with the codes of honor of secular societies and with the commandments of religious societies. For example, a woman's bare ankle is hardly noticed in the US. It is cause for a public beating in Taliban Afghanistan Also, the interpretation that different wise give to one and the same ethical premise may differ. For example, act-Utilitarians disagree with rule-Utilitatrians about whether one ever can justify lying about illicit sexual relations. Similarly, proponents of Natural-Law ethics disagree with Utilitarians about justifiable reasons for killing a fetus to save the mother's life.

The ethical premises that concern matters of political philosophy prescribe basic rights of human beings and essential characteristics of a just society. Examples of fundamental human rights are "freedom of thought and worship" and "freedom of speech and assembly." Examples of the ingredients of a just society are "equality before the law" and "equality of opportunity." Philosophers agree that there are such things as fundamental human rights. However they disagree as to whether to look for the origin of such rights in natural law or social-contract theory. Philosophers are equally at odds about how to characterize a just society. For example, Aristotle accepted the subjugation of women to men, slaves to citizens, and Barbarians to Helenes. He reserved justice for those who were "free and either proportionately or arithmetically equal" (the Nichomachian Ethics, pp. 106-125). In contrast, Rawls insists that a just society is a society in which equality of opportunity reigns and in which each person has an equal claim to the basic rights and liberties. In a just society social and economic inequalities will arise only if they contribute to the welfare of the least advantaged members of society (Rawls 1971, pp. 54-83).

The ethical premises determine what is right or wrong in a person's relation to other human beings. They also structure a rational animal's appetition for the good. *It is quite clear that different secular as well as religious societies will adopt different families of basic ethical premises. Still there may be a core of ethical premises that reasonable secular and religious societies will accept so that their respective nations can survive as free democratic societies.* If such principles exist, we may find them among the principles of justice that Rawls' reasonable and rational persons in a Hobbesian Original State would adopt for a democratic society of free and equal citizens. These reasonable and rational persons, supposedly, have a sense of justice, a conception of the good, and the intellectual powers of judgment, thought and inference. Moreover, in the Original State they search for principles of justice that specify fair terms of social cooperation between free and equal citizens and ensure the emergence of just institutions in a democratic society. Rawls believes that the resulting principles would enable even a society of individuals who are profoundly divided by reasonable religious, philosophical and moral doctrines to exist over time as a just and stable society of free and equal citizens (Rawls, 1996, pp. 47-88). There are in our social reality examples of just and stable democratic societies with free and equal citizens. Norway is one of them, I believe that Rawls' reasonable and rational persons would accept the principles that the Norwegian constitution incorporates. I also believe that it is fair to say that Norwegians today are "profoundly divided by reasonable religious, philosophical and moral doctrines." It will be interesting to see how the tolerance of the doctrines fares as the population of African and Asian immigrants in Norway grows.

II.4 **Right Desires and Rational Choice in our Social Reality**

We have seen that it is possible that a given society may have a set of basic scientific principles that all its members, if prodded, would consider valid. Still, it is not certain that two of the same society's members in a given choice situation would reason with the same basic principles. We have also seen that in a society in which members are at odds about fundamental religious, philosophical and moral issues only the principles that concern poliitical justice have a good chance of being accepted by all. But if that is so, there are better ways to think of a rational animal's proper premises than to identify them with the basic principles we discussed above. Here is one such way.

Scientific and ethical premises influence a rational animal's acts and judgements in interesting ways. To see how, take a second look at my examples. Whatever are the scientific premises on which a given person bases his acts and judgements, they express facts. Some of these originate in scientific classifications and blue prints. Others are determined by institutional constraints. All of them express factual aspects of the social reality which the person is experiencing. Similarly, whatever are the ethical premises on which a given person bases his acts and judgements, they and the sanctions that the person associates with them determine institutional facts. These institutional facts also express factual aspects of the social reality which the person is experiencing. Both the scientific and the ethical premises vary with individuals and with the situations that different individuals face. For a given US farmer some of the premises may describe ways to produce pertinent farm products. Others may inform him how to rank the same products according to their profitability and insist that he not employ illegal immigrants. For a given physicist at CERN some of the premises may instruct him how to read tracks in a cloud chamber. Others may inform him how to report his results and insist that he do it truthfully.

In the remainder of the paper we shall think of a person's basic principles as facts in the social reality that he is experiencing. Some of these facts he carries with him as easily accessible explicit or tacit knowledge. Others he can, if need be, acquire by reading books and journals or simply by picking the brains of friends and foes. The sentences that he, in virtue of these facts, believes to be true may harbor contradictions. They may also express facts that are not accepted as facts by others, In each choice situation he will make his decisions on the basis of pertinent facts only. If the latter harbor contradictions, and if he becomes aware of it, we expect that he will make changes in his basic principles locally and, if possible, adjust his choice or judgement accordingly. **The facts that determine a person's choice and judgements in a given situation depend on the person's desires and vary with the situation he faces.**

A person's desires are right if they reflect an appetition for the good. In the history of moral philosophy the 'good' has been interpreted to be many different things. To Plato the 'good' was a form in a world of ideas whose reference in Plato's social reality consisted of all those things that one truthfully could describe as 'good.' To medieval Christian philosophers the 'good' was God, and eternal life with God was the goal of all right desires. To nineteenth century Utilitarians the 'good' was the sum total of happiness that the members of a given population experienced, and happiness was pleasure and freedom from pain. Here I shall give 'good' the interpretation that Aristotle gave to the term **The 'good' is that at which every art and inquiry and every action and pursuit aim. Also the 'good' is something people search for its own sake.** Aristotle identified this 'good' with 'happiness', and I shall insist that the happiness in question is the 'good' that any given individual experiences. As such, happiness is an undefined term that economists usually designate by 'utility.'

Aristotle insisted that the 'good' was an activity of the soul in accordance with moral and intellectual virtues. This connection between the 'good' and all the virtues that he listed in his Nichomachean Ethics was an essential characteristic of Aristotle's idea of the 'good.' We shall obtain an analogous connection by insisting that happiness; i.e., **the good is a function of variables some of which we associate with Aristotle's virtues, and some of which we associate with members of Rawls' list of primary goods.** Examples are knowledge, esteem, friendship, justice, basic rights and liberties, and income and wealth. The function may but need not be additively separable, and the arguments as well as the function itself may vary from one individual to the next. Here it is important to observe that an appetition for the good may lead to increased values of some factors affecting happiness and to decreased values of others. Thus it is perfectly possible that the act of a Norwegian who fails to pay a buss ticket and the act of a North Carolina business man who sees fit to employ an illegal immigrant may reflect an appetite for the 'good.'

With the preceding observations in mind I can conclude this section with the following characterization of rational choice and judgements: **Rational choices and judgements are good choices and judgements that, in accordance with the rules of logic, follow by necessity from pertinent facts and right desires.** The facts that determine a person's choice and judgement in a given situation depend on the person's desires and vary with the situation he faces.

III *Rationality in Economics*

In Section I I insisted that a human being is a rational animal, and that a rational animal is an animal with deliberative imagination who has beliefs and is able to opine and reason. These rational animals constitute the populations whose characteristics we study in econometrics. In Section II I insisted that rational choices and judgements were good choices and judgements that followed by necessity from pertinent facts and right desires. The desires, supposedly, varied with persons and the pertinent facts varied both with persons and with the situations that the persons faced. To me this characterization depicts characteristic features of the choices and judgements that members of the populations we study in econometrics make.

In my characterization of rational choices and judgements the terms 'good choice,' 'good judgement,' 'pertinent fact,' and 'right desire,' are undefined terms. Hence, the characterization notwithstanding, 'rational choice' and 'rational judgement' have no definite meaning. In econometrics the latter terms are given interpretations that seem appropriate for the studies on hand. Such interpretations econometricians usually extract from various economic theories. We shall now take a close look at some of these interpretations and their empirical relevance.

III.1 **Consumer Choice under Certainty**

There are all sorts of economic theories that are relevant in this context. Some pertain to choice under certainty. Others concern choice under uncertainty. Still others delineate strategies in various game-theoretic situations. Economists use these theories to describe the behavior of organizations as well as the behavior of individuals. Econometricians can use them to search for empirically relevant interpretations of the undefined terms in my characterization of rational choices and judgements.

It is important to note that econometricians cannot use economic theories to test the rationality of members of a given population. The members are rational. It is also the case that econometricians can never know from *a priori* reasoning alone in what situations a particular economic theory might have empirical relevance. This may sound strange. So here is an example to illustrate what I have in mind.

E.10 Consider my daily shopping for food in the neighborhood grocery store and the standard theory of consumer choice under certainty. I usually buy a loaf of bread and I might buy coffee if the price is right. Some times the store is out of my family's favorite brand of bread. Then I buy another brand that I believe my little daughter will like. I will do that even if the other brand costs twice as much, and even if there is another grocery store, ten minutes away, that might have the brand I want. As to coffee, I will buy coffee only if I judge the price to be low. Then I buy many more bags of coffee than I need, and I store them for later use. The theory of consumer choice insists that a rational consumer in each period chooses the commodity bundle that maximizes his utility subject to his budget constraint. This theory cannot be used to question the rationality of my shopping in the neighborhood grocery store. There are three good reasons for that. The theory does not account for the costs of search. It provides no opportunity for storing goods. And it denies current prices the ability to convey information about future prices.

The moral of E.10 is not that the theory of consumer choice under certainty is wrong. Instead it is that we must choose its applications with care. Also, in searching for applications, we must keep in mind that the main purport of the theory is to delineate two characteristic features of consumer choice and exhibit how they are reflected in consumer behavior. In this theory 'good judgement' is taken to mean that the consumer can order the commodity bundles he faces. Also 'good choice' means choosing among the available bundles the one the consumer ranks the highest. In the intended interpretation of the theory, a consumer is an individual living alone or a family living together and having a common budget. A commodity bundle is an ordinary commodity vector. It may also be a vector of safe and risky assets or a life-cycle plan of consumer expenditures. Finally, an available commodity bundle is a commodity bundle that satisfies the budget constraint which the consumer faces. Econometricians have used the theory successfully to study how consumers' expenditures on various categories of commodities vary with their income (Engel, 1857 and Aasnes et al, 1985). They have also used it successfully to determine how a consumer's choice of safe and risky assets varies with his net worth (Arrow, 1965 and Stigum, 1990), and how his consumption-savings decision varies with his life-cycle income stream (Modigliani and Brumberg, 1955, Friedman, 1957, and Stigum, 1990).

III.2 **Choice under Uncertainty**

Most economic theories of choice agree that 'good judgement' is synonymous with 'ability to rank available options.' The characteristics of the rankings in question, however, vary over theories as well as over the situations the decision maker faces. A careful look at the theory of choice under uncertainty will bear witness to that.

An uncertain option can be many things; e.g., a gamble, an insurance policy, or an investment in the stock market. We shall distinguish between two kinds of uncertain options - those that pertain to risky situations and those that pertain to uncertain situations. Here a **risky situation** is taken to be a situation in which the likelihoods of all possible events are known or can be calculated by reason alone. An **uncertain situation** is a situation in which the likelihoods of all possible events are not known and cannot be calculated with reason alone. I begin by discussing choice in risky situations.

III.2.1 **Risky Situations**

To make my discussion of choice in risky situations as simple as possible I shall think of an uncertain option in such situations as a prospect. A *prospect* is an n-tuple of pairs, $\{(x_1,p_1),...,(x_n,p_n)\}$, where $x_i \in R$ is an outcome, $p_i \in R_+$ is a measure of the likelihood of x_i happening, and $p_1 + ... + p_n = 1$.

Showing 'good judgement' in risky situations involves carrying out two successive tasks. The first task consists in assigning numbers to the p_i in the prospects which the decision maker is facing. That task is some times easy and other times not so easy. Also, a given task may be easy for some and much too difficult for others. Examples E.11 – E.12 below will illustrate what I have in mind.

According to Laplace, the probability of an event is the ratio of the number of cases that are favorable to it, to the number of possible cases, when there is nothing to make us believe that one case should occur rather than any other, so that the cases are, for us, equally likely (Laplace, 1951, pp. 6-7 and 11). With this definition in mind and, if need be, with a little bit of coaching I should think that most people would agree with the probability I assign in E.11.

E.11 A blindfolded man, A, is to pull a ball from an urn with k red balls and $(100-k)$ white balls. The urn is shaken well. So the probability of A pulling a red ball from the urn is $k/100$.

With coaching most people might be able to determine the probability of more complicated events; e.g., the probability of E_1, four red balls in four draws with replacement from an urn with $k = 84$, or the probability of E_2, at least four red balls in ten draws with replacement from an urn with $k = 50$. However, without coaching it is unlikely that the majority of people would manage to figure such probabilities.

D. Kahnemann and A. Tversky believe that most people in assessing likelihoods rely on a limited number of heuristics which help them reduce complex computational tasks to manageable proportions (Kahneman and Tversky, 1972). One such heuristic, *anchoring*, leads people to overestimate the probability of conjunctive events and to underestimate the probability of disjunctive events. Thus chances are that most people would overestimate the probability of E_1 and underestimate the probability of E_2. Much experimental evidence bears out this prediction (cf. for example Bar-Hillel, 1973).

What untutered people might do in the case I describe in E.12 is anybody's guess.

E.12 Consider a mechanical system of r indistinguishable particles. The phase space has been subdivided into a large number n of cells so that each particle is assigned to one cell. There are n^r different ways in which r particles can be arranged in n cells. I have no reason to believe that one way is more likely than another, So I take all ways to be equally likely. Since there are $r!/r_1!r_2!...r_n!$ indistinguishable ways in which we can arrange r particles such that r_i particles are in cell i, i=1,...,n, I conclude that the probability that cells 1,...,n contain $r_1,...,r_n$ particles with $r_1 + ,,, + r_n = r$ is $(r!/r_1!r_2!...r_n!)n^{-r}$.

The interesting aspect of E.12 is that I have used Laplace's definition of probability correctly and come up with a wrong probability assignment. According to W. Feller, numerous experiments have shown beyond doubt that the probabilities I calculated in E.12 are not the true probabilities of any known mechanical system of particles. For example, photons, nuclei, and atoms containing an even number of elementary particles behave as if they only considered distinguishable arrangements of the pertinent system's particles. Since there are just (n+r-1)!/(n-1)!r! distinguishable arrangements of r particles in n cells, and since all of them seem to be equally likely, the true probability of the given event for photons, nuclei, and atoms containing an even number of elementary particles is $[(n+r-1)!/(n-1)!r!]^{-1}$.

Few if any would question Feller's authority in E.12. So from E.11 and E.12 we conclude that we shall know only in trivial cases what are the prospects among which a given decision maker chooses. This is true even if we help him determine the values of the pertinent probabilities. The values we assess may be very different from the values that the decision maker perceives. For example, the high probabilities may be higher than the corresponding perceived probabilities, and the low probabilities may be lower than the corresponding perceived probabilities. Many experimental studies bear witness to such a possibility (cf. for example Mosteller and Nogee, 1951 and Preston and Baratta, 1948).

To show 'good judgement' in risky situations the decision maker must carry out a second difficult task: determine his ordering of the prospects he faces. For that purpose, consider a decision maker, B, whose perceived probabilities often differ from the true probabilities in the prospects he faces. In a given situation a prospect is to B like a measurable function on a probability space, $(\aleph, \Re, \wp)$, where $\aleph$ is a finite set of states of nature, $\Re$ is the field of all subsets of $\aleph$, and $\wp$ is a probability measure. The functions take only a finite number of values all of which belong to a set of real numbers, X, and the value of $\wp$ at any $A \in \Re$ equals the likelihood of A happening that B perceives. Also, B's ordering of the pertinent prospects induces an ordering of functions in which B orders indicator functions of subsets of $\aleph$ in accordance with the sets' $\wp$-values. Finally, with a slight modification of axiom SSA 4 B's ordering of prospects satisfies the axioms concerning $\aleph$ and B's ordering of measurable functions that I listed in (Stigum, 1990, pp. 434-439). That ensures the existence of a function, $U(\cdot): X \rightarrow R$, that is determined up to a positive linear transformation, and is such that if x and y are any two of the prospects B faces, B will prefer x to y if and only if

$$\sum_{\eta \in \aleph} U(x(\eta)) d\wp \; > \; \sum_{\eta \in \aleph} U(y(\eta)) d\wp \, .$$

Thus, if $\aleph$ and B's ordering of measurable functions satisfy my axioms, B will order the pertinent prospects according to their perceived expected utilities.

The theory of mine is controversial in several respects. My axioms represent a modification of L.J. Savage's axioms for choice in risky situations (cf. Savage, 1954). In Savage's theory $\aleph$ contains a nondenumerable number of states of nature. Also, both the utility function and the subjective probability measure of the decision maker are determined by his or her risk preferences. Finally, Savage seems to believe that the utility function and the subjective probability measure are determined once and for all for all the risky situations that the decision maker might face. I believe that $U(\cdot)$, $\aleph$, and $\wp(\cdot)$ may vary from one choice situation to the next. Also, in the situations I envision above 'good judgement' is obtained sequentially in two steps. We first determine B's perceived probabilities and then his or her ordering of the prospects in question. That allows me to rephrase my SSA 4 axiom in the obvious way such that the α_i in equation 19.22 on p. 435 of (Stigum, 1990) can be interpreted as B's perceived probability of the i^{th} state of nature.

The empirical relevance of my theory is also uncertain. To see why consider the following example.

E.13 Consider an urn with 100 balls that differ only in color and assume that there are 89 red balls, 10 black balls, and 1 white ball. The urn is shaken well and a blindfolded man is to pull one ball from it. We ask a decision maker, B, to rank the components of the following two pairs of prospects in which he will receive

α_1: \$1000 regardless of which ball is drawn;

α_2: nothing, \$1000, or \$5000 according as the ball drawn is white, red, or black,

β_1: nothing if the ball is red and \$1000 otherwise;

β_2: nothing if the ball is either red or white and \$5000 if the ball is black.

If B's preferences satisfy my axioms, B will prefer α_1 to α_2 if and only if he prefers β_1 to β_2 . Also, B will prefer α_2 to α_1 if and only if he prefers β_2 to β_1.

Ever since Maurice Allais in 1952 started dreaming up examples like E.13 (cf. Allais, 1979), numerous individuals have been asked to rank similar pairs of prospects. Judging from the experiments of which I am aware, roughly 60% of the subjects answer in accordance with the prescriptions of my theory. Those who "fail" the test usually prefer α_1 to α_2 and β_2 to β_1. Their preferences seem to reveal an aversion to uncertainty that is characteristic of individuals who shade their probabilities in uncertain situations. More on that below.

III 2.2 **Uncertain Situations**

In discussing choice of options in uncertain situations we shall again, for simplicity, think of an option as a prospect, $\{(x_1,p_1),...,(x_n,p_n)\}$. In this case the x_i are known, but the p_i are not. Also the true values of the p_i cannot be calculated by reason alone. We shall consider two prominent ways of dealing with such prospects. In one of them the decision maker assigns values to the p_i in accordance with Baysian principles and chooses among options according to their expected utility. In the other the decision maker assigns values to the p_i in accordance with ideas that A. P. Dempster and G. Shafer developed in (Dempster, 1967) and (Shafer, 1976). Also, he chooses among options according to the values of a Choquet integral that he associates with them. I begin with the Baysians.

Consider the following example, and take special note of the forty-five subjects who were indifferent in their choice of urns.

E.14 Two urns, A and B, contain 100 balls that are either red or white. There are 50 red balls in A, but nobody knows how many there are in B. A blindfolded man is to pull a ball from one of the urns, and you are to choose the urn for him. If he pulls a red ball, you will receive \$100, otherwise nothing. Of 140 colleagues, students, and friends in Evanston and Oslo who were faced with the given choice, 82 chose A, 45 could not make up their minds, and 13 chose B.

Each one of the 45 indifferent persons may have argued like true Baysians. The probability of pulling a red ball from A is 1/2. Also, there are 101 possible combinations of red and white balls in B, and there is no reason why one combination is more likely than another. So, we should assign prior probability $(101)^{-1}$ to each of them and insist that the probability of picking a red ball in B equals $\sum_{i=0}^{100} (i/100)\cdot(101)^{-1} = (101)^{-1}\cdot[100\cdot101/2\cdot100] = 1/2$. If the Baysian arguments are right, there is no reason to prefer one urn to the other.

A Baysian prior is supposed to reflect the decision maker's knowledge about the p_i in a given prospect. Assigning such priors can be problematic. Here is a case in point.

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E.15 Consider two deseases, X and Y, that require different treatments, and that are equally fatal if untreated. A person, A, is taking a test to determine whether he is suffering from X or Y. He knows that the probability that the test result will be accurate is 4/5. He also knows that X for a variety of demographic reasons is nineteen times as common as Y. The test reports that A suffers from Y. From this A deduces that the probability is 4/23 that he is suffering from Y and 19/23 that he is suffering from X. So he asks his doctor to treat him for X.

I became aware of this example from reading an article of L. J. Cohen (Cohen, 1981, p. 329). Cohen insists that A has used a prior concerning the relative prevalence of the two diseases and computed the probability that an instance of a long run of patients that take the test will suffer from disease X. He should have used, instead, a prior that assesses A's own predisposition to the two diseases. If A has no known predisposition to either disease, he should have concluded from the test results that the probability is 4/5 that he is suffering from Y and ask his doctor to treat him for Y rather than X. Results of experimental tests in comparable situations suggest that subjects tend to judge the values of the pertinent probabilities in the way Cohen suggests is right (cf. for example, Hammerton, 1973).

We have seen above that learned people may disagree on what is the appropriate prior to use in evaluating a given option. It is also the case that Baysians argue among themselves what is the best way to model ignorance. They worry when their way of modeling ignorance of a parameter, p, suggests that they are not that ignorant about the value of 1/p. Also they are concerned when use of a diffuse prior to model ignorance of a parameter,θ , may swamp the information that a researcher can obtain from sample information about θ . For us the important thing to observe is that not everybody is a Baysian - at most 45 of the 140 subjects in E.15 Also, even one who tends to think like a Baysian will have difficulties calculating the posteriors that are required for proper use of Baysian ideas. Finally, however a Baysian chooses his priors and calculates his posteriors, both the priors and the posteriors are honest probabilities. They are nonnegative and their sums or their integrals, as the case may be, equal 1.

To see how Baysians order prospects, consider a decision maker, D, who calculates the probabilities in any prospect that he faces the way a Baysian would calculate them. To D a prospect x is like a probability distribution, $F_x(\cdot): R \rightarrow [0,1]$. D's ordering of prospects induces an ordering of such probability distributions. Von Neumann and Morgenstern in (von Neumann and Morgenstern, 1948) gave sufficient conditions on D's ordering of probability distributions that there exists a function, $U(\cdot): R \rightarrow R$, that is determined up to a positive linear transformation, and is such that if x and y are any two prospects, D will prefer x to y if and only if

$$\int U(t)F_x(dt) > \int U(t)F_y(dt).$$

Thus if $x = \{(x_1,p_1),\ldots,(x_n,p_n)\}$, $y = \{(y_1,q_1),\ldots,(y_m ,q_m)$, and the p_i and the q_j are the probabilities that D has calculated, then D will prefer x to y if and only if

$$\sum_{i=1}^{n} U(x_i)\, p_i \quad > \quad \sum_{j=1}^{m} U(y_j)\, q_j \ ;$$

i.e., if and only if the expected utility of x is larger than the expected utility of y.

M. Allais and his followers are as critical of von Neumann and Morgenstern's theory as they are of Savage's theory. Also, many of the tests that they have conceived have been good tests of both theories (cf. MacCrimmon and Larsson, 1979). The dismal results of these tests have motivated researchers to look for alternative theories. We shall next discuss the most promising of them.

Many knowledgeable persons will insist that the 82 subjects in E.16, by choosing A over B, have revealed an aversion to uncertainty. Specifically, so the argument goes. For any one of these subjects, we can find a number $k < 50$ and an urn, $C(k)$, with k red balls and $(100-k)$ white balls such that the given person would be indifferent between having the blindfolded man pull a ball from B or $C(k)$. This indifference indicates that the person is assigning probability $(k/100)$ to the event that the blindfolded man might pull a red ball from B. By a similar argument, he would assign probability $(k/100)$ to the event that the blindfolded man might pull a white ball from B. But if that is true, the given subject is a person who reacts to uncertainty by shading his probabilities. Chances are good that the other members of the group of 82 in E.15 react to uncertainty in the same way.

When people shade their probabilities, they assign superadditive probabilities to the possible events in an uncertain situation. Such probabilities have interesting properties. To study them I assume, for simplicity, that there are only a finite number of states of nature; i.e., that $\aleph = \{\eta_1,\ldots,\eta_n\}$ for some n. Also, I take $\Re$ to be the set of all subsets of $\aleph$, and I let $Bel(\cdot): \Re \to [0,1]$ be a function that records the values that a given decision maker, D, assigns to the subsets of $\aleph$. D shades his probabilities in the face of uncertainty. Hence, for any two disjoint events, A and C, we find that $Bel(A) + Bel(C) \leq Bel(A \cup C)$. Finally, I assume that there exists a function, $m(\cdot): \Re \to [0,1]$, with $m(\emptyset) = 0$, $\sum_{A\subset\aleph} m(A) = 1$, and $Bel(A) = \sum_{C\subset A} m(C)$ for all $A \in \Re$.

In his book Shafer develops interesting ways for D to combine belief functions so as to update his beliefs. However, he has little to say about how D should order the options he faces. There are several alternatives (cf. Jean-Yves Jaffray, 1989, Chateauneuf, 1986, and Gilboa, 1987). I shall use a method that I once learned from Kjell Arne Brekke in 1986 (cf. Stigum,1990, pp. 445-455). Let $\aleph$, $\Re$, $Bel(\cdot)$,and $m(\cdot)$ be as above, and think of a prospect as a function, $x(\cdot): \aleph \to X$, where $X = \{x_1,\ldots,x_n\}$ is the set of all consequences of the prospects that our decision maker, D, faces. I assume that D orders consequences according to the values of a function, $U(\cdot): X \to R$. Also, for each prospect, $x(\cdot)$, and every $A \in \Re$, I let $W_x(A) = \min_{\eta\in A} U(x(\eta))$ Finally, I insist that the utility which D receives from prospect $x(\cdot)$ equals V(x), where

$$V(x) = \sum_{A\subset\aleph} W_x(A)m(A).$$

The ordering of prospects that $V(\cdot)$ induces has many interesting characteristics. The ordering cannot be rationalized by an expected utility index. Instead,

$$V(x) = \int_0^\infty Bel(\{\eta\in\aleph: U(x(\eta))\geq t\})\, dt,$$

where the integral reduces to an expected utility index only when $Bel(\cdot)$ is additive. Also, the ordering exhibits a remarkable aversion to uncertainty that we can document in the following way. Let

$$P = \{p(\cdot): \aleph \to [0,1]: \sum_{i=1}^n p(\eta_i) = 1\} \text{ and,}$$

for each $p \in P$, let $Pp(\cdot): \Re \rightarrow [0,1]$ be such that $Pp(A) = \sum_{\eta \in A} p(\eta)$ for each $A \in \Re$. Also, let

$$C = \{p \in P: \text{for all } A \in \Re, Pp(A) \geq Bel(A)].$$

Then

$$V(x) = \min_{p \in C} \sum_{i=1}^{n} U(x(\eta_i))p(\eta_i).$$

Finally, the ordering satisfies neither Savage's axioms nor mine. The following example bears witness to that.

E.16 Suppose that we ask an individual, Peter, to rank the components of the two pairs of prospects that we described in E.15. Peter lets $\aleph = \{\eta_1, \ldots, \eta_{100}\}$ and insists that η_i is the name of a red, black, or white ball according as $0 < i \leq 89$, $90 \leq i \leq 99$, and $i = 100$, respectively. Moreover, he lets the set of consequences, X, be {0, \$1000, \$5000 }, and notes that his utility function on X is given by U(0) = 0, U(\$1000) = 0.85, and U(\$5000) = 1. Finally, Peter decides for himself that the expression, "the urn is shaken well," is vague and assigns the following basic probabilities to the subsets of $\aleph$: M(A) = 0.7, 0.2, and 0.02 according as A is the set of red, black, and white balls, respectively. Also, M(A) = 0.05, 0.03, and 0.01 according as A is the set of balls that are, respectively, either red or black, either red or white, and either black or white. In all other cases M(A) = 0. Now, Peter ranks prospects in accordance with the values of the function $V(\cdot)$ that his $U(\cdot)$ and his $m(\cdot)$ determine. Therefore, he needs little time to figure that $V(x_{\alpha 2}) < V(x_{\alpha 1})$ and that $V(x_{\beta 1}) < V(x_{\beta 2})$. Peter's ranking of α_1 and α_2 and of β_1 and β_2 violates the prescriptions of mine and Savage's theory.

III.3 **Game-Theoretic Situations**

In Consumer Choice under Certainty the consumer knows the values of all relevant current and future prices. In Consumer Choice under Uncertainty the consumer knows all relevant current and past prices and he uses the information they convey to form his ideas about the probability distributions of future prices (cf. Stigum, 1969 and Stigum, 1990, pp. 765- 800). In both theories the consumer forms his judgements and makes his choices independently of the judgements and choices of other consumers. In particular, he does not take into account that his ability to implement his choices depends on the choices of other consumers. Various sufficient conditions on preferences and expectations that ensure the existence of prices at which all consumers can implement their choices exist. The reader can find examples of such conditions for the certainty case in (Debreu, 1959). Analogous conditions for the uncertainty case are given in (Stigum, 1969 and 1972).

In game-theoretic situations each participant has on hand a set of pure strategies and faces a set of consequences each member of which results from the particular combinations of pure strategies that the participants choose. Also, each participant orders consequences according to the values of a utility function and may use mixed as well as pure strategies. Finally, each participant knows the rules of the game, knows his own and his opponents’ sets of pure strategies, knows the consequences for him and the others of his and their choice of strategies, knows his own utility function, and knows the families of functions to which his opponents’ utility functions belong. Game theorists usually add to this that it is common knowledge among participants that each participant possesses such knowledge.

There are all sorts of games. Some are non-cooperative. Others are cooperative. Some are static. Others are dynamic. Whatever the pertinent game is, the novel aspect of a game-theoretic situation is that each participant in his search of good choices must take into account the possible choices of his opponents. Game theorists agree that a good choice of strategy for a given player must be a best response to the strategies of his opponents. However, it is often hard to determine what constitutes a best response in situations in which an opponent’s choice of strategy is not well defined. Also, even in situations where all the participants' best responses are common knowledge, it may be impossible for a participant to single out a good choice of strategy before he knows what his opponents will do. I shall use the following example of a non-cooperative static game to illustrate what I have in mind.

E.17 Consider a game with two players, A and B, in which A has four pure strategies, a,b,c, and d, B has three pure strategies, α,β, and γ, and the payoff matrix is as follows:

		B’s strategies		
		α	β	γ
	a	150, 60	30, 200	150, 50
	b	200, 75	40, 300	100, 50
A’s strategies	c	50, 300	200, 65	100, 30
	d	100,50	100, 45	100, 30

Thus if A chooses b and B chooses γ, A will receive utility 100 and B will receive utility 50.

We take for granted that A and B are rational animals and that that is part of the players' common knowledge. A rational animal in B's situation would never choose γ since the utilities he may receive from γ are all smaller than the respective utilities that he can obtain by choosing α or β. Also, if B will never choose γ, then a rational animal in A's situation will never choose a since he can obtain higher utilities by choosing b. Consequently, we can without loss in generality reduce A and B's game to a game where A has three strategies, b,c, and d, B has two strategies, α and β, and the payoff matrix is as follows:

		B’s strategies	
		α	β
	b	200, 75	40, 300
A’s strategies	c	50, 300	200, 65
	d	100, 50	100, 45

In the last description of the game B’s best responses to A’s pure strategies, b, c, and d, are β, α, and β, respectively. Similarly, A's best responses to B’s pure strategies, α and β, are b and c, respectively. Also, the utility that A can gain from playing d is smaller than the expected utility that he would obtain by playing b with probability 1/2 and c with probability 1/2. So most game theorists would insist that a rational animal in A's situation would never play d. However, this need not be so. Unless it is common knowledge that players in a game rank uncertain prospects according to their expected utility, we cannot take for granted that A will never employ d. More on that below.

A Nash equilibrium in a game is a combination of strategies in which each participant has played his best response against the chosen strategies of the other players. In the given game there is no Nash equilibrium in pure strategies. If it is common knowledge that A and B rank uncertain options according to their expected utility, there is, instead, a Nash equilibrium in mixed strategies. In this equilibrium A plays b with probability 47/92 and c with probability 45/92 and B plays α with probability 16/31 and β with probability 15/31. There is no other Nash equilibrium.[3]

There are many interesting aspects of the preceding example. For example, a Nash equilibrium in pure strategies is an equilibrium in which each participant in the game, after having learned to know the strategies his opponents chose, is satisfied with the strategy he chose for himself. A Nash equilibrium in mixed strategies is nothing of the sort, since in such an equilibrium the game participants shall never know what strategies their opponents have adopted. Game theorists seek to ameliorate this deficiency by adding two conditions to our characterization of games. They insist that it must be common knowledge that each participant is ‘rational’ and that 'rational' individuals limit their choice of good strategies to Nash equilibrium strategies. In a game with a unique Nash equilibrium, therefore, the players need not observe the strategies of their opponents. They can calculate the strategies of their opponents and choose their own strategies accordingly. What the participants are supposed to do in games with multiple Nash equilibria, however, is problematic to say the least.

It is one thing to argue that it is 'rational' for the players of a game to adopt mutually consistent Nash equilibrium strategies. It is another thing to insist that in game-theoretic situations rational animals tend to choose mutually consistent Nash equilibrium strategies. The former is an assertion about what rational animals **ought** to do when they participate in games. The latter is an assertion about what rational animals **actually** do. Game theorists' arguments in support of Nash equilibrium strategies make wonderful sense for a **normative** theory of games. However, they are of little help in the search for ways to develop a **positive** theory of games. To describe actual behavior in games we must introduce new ideas.

From the point of view of a positive theory of games it is awkward to insist that players are "rational' and that 'rational' players always choose Nash equilibrium strategies. Players are rational animals, but it is far from evident that rational animals necessarily choose Nash equilibrium strategies. To describe the behavior of rational animals in game-theoretic situations, we must introduce ideas about the players' expectations and about their risk preferences. To see how, take another look at the game in E.17. From the point of view of A, his three pure strategies are three prospects with known consequences and unspecified probabilities. The probabilities specify A's ideas as to how B goes about choosing his two strategies. Similarly, to B his two strategies are prospects with known consequences and unspecified probabilities. The probabilities describe B's ideas as to how A goes about choosing his three strategies. A and B must use the information they possess about each other to evaluate the pertinent probabilities, rank the resulting prospects, and determine their respective good choices. A's and B's probability assignments and risk preferences are not common knowledge. The next example elaborates on these ideas.

E.18 Consider the game in E.17 without the dominated strategies, and assume that both A and B assign probabilities to the strategies of their opponent in accordance with the principles of Dempster and Shafer. A argues that B has no good reason for preferring α to β. So he assigns the following basic probabilities to B's choice of strategies:

$$m_A(\{\alpha\}) = ¼ = m_A(\{\beta\}) \text{ and } m_A(\{\alpha,\beta\}) = ½.$$

B on his side argues that A has no good reason for preferring b to c and that there is a chance that A is averse to uncertainty. So he assigns the following basic probabilities to A's choice of strategies:

$m_B(\{b\}) = ¼ = m_B(\{c\}), m_B(\{b,c)) = ¼, m_B(\{d\}) = 1/6$, and $m_B(\{b,c,d\}) = 1/12$.

Also, both A and B in uncertain situations rank their prospects according to the prescriptions of Kjell Arne Brekke that I detailed above. Thus, for A we find that

$W_A(\{\alpha\}|b) = 200, W_A(\{\beta\}|b) = 40, W_A(\{\alpha,\beta\}|b) =40$, and $V_A(b) = ¼(200+40) + ½\ 40= 80$;

$W_A(\{\alpha\}|c) = 50, W_A(\{\beta\}|c) = 200, W_A(\{\alpha,\beta\}|c) = 50$, and $V_A(c\ ) = ¼(50+200) + ½\ 50= 87.5$; and

$W_A(\{\alpha\}|d) = 100,\ W_A(\{\beta\}|d) = 100, W_A(\{\alpha,\beta\}|d) = 100$, and $V_A(d) = 100$.

For B we find that

$W_B\ (\{b\}|\alpha) = 75, W_B\ (\{c\}|\alpha) = 300, W_B(\{d\}|\alpha) = 50, W_B(\{b,c\}|\alpha) = 75, W_B(\{b,c,d\}|\alpha) = 50$; and

$W_B(\{b\}|\beta) = 300, W_B(\{c\}|\beta) = 65, W_B(\{d\}|\beta) = 45, W_B(\{b,c\}|\beta) = 65, W_B(\{b,c,d\}|\beta) = 45$ with

$V_B(\alpha) = ¼(75+300+75) + 1/6\ 50 + 1/12\ 50 = 1500/12 = 125$, and

$V_B(\beta) = ¼(300+65+65) + 1/6\ 45 + 1/12\ 45 = 1425/12 = 118.75$.

From this we conclude that A will choose strategy d, and that B will choose strategy α. Neither one of them has reasons to regret their choices. This solution shows why A's d strategy in the E 3.8-game cannot be eliminated unless it is common knowledge that A is an expected utility maximizer.

The game-theoretic situations we considered above are prototypes of a little part of the game situations we face in economics. In other cases we must consider the possibility of preplay communication, the strategic aspects of threats, and the advantages of cooperation. Economists have devised all sorts of theoretical models to go with such possibilities. Some are easy to grasp and others are quite fancy. Here the important thing to notice is that the rationality that these theoretical models prescribe need not have much in common with the rationality of rational animals. The empirical relevance of the characteristics of rational choice in these models must be confronted with data before we can accept them. I shall give two examples to illustrate what I have in mind.

A long time ago, 1950, John Nash insisted that any solution of a two-person bargaining problem must satisfy four, supposedly, reasonable conditions: Pareto-optimality, symmetry, independence of irrelevant alternatives, and invariance to linear transformations of utility. Ingenious experiments in economic laboratories have demonstrated that most experimental bargaining outcomes satisfy the first three conditions and fail to satisfy the fourth (cf. Davis and Holt, 1993, pp. 242-275). The failure of the fourth condition is problematic for game theorists who insist that it is common knowledge in games that players rank mixed strategies according to their expected utility.

The utility function of an expected-utility maximizer is determined up to a positive linear transformation.

Multistage games usually have many Nash equilibria. Multiple equilibria are problematic for a positive theory of games, and have led game theorists to look for refinements of Nash equilibria. One such refinement is Reinhard Selten's idea of a subgame perfect Nash equilibrium in which the players' strategies establish Nash equilibria in each and every subgame (cf. Selten, 1965). Subgame perfect Nash equilibria have interesting characteristics one of which is that they rule out noncredible threats off the equilibrium path. They also carry with them intriguing questions for rational choice in multi-stage games. Here is why. In any given case we find Selten's equilibria by so-called backward induction, and backward induction arguments are questionable. A number of laboratory experiments have demonstrated that subjects are not good at backward induction (cf. Davis and Holt, 1993, pp. 102-109). Also, as evidenced by Selten's own 'chain-store paradox,' they can lead to unreasonable equilibria. Finally, one aspect of rational choice, on which Selten insists, is dubious: If your opponent makes a draw that seems foolish to you, do not question his rationality! Leading game theorists agree that such advice is controversial, and some of them are looking hard for a concept of rationality that will rescue the backward induction argument (cf. Asheim,1999).

IV *Modeling Rationality in Econometrics*

Aspects of rational choice surface in econometrics in two very different ways. We find ideas of rationality in the models that econometricians estimate. These ideas usually originate in the economic theories on the basis of which the models are constructed. Notions of rational choice also enter the way econometricians choose their strategies in the game-theoretic situations they face. The games econometricians play are of two kinds. There are games 'against' the profession in which an econometrician's academic success is at stake. Besides, there are games 'against' Nature in which econometricians search for strategies that will minimize their expected losses. In this section we shall discuss both how econometricians model rationality. How they choose good strategies in game-theoretic situations will be the topic of Section V.

From our discussion of rational animals and rationality in economics it follows that we cannot put stringent requirements on the characteristics of rational animals' good choices. We can insist that in a given choice situation a rational animal will rank available alternatives and choose one that he ranks the highest. We can probably also insist that a rational animal in similar choice situations will make the same choices. If we insist on more than that, we are likely to find that there are sample populations in which these requirements have little empirical relevance.

As we have seen, the preceding facts do not deter economists from imposing severe requirements on rational choice in the situations that they consider. The requirements on which economists insist find their way into the models of individual behavior that econometricians construct. I shall show how in two different cases, rational consumers and rational expectations in macroeconomics.

IV.1 **Rational Consumers**

In the theory of consumer choice under certainty the consumer ranks commodity bundles according to the values of a utility function. The available bundles are the ones that satisfy the consumer's budget constraint. And the consumer's good choice is the commodity bundle that among all available bundles maximizes his utility. The theory's demands on rational behavior come as a consequence of the conditions it imposes on the domain and functional characteristics of the utility function. For example, suppose that we identify a commodity bundle with a vector in R_+^n and assume that the utility function, U(.), is real valued, continuous, strictly increasing, and strictly quasi-concave with domain R_+^n. Then we can show that the consumer's choice of commodity bundles, x, varies with his total expenditure, c, and with the commodity prices he faces, p, in accordance with the values of a well defined vector-valued function, $f(.): R_{++}^n \times R_+ \to R_+^n$. The function, $f(\cdot)$, is the consumer's demand function. It is (cf. Stigum, 1990, pp.184-189) continuous, homogeneous of degree zero, and satisfies three interesting conditions: Samuelson's Fundamental Theorem of Consumer Choice, Houthakker's Strong Axiom of Revealed Preference, and

(i) $pf(p,c) = c.$

If we add the assumption that U(.) is twice differentiable, we can deduce that almost everywhere in $R_{++}^n \times R_+$ the ith and jth component of f(.) satisfy the equations,

(ii) $\partial f_i(p,c)/\partial p_i + f_i(p,c)\partial f_i(p,c)/\partial c < 0$ for i = 1,...,n; and

(iii) $\partial f_i(p,c)/\partial p_j + f_j(p,c)\partial f_i(p,c)/\partial c = \partial f_j(p,c)/\partial p_i + f_i(p,c)\partial f_j(p,c)/\partial c$ for $1 \leq i,j \leq n$.

We can also show that, where the derivatives exist,

(iv) the $n \times n$ Slutzky matrix, $[\partial f_i(p,c)/\partial p_j + f_j(p,c)\partial f_i(p,c)/\partial c]$, is negative semi-definite.

The preceding restrictions on a consumer's good choices enter in various ways the models of individual behavior that econometricians build. *Some econometricians assume that the pertinent utility functions belong to a certain class of functions and derive explicit expressions for the components of f(·).* Their demand functions will satisfy conditions (i)-(v) by construction. Unfortunately, the known classes of utility functions from which we can derive explicit expressions for f(·) are rather limited. Also, their members yield demand functions that, in addition to satisfying conditions (i)-(v), have properties that seem arbitrary and of little empirical relevance.

E.19 Consider the Klein-Rubin utility function,

$$\log U(x) = \sum_{i=1}^n \beta_i \log(x_i - \gamma_i), \text{ where } x_i > \gamma_i,\ \gamma_i \geq 0,\ \beta_i > 0, \text{ and } \sum_{i=1}^n \beta_i = 1.$$

The corresponding demand function,

$$F_i(p,c) = (\beta_i/\sum_{j=1}^n \beta_j)([c - \sum_{j=1}^n p_j\gamma_j\ ]/p_i) + \gamma_i\ ,\ i = 1,...,n$$

satisfies conditions (i)-(v) above for p > 0 and $c > \sum_{j=1}^n p_j\gamma_j$. In addition, it has a Slutzky matrix whose off-diagonal elements are all positive. The latter property has probably little empirical relevance.

Other econometricians formulate systems of demand functions the estimated versions of which they believe will approximate arbitrarily closely the true demand function. This can be done in several ways. Here is one of them: Consider a twice differentiable, strictly increasing, strictly quasi-concave utility function, U(·), the associated demand function, f(·), and the cost function,

(v) $C(u,p) = \min\{px: x \in R_+^n, \text{ and } U(x) \geq u\}$, $u \in$ range of U(·), and $p \in R_{++}^n$.

We can show that $C(\cdot)$ is (1) continuous and almost everywhere twice differentiable in (u,p), (2) increasing in u, and (3) nondecreasing, homogeneous of degree one, and concave in p. Moreover, if we let $c = C(u,p)$ and solve the equation for u to get u as a function of c and p, $u = U^M(c,p)$, we can show that $U^M(\cdot)$ is the indirect utility function; i.e., that

(vi) $$U^M(c,p) = U(f(c,p)),$$

and that, whenever the pertinent derivatives exist,

(vii) $$w_i = p_i f_i(c,p)/c = \partial \log C(u,p)/\partial \log p_i\ , \ i=1,...,n.$$

(viii) $$w_i = p_i f_i(c,p)/c = - (\partial U^M(c,p)/\partial \log p_i\ )/(\partial U^M(c,p)/\partial \log c),\ i=1,...,n.$$

Now, it is a fact that if a function, $C(\cdot): R_+ \times R_{++}^n \to R_+$, has the properties we ascribed to the cost function above, there is a twice differentiable, strictly increasing and strictly quasi-concave utility function, $U(\cdot): R_+^n \to R_+$, relative to which $C(\cdot)$ is a proper cost function. This and equation vii above suggest an interesting way to generate approximate demand functions: Delineate a family of functions, $\{C^\alpha(\cdot): \alpha \in$ some index set,$\Im\}$, the members of which can be used to approximate a cost function, $C(\cdot)$, in a neighborhood of any given pair, $(u,p) \in$ domain of $C(\cdot)$, and proceed to estimate the pertinent version of

(ix) $$w_i = \partial \log C^\alpha(\cdot)/\partial \log p_i\ , \ i=1,...,n.$$

E.20 A good example of this procedure is Angus Deaton and John Muelbauer's generation of their Almost Ideal Demand System (AIDS) (cf. Deaton and Muelbauer, 1980). They delineate the family of approximating functions by the equations,

(x) $\log C^\alpha(u.p) = u \log b(p) + (1-u) \log a(p),\ u \in [0,1]$, and $p \in R_{++}^n$,

(xi) $\log a(p) = \alpha_0 + \Sigma_{i=1}^n \alpha_i \log p_i + (1/2) \Sigma_{i=1}^n \Sigma_{j=1}^n \gamma_{ij}^* \log p_i \log p_j$, and

(xii) $\log b(p) = \log a(p) + \beta_0 \Pi_{i=1}^n p_i^{\beta i}$.

From these equations they derive the following version of equation (ix):

(xiii) $w_i = \alpha_i + \beta_i \log (c/P) + \Sigma_{j=1}^n \gamma_{ij} \log p_j$, where

(xiv) $\log (P) = \alpha_0 + \Sigma_{i=1}^n \alpha_i \log p_i + (1/2) \Sigma_{i=1}^n \Sigma_{j=1}^n \gamma_{ij} \log p_i \log p_j$

with $\gamma_{ij} = (1/2)(\gamma_{ij}^{*} + \gamma_{ij}^{*})$.

An approximating function need not have all the properties of the function it approximates. The $C^{\alpha}(\cdot)$ of Deaton and Muellbauer satisfy conditions (1) and (2) of a cost function. They also satisfy the homogeneity requirement in (3) if we insist that

(xv) $$\sum_{i=1}^{n} \alpha_i = 1;\ \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} = 0; \text{ and } \sum_{i=1}^{n} \beta_i = 0.$$

However, they need not be everywhere nondecreasing and concave in p. With the restrictions (xvi) below added to equations (xv),

(xvi) $$\sum_{k=1}^{n}\gamma_{ij} = 0,\ i = 1,\dots,n,$$

(xv)

the share equations that Deaton and Muellbauer's $C^{\alpha}(\cdot)$ generate are homogeneous of degree zero and satisfy conditions (i) and (iii) above. However, they need not satisfy condition (ii). Hence their Slutzky matrix need not be negative semi-definite. Also, for large enough c, the estimated versions of the share equation, (xiii), will violate the conditions,

(xvii) $w_i \in [0,1],\ i = 1,\dots,n.$

The preceding observations imply that the share equations in (xiii) cannot be used in a theory-data confrontation to test the theory of consumer choice. That fact has prompted many econometricians to look for alternatives to Deaton and Muelbauer's AIDS. R. Cooper and K. McLaren are two of them (cf. Cooper and McLaren, 1996). They claim that it is not necessary to generate estimable functions that satisfy the conditions of the consumer demand functions everywhere. For example, the demand function of the Klein-Rubin consumer satisfies the required conditions only in the region, $p > 0$ and $c > \sum_{i=1}^{n} p_i\gamma_i$. Also the estimated share equations in (xviii) will satisfy the conditions in (xxii) for all (c,p) in the sample. Such observations suggest to Cooper and McLaren that it suffices to look for families of generating functions that produce functions the estimated versions of which should have the characteristics of demand functions at all pairs, (c,p) in a given sample. The empirical relevance of the theory will then depend on the estimated functions having the required properties.

E.21 One of the families of generating functions that Cooper and McLaren have described is the following family of indirect utility functions,

(xvii) $U^M(c,p) = ([(c/kP1)^{\mu} - 1]/\mu) \bullet (c/P2)^{\eta}$.

Here $0 \leq \eta \leq 1$, $\mu \geq -1$, $k > 0$, and Pj(p), j=1,2, are two price indexes that are continuous, homogeneous of degree one, nondecreasing and concave and satisfy the condition,

(xviii) $Pj(1) = 1$, and $Pj(p) > 0$ for $p \in R_{++}^{n}$, j = 1,2.

The members of this family have all the properties of an indirect utility function in the region, {(c,p): c > kP1(p)}. The demand functions that the members generate satisfy conditions (i)-(v) in the same region. If the theory is correct, then the estimated versions of these demand functions should also have the required properties at all points in the sample.

IV.2 **Rational Expectations**

Consumer expectations play no role in the theory of consumer choice under certainty. As a consequence, expectations have no representation in most econometric models of consumer choice. That is disconcerting in as much as expectations in the theory of consumer choice under uncertainty play havoc with many of the theorems that we have recanted above. For example, both Samuelson's Fundamental Theorem (condition (iv) above) and Houthakker's Strong Axiom of Revealed Preference are invalid. The semi-definiteness of the Slutzky matrix (condition (v) above) is also suspect. The reader can find details concerning these matters in my 1969 article on "General Equilibrium under Uncertainty" and in chapter 30 of Stigum, 1990.

We encounter expectations in econometrics both in the form of an adaptive expectations hypothesis and in the form of a rational expectations hypothesis. Here we shall discuss the latter. In 1961 John Muth suggested that we consider individual decision makers' expectations to be informed predictions of future events that coincide with the predictions of the relevant economic theory. In doing that we could ensure that our theories' descriptions of individual behavior would be consistent with the pertinent decision makers' beliefs about the behavior of the economic system. Muth's idea of expectations formation constitutes the essential ingredient of the *Rational Expectations Hypothesis* (REH) in econometrics.

In the 1970's the REH surfaced in macroeconomic studies of inflation and the natural rate of unemployment. Economists observed that it was in the best interests of workers to be able to predict the rate of inflation as well as possible. To be successful in doing that the workers needed to take into account all available information, including forecasts of changes in monetary and fiscal policies. Such information notwithstanding, individual workers might not make accurate assessment of probable changes in the price level and different individuals were likely to make different forecasts. Still, so economists hypothesized, on the average the workers' predictions would be right. Also the mistakes which the workers as a group made in forecasting the rate of inflation would be random and uncorrelated with the information they possessed. These ideas find expressions in the macroeconomic model I describe in E 3.13 I have learned of the model from Hashem Pesaran who ascribes it to Robert E. Lucas (cf. Pesaran, 1987, pp. 26-29)..

E.22 Suppose that the behavior over time of an economy can be described by the following system of equations:

(1) $y_t^d + p_t = m_t + v$, t= 1,2,...

(2) $y_t^s - ÿ = \alpha(p_t - p_t^*) + \varepsilon_t$, t = 1,2,...

(3) $y_t^s = y_t^d$, t = 1,2,...

Here y^d , y^s , and ÿ are the logarithms , respectively, of the demand (for), supply, and natural level of aggregate output. Also, p and p* are the logarithms, respectively, of the actual price level and the price level that the members of the economy in one period expected to prevail the next period. Finally, m and v are the logarithms, respectively, of the money supply and the velocity of money, t records pertinent periods, and $\{\varepsilon_t$; t=1,2,...} constitutes a purely random process with mean zero and finite variance.

Suppose next that m is an exogenous policy variable and adopt the rational expectations hypothesis that, for each t, p_t* equals the economic system's best prediction of the value of p_t in period t-1. By solving equations (1)-(3) for p_t we find that

(4) $p_t = (\alpha/(1+\alpha))\, p_t^* + (1/(1+\alpha))\, (m_t + v - ÿ - \varepsilon_t)$.

Also. by taking expectations of both sides of (4) conditional on the information set in period t-1, Ω_{t-1} , we find that the best predictor of p_t, $þ_t$, is given by the equation,

(5) $þ_t = (\alpha/(1+\alpha))\, p_t^* + (1/(1+\alpha))\, E\{m_t + v - ÿ - \varepsilon_t | \Omega_{t-1}\}$,

It follows from (5) and the REH that

(6) $p_t^* = E\{m_t + v - ÿ - \varepsilon_t | \Omega_{t-1}\}$.

If we now combine (6) and (4) and take expectation of both sides in (4) with respect to Ω_{t-1}, we can conclude that

(7) $E\{p_t \mid \Omega_{t-1}\} = E\{m_t + v - ÿ - \varepsilon_t | \Omega_{t-1}\}$,

and hence that

(8) $p_t^* = E\{p_t | \Omega_{t-1}\}$.

The last equation shows that the price expectations of the members of the economy are valid on the average. Also, since $E\{p_t - p_t^*| \Omega_{t-1}\} = E\{p_t| \Omega_{t-1}\} - p_t^* = 0$, the equation implies that the prediction error in one period is uncorrelated with the variables in the preceding period's information set.

E.22 gives a good idea of the role the REH plays in macroeconomic models. In reading the example the reader ought to note the severe knowledge requirements that the hypothesis places on the members of the economy. They are supposed to know both the true structural model of the economy and the data generating process (DGP) of endogenous and exogenous variables alike.

In E.22 we did not characterize the DGP of the exogenous variables. One way to accomplish that is to assume that the m_t and the ε_t in equations (1) and (2) satisfy the following set of equations:

(9) $m_t = \rho m_{t-1} + \xi_t$, $t=1,2,...,$

where $\rho \in (0,1)$, and $\{\xi_t : t=1,2,...\}$ is a purely random process with mean zero and finite variance that is distributed independently of the ε_t .

(10) $E\{\varepsilon_t|\Omega_{t-1}\} = E\{\xi_t| \Omega_{t-1}\} = 0$, $t=1,2,...$

When we add equations (9) and (10) to (1)-(3), we can show that

(11) $y_t = \ddot{y} + (1/(1+\alpha))(\alpha\xi_t - \varepsilon_t)$, $t= 1,2,...$, and

(12) $p_t = \rho m_{t-1} + v - \ddot{y} + (1/(1+ \alpha))(\xi_t - \varepsilon_t)$, $t = 1,2,...$.

Thus, if the postulates, (1) - (3) and (9) -(10) are valid, it follows from equation (11) that only unforeseen changes in the money supply affect the equilibrium level of y. Also, equation (12) provides an explicit form for the linear best predictor of the price level.

In all likelihood there are few rational animals with the kind of knowledge that the REH requires in macroeconomics. It is, therefore, interesting that in financial markets and in markets for foreign exchange the arbitrage activities of a small number of knowledgeable traders might suffice to bring about rational expectations equilibria. Supposedly, so economists argue, in their pursuit of profits the knowledgeable traders will push their arbitrage activities to the point where the errors in their forecasts of the rate of return on holding securities or foreign exchange are uncorrelated with the information they possess. The next example illustrates the dynamics of a market for shares in rational expectations equilibrium.

E.23 Consider the shares of some US Company, and let P_t and d_{t+1}, respectively, denote the price of a share at the beginning of period t and the dividends per share that the company pays at the end of period t. Also, let z_{t+1} denote the rate of return on holding a share during period t. Then $z_{t+1} = [d_{t+1} + (P_{t+1} - P_t)]/ P_t$.

Next, let y_{t+1} and λ_t, t = 0,1,… , respectively, be a vector of rates of return on holding a share in each of n different US companies during period t and the vector of the market's best prediction of the values of the components of y_{t+1} at the beginning of period t. Also, let

$$\eta_{t+1} = y_{t+1} - \lambda_t,\ t = 0,1,\ldots ,$$

and assume that the random process, $\{\eta_t: t=1,2\ldots\}$ is an orthogonal wide-sense stationary process that satisfies the conditions,

$$E\{\eta_{t+1}|y_t,\ldots,y_0\} = 0,\ t = 0,1,\ldots ,\text{ and } E\{\eta_t| y_t,\ldots,y_0\} = \eta_t,\ t = 1,2,\ldots .$$

Finally, let φ be an n×n matrix whose *eigenvalues* have absolute value less than 1; let y_{t+1}^e denote investors' expectations of the value of y_{t+1} at the beginning of period t; and assume that

$$y_{t+1}^e = y_t + \varphi(y_t - y_t^e),\ t =0,1,\ldots.$$

If we now let I be n×n identity matrix and insist that $\lambda_0 = (I+\varphi)y_0$, we can invoke the REH,

$$\lambda_t = y_{t+1}^e,\ t = 0,1,2,\ldots ,$$

and deduce from the preceding equations that

$$y_{t+1} = \Sigma_{0\leq s\leq t} (-\varphi)^s(I+\varphi)y_{t-s} + \eta_{t+1},\ t = 0,1,2,\ldots,\text{ and}$$

$$y_{t+1} = y_t + \eta_{t+1} + \varphi\eta_t,\ t = 0,1,2,\ldots ..$$

We can also show that the components of y_t are cointegrated if the rows of φ are linearly dependent.

V *Rational Choice in Econometrics*

Econometricians make choices in all sorts of situations. In some of them they choose among possible research projects. In others they single out good ways to solve the analytical problems that come with the chosen projects. In still others they decide on how to present their results and where to publish them. We shall see what are the characteristics of rational choice in these situations.

V.1 **Research Projects**

A project is a plan or a proposal. In econometrics a research project is an undertaken that involves a concerted effort to solve a theoretical problem or to carry out an empirical analysis. The theoretical problem may be a problem in mathematical statistics or a problem in economic theory. Examples are the asymptotic properties of a test statistic and the salient properties of an aggregate production function. The empirical analysis may consist in establishing the empirical relevance of a given theory, providing economic forecasts for government policy makers, or giving a scientific explanation of regularities that certain data display. There are many other possibilities.

In choosing among research projects an econometrician must take into account his own technical expertise and the ideas about which he would like to learn. He must also consider the availability of data and funding, and the possible use of collaborators. Finally, he must take into account various strategic aspects that concern his career; e.g., the general interest of each project, the required time for their completion, and his tenure situation. Certainly, it makes little sense for him to choose a research project that involves technical expertise beyond his present capacity unless he is interested in acquiring the required extra knowledge or can count on the collaboration of an expert. Similarly, it makes little sense for him to choose a research project that requires data collection and long time to complete unless his tenure position and salary are not affected by the required completion time.

At a given point in time, therefore, what constitutes a rational choice among research projects, depends on the pertinent econometrician's research interests, his knowledge, and his awareness of possible research projects. It also depends on the environment in which he works and on the financial resources of his academic community. Finally, it depends on the place in the career ladder in which he operates. A Ph.D student may find it advantageous to work on problems that interest his thesis adviser since that may provide him easy access to funding and expert advice. Ease of funding and the research interests of colleagues may also be the deciding factors in a young econometrician's choice of research activities. For a more established researcher the search for knowledge and the desire to help solve his country's pressing economic problems may be the overriding reasons for his selection of research topics.

Choosing a research project is a multifarious process. Thomas Kuhn envisions it as a choice between normal-science puzzles that might throw light on relevant aspects of some given scientific paradigm (Kuhn, 1970). Imre Lakatos thinks of it as a choice between positive heuristics of a research program that needs development (Lakatos, 1978). Finally, Karin Knorr-Cetina insists that it is part of a social process of negotiation that is situated in time and space and should not be analyzed with the logic of individual decision making (Knorr-Cetina, 1981, p. 152). It is quite clear that many of the theoretical problems that econometricians tackle can be viewed as normal-science puzzles. It is also evident that it is possible to think of an econometrician's empirical work as providing the right of existence for positive heuristics of a pertinent research program. However, I do not believe that Knorr-Cetina's view of the products of science can be used to describe an econometrician's choice among research projects. For the average econometrician his own evaluations are as important as the social process of negotiation in which he at times may find himself.

I have no case study other than my own on which to base my opinion about the way that econometricians choose their research projects. Except for details and a good bit of luck, my case study is probably like the case study of any representative US econometrician. It will, therefore, serve to illustrate the ideas that I have tried to express above.

E.24 As a Ph.D. student I decided on my own the thesis topics on which I wanted to write, and I chose my thesis advisor accordingly. My thesis was accepted in 1962, and after further studies, its three parts appeared as journal articles in 1964, 1967, and 1969.

In my first real job as an assistant professor of economics I found myself less than a hundred yards from an outstanding math department with an extraordinary group of probabilists and mathematical statisticians. They invited me to participate in a seminar in which we were to read a newly published book on a certain family of random processes. The seminar discussions were very inspiring and, for me, resulted in four published papers. The first I wrote alone. The other three were joint work with one of the other seminar participants.

In 1968 I accepted a job as an associate professor of economics at a new university. There I decided that it was time to write an applied econometrics paper. A few years back, two distinguished professors had published interesting studies concerning risk aversion and choice of safe and risky assets. Also, researchers at a government institution had collected data that, on the surface of things, looked suitable for a test of the professors' ideas. So I reformulated the theory on choice of safe and risky assets for the empirical analysis, received copies of the given data, found a willing expert on computers at a neighboring University to help me out with the data analysis, and was all set to start the test. Then I discovered an aggregation problem in the theory that had to be solved before I could carry out the empirical analysis. The solution to the aggregation problem I found first ten years later with the help of results that colleagues at other universities published in 1974 and 1976.

In the latter part of the seventies I moved as a full professor of economics to a third university. There my salary was determined independently of my publications. That gave me a chance to devote most of my free time to writing a monograph on methodology that I thought my profession needed badly. In the process I also managed to finish the empirical analysis of risk aversion and the choice of safe and risky assets. The reults form two long chapters in my methodology book. The latter a distinguished university press was kind enough to publish in 1990.

To all this I should add that when the opportunity presented itself in 1968, I chose to move from one economics department to another that had more people working on problems of interest to me. The new department was a very inspiring and congenial place in which to work. So, while waiting for a solution to my aggregation problem, I published roughly two papers a year. I resigned in 1978 for family reasons alone not knowing that *Frau Fortuna* was still looking out for me. My third university had a math department with an excellent group of logicians and two extraordinarily helpful computer experts. Without them I might still be working on my monograph.

Over time the choices of research projects that the members of an academic community make have an interesting dynamics of its own. In seminars and conferences, econometricians present their results, exchange ideas about each others research projects, and share information about theoretical and applied results in related areas. In that way the current results of some may provide inspiration and important inputs for future studies of others. Also, the cumulative efforts of many may provide knowledge for government agencies and central statistical offices that will lead to better ways of collecting statistical information and to the collection of new and interesting data. Finally, the shared information about theoretical and applied developments in related areas may suggest new and exciting research opportunities for the seminars and conference participants.

Three examples of the dynamic process that I have in mind are the elusive Phillip's curve, the current estimates of vintage price functions and depreciation profiles for the US National Income and Product Accounts (NIPA), and the many arch- and garch models of financial markets. The first originated in E.B. Phillips' study of "Inflation and Unemployment in Great Britain during the period 1881-1957" (Phillips, 1958). It resulted in decades of concerted efforts by econometicians to establish a stable relationship between the rate of inflation and the rate of unemployment in economically developed countries. The second can be seen as the result of four decades of joint work by Dale Jorgenson and many others to provide the US NIPA with internally consistent measures of capital stocks and depreciation profiles (Fraumeni, 1996). The third was the result of Rob Engle's discovery that characteristics of the conditional variance of a wide-sense stationary process have an interesting bearing on the efficient market hypothesis (Engle, 1982).

For all I know, it might be a good way to view an econometrician's choice of research project as the choice of a puzzle that needs a solution. Then we could think of E.B. Phillips' paper as a search for a deep parameter in Keynesian macroeconomics. Similarly, we could consider Dale Jorgenson's et al's work as a joint venture to determine how best to measure the depreciation of various kinds of capital assets. However, I believe that such a view of these research activities is missing a very important point: Good econometric work has three sides, an economic side, a statistical side, and an applied side. These sides interact in interesting ways in the dynamics of research choice. Looking back at the development of econometrics during the last fifty years is like watching a beautiful mountainside full of crisscrossing paths that aim for the top. If we were to take a hike along one of the paths that emanate from E.B. Phillips' paper, we might meander through various applied papers, discover papers by Ed Phelps and Milton Friedman on the natural rate of unemployment, run across Robert Lucas' introduction of the REH in macro economics, and wade through papers on the statistical problems of analyzing REH models that Hashem Pesaran describes so well in his book on **The Limits of the Rational Expectations Hypothesis**. At some point the path would come to a halt in waiting for another econometrician's choice of research project to carve out the next few meters. If we should come back to this point next year, we might find that the choice has resulted in a discourse on the dire effects of multiple equilibria in the labor market, the discovery of a new deep parameter in macro economics, or, simply, a novel way of applying the GMM estimation method.

V.2 **Model Selection**

Each research project comes with problems that require solutions. Solving them involves choices of a varied kind. Here the problems that an econometrician encounters in selecting models for his empirical work are of special interest. We shall, next, discuss characteristics of rational choice in solving model selection problems.

Model selection in econometrics is not an easy task. A long time ago, Edward Leamer observed that the data banks of the National Bureau of Economic Research (NBER) contained time-series data on two thousand macroeconomic variables. With these data, a given econometrician who restricts himself to exactly five explanatory variables, can estimate 2.65×10^{14} different linear equations (Leamer, 1983). Using one second on each equation, it would take him more than thirty-one million years to estimate all of them. The econometrician does not have that many years. So, if he is to use the NBER data, he must select a subset of the available variables and search for a useful linear or nonlinear relation among them. Model selection in econometrics is about how best to choose variables for an empirical analysis and about how to search for the linear or nonlinear relations among them that will best serve the purposes of the analysis.

V.2.1 **Bridge Principles**

To me an empirical econometric analysis is an instant of an economic theory-data confrontation. So, in our discussion of model selection we shall take for granted that the pertinent econometrician has a theory that he intends to confront with data. We shall also assume that the data he will use belong to a data set that he or somebody else has collected. Finally, we shall assume that he has already chosen a data set for his empirical analysis. Then, the first problem our econometrician must solve is how best to relate the variables in his theory to variables in his data universe; i.e., to variables and constants whose values he has observed and to function- and predicate variables that he has created with them.

Relating theoretical variables to variables in a data universe in a meaningful way is a tricky matter that requires both a good grasp of economic theory and knowledge of related empirical work. For example, in studying consumer choice we must decide on the reference of 'consumer' and find good ways to measure some of his or her characteristics. We must also account for the fact that the theory is about choice of single commodities and assets while our data usually refer to aggregates of commodities and assets. Finally, we must search for reasonable measures of prices of aggregates and decide whether to to treat variables like permanent income as unobservables or look for good proxies. Whatever choices we make, we must provide justification for them with reference to theory and related empirical work. We must also account for them in explicitly formulated bridge principles that relate the theoretical variables to variables in the data universe. Without the bridge principles the results of the empirical analysis will be hard to interpret.

E.25 How best to relate variables in a theory universe to aggregates in a data universe is a problem that arises in many situations in econometrics. The role that economic theory might play in the search for solutions to such problems is interesting. Here are three examples that illustrate what I have in mind.

The theory of consumer choice under certainty concerns choice among single commodities. What the theory says about demand for single commodities need not be true of demand for composite commodities. It is true, according to the Hicks-Leontief aggregation theorem, if the prices of the components of each composite always vary proportionately among themselves. (Stigum, 1990, ch. 10) contains proof of this theorem.

Kenneth Arrow's interesting theorems concerning consumer choice among one safe and one risky asset need not be valid for choice among one safe asset and one aggregate of risky assets. They are valid if the consumer's utility function belongs to one of two classes of such functions that D. Cass and J. Stiglitz delineated in 1970. For a discussion and proof of this fact cf. (Stigum, 1990, ch. 12).

In the standard theory of entrepreneurial choice the firm produces one output with the help of capital, labor, and an intermediate product. Usually, the theory faces data on firms that produce several different products with the help of various intermediate products and many different kinds of capital and labor. It is, therefore, relevant that a firm's production function must satisfy stringent requirements in order that we be able to write its output as an aggregate whose production is a function of a capital aggregate, a labor aggregate, and total expenditures on intermediate products. Some of these requirements are detailed in (Fisher, 1968) and (Stigum, 1967b and 1968).

The theoretical variables and the variables in the data universe are jointly distributed random variables. Hence, the econometrician can specify properties of the probability distribution of the theoretical variables and use them and his bridge principles to derive salient characteristics of the process that generated his data - the DGP. If he proceeds in that way, he can test the empirical relevance of his theory by checking whether the theory, when translated by his bridge principles, makes statistically valid predictions about characteristics of the data.

Our econometrician can also delineate properties of the DGP from scratch and, then, use them and his bridge principles to deduce salient characteristics of the probability distribution of the theoretical variables - the TGP. If he proceeds in that way, he can test the empirical relevance of his theory by checking whether the properties of the TGP are in accord with the prescriptions of his theory.

It must not go unnoticed here that in the first case all tests concerning the empirical relevance of the theory are carried out in the data universe. In the second case all tests of the empirical relevance of the theory are carried out in the theory universe - the universe of the theoretical variables. Whether the econometrician has a choice as to in which universe to carry out his tests, depends on the way he chose to formulate his bridge principles. Since this is an important fact, I shall give several simple examples below to illustrate what I have in mind.

E.26 Consider the five-tuple of real-valued random variables, (y,u,c,x,z), and assume that the first three components reside in the theory universe and that the last two roam around in the data universe. Suppose also that, according to theory, TH is valid.

TH: There is a finite, positive constant, k, such that c = ky.

Finally, assume that the variables in the two universes are related as follows:

B1: y + u = x, and B2: c = z.

We shall consider three different cases for the analysis: I, II, and III. In each case we assume, without say, that a deamon has produced N independent draws of the values of y,u,c,x, and z, and that he has only revealed to us the N values of the pair, (x,z).

I. I. In this case we impose conditions on the probability distribution of the theoretical variables and use these conditions and the bridge principles, B1 and B2, to deduce restrictions on the probability distribution of the pair, (x,z). Specifically, we assume that A is valid.

A. The means and variances of y, u, and c, are finite and satisfy the conditions, Ey >0, Eu = 0, Ec > 0, $\rho_{yu} = 0$, $\sigma_y^2 > 0$, $\sigma_u^2 > 0$, and $\sigma_c^2 > 0$.

From these conditions and the bridge principles we can deduce the validity of TI 1.

TI 1: If A, B1, and B2 are valid, then the means and variances of x and z are finite and positive.

Those are the only restrictions on the distribution of x and z that we must heed when we search for a distribution, CGDI, of x and z that, in David Hendry's terminology, generates a congruent model of our data.

The theory that we formulated in equation TH and the bridge principles impose further restrictions on the distribution of x and z that the CGDI might not satisfy. Here they are:

TI 2: If TH, A, B1, and B2 are valid, then it must be the case that z/k = y, x - z/k = u, Ez/Ex = k, $\sigma_z^2/k^2 = \sigma_y^2$ and $\sigma_x^2 - \sigma_z^2/k = \sigma_u^2$.

The interesting part of TI 2 is that the conclusion holds only if the covariance of x and z satisfies equation HI.

HI. $\rho_{xz} = \sigma_z^2/k^2$.

Checking whether HI with k = Ez/Ex is valid, provides us with ***a test of TH and A in the data universe.***

II.

II. In this case we start by deriving the distribution of x and z that generates the pertinent gongruent model of our data without making any assumptions about the distribution of y, u, and c. With this distribution and the bridge principles on hand we can establish the following theorem:

TII.1: Let CGDII be the distribution of (x,z) that in case II generates the pertinent congruent model of our data, and assume that B1 and B2 are the bridge principles that relate the components of (y,u,c) to (x,z). Then the following equations must be valid: $Ey + Eu = Ex$, $Ec = Ez$, $\sigma_y^2 + 2\rho_{yu} + \sigma_u^2 = \sigma_x^2$, and $\sigma_c^2 = \sigma_z^2$.

TII.1 and TII.2 below demonstrate that we cannot construct ***a test of TH in the theory universe*** without making appropriate assumptions about the distribution of (y,u,c).

TII.2: If TH, B1, and B2 are valid, and if (x.z) is distributed in accordance with CGDII, then the following equations must hold: $y = z/k$, $u = x - z/k$, $Ey = Ez/k$, $Eu = Ex - Ez/k$, $\sigma_y^2 = \sigma_z^2/k^2$, $\rho_{yu} = \rho_{xz}/k - \sigma_z^2/k^2$, and $\sigma_u^2 = \sigma_x^2 - 2\rho_{xz}/k + \sigma_z^2/k^2$.

III. In this case we proceed the way we did in Case II, and we let CGDII be as we described it there. To obtain a test of TH in the theory universe we must add to TH a condition on the distribution of (y.u.c). The required condition is stated in TIII.1.

TIII.1: Suppose that TH, B1, and B2, are valid. Suppose also that $Eu = 0$ and $\rho_{yu} = 0$. If (x,z) is distributed in accordance with CGDII, then the conclusion of TII.2 is valid. In addition, it must be the case that H* is valid as well.

H*: $k\sigma_y^2 = \rho_{xz}$ with $k = Ez/Ex$.

Then H* provides us with ***a test of TH, Eu =0, and*** $r_{yu} = 0$ ***in the theory universe.***

There are two aspects of the preceding example that are special. First of all, in analyzing the three cases we implicitly assume that we are engaged in an analysis of N independently and identically distributed observations on the pair, (x,z). That allows us to coach the discussion in terms of properties of the distribution of (y,u,c,x,z) instead of the distribution of the demon's N independent draws of the given five-tuple of random variables. Except for the simplicity in presentation, the ideas that the example illustrates generalize to analyses of panel data and time-series data as well.

Secondly, the bridge principles involve all the components of (y,u,c,x,z). That is not generally the case. In a theory-data confrontation often only a subset of the theory's variables are involved. Who they are depends both on the data and on the kind of confrontation the econometrician has in mind. Also the data we use are often constructs that we have created with the help of many auxiliary variables. Usually, only a few of the latter play an independent role in the theory-data confrontation. Those that do not and the left-out theoretical variables are not related to each other and to other variables in the bridge principles.

V.2.2 **Data Analyses**

Choosing variables for the data universe in a prospective theory-data confrontation is difficult. It is, therefore, important to be aware that the choice might have fundamental consequences for the relevance of the empirical results. Left-out variables can confound causal relationships and lead to misrepresentations of dynamic characteristics. To see how, just envision yourself studying the effect of a retraining program for unemployed workers on their employment possibilities. You have observations from two disjoint groups of individuals one of which has been exposed to the retraining program. To avoid confounding causal effects, you must find a way of rendering the groups observationally equivalent. That requires having observations on a number of the salient characteristics of each group. To avoid misrepresenting dynamic characteristics of the labor market, you must also find appropriate numbers of lags for the variables that end up being included in your analysis.

The idea of confounding causal relationships is related to the idea of noncollapsibility in regression analysis. Suppose that the theory in the theory-data confrontation insists on a causal relation between two variables, x and z, in which x is the dependent variable and z is the independent variable. Suppose also that we have independently distributed observations in the data universe on x and z, and on the variable, v. Finally, suppose that that the three data variables have finite means and variances, and that there exists a function, $G(\cdot): R \rightarrow R$, such that

$$G(E\{x|z\}) = \alpha + \beta z, \text{ and } G(E\{x|z,v\}) = a + bz + cv.$$

Then the regression of x on z,v is *collapsible* for b over v if $b = \beta$ (cf. Greenland et al, 1999, p. 38). If it is, we may ignore v in our search for the causal relationship between x and z.

It is not the place here to discuss sufficient conditions for collapsibility. However the relationship between collapsibility and rationality in model selection is relevant. For that purpose observe that, in the case considered above, there exist vectors of constants, (a_1,b_1) and (a_2,b_2,c_2), and variables, u_1 and u_2 , with finite means and variances that satisfy the conditions:

$$x = a_1 + b_1 z + u_1,\ E\{u_1\} = 0, \text{ and } E\{zu_1\} = 0;$$

$$x = a_2 + b_2 z + c_2 v + u_2,\ E\{u_2\} = 0,\ E\{zu_2\} = E\{vu_2\} = 0.$$

With our data the least squares estimates of the constants in these equations are consistent, but might be biased. They will be biased in the first (second) equation if $E\{u_1|z\}$ ($E\{u_2|z,v\}$) varies with z ((z,v)). A heteroscedastic u_1 (u_2) indicates that $E\{x|z\}$ is nonlinear. Therefore, many econometricians will insist that we look for a linear or nonlinear relationship between x and z and other variables in the data universe in which the pertinent conditional mean of the error term does not vary with the values of the independent variables. E.27 gives an example of a successful search for such a relation. The example is a freely recanted case study based on pp. 480-481 in (Anderson and Vahid, 1997).

E.27 A and V have James Tobin's data on food consumption, household income and household size in the US in 1941. To determine the causal relationship between these variables they begin by estimating a linear equation. The result is as follows:

$$\log FOODCON_i = \underset{(0.1)}{0.82} + \underset{(0.03)}{0.56} \log HINC_i + \underset{(0.04)}{0.25} \log AHSIZE_i + u_i,\ i \in N$$

Diagnostic specification tests show strong evidence of heteroscedasticity and nonnormality in the residuals. Also an LM test suggests that the variance of the residuals varies with the size of the sample's (income/houshold size) groups. This indicates that a weighted regression might be appropriate. Running the pertinent weighted regression, they obtain

$$\log FOODCON_i = \underset{(0.07)}{0.73} + \underset{(0.02)}{0.59} \log HINC_i + \underset{(0.03)}{0.23} \log AHSIZE_i + u_i .\ i \in N.$$

Now tests for heteroscedasticity and non-normality in the residuals fail to find evidence of misspecification. However, an LM test for omitted nonlinearity indicates that the log-linear specification they are using is inappropriate. A and V end up choosing the following (weighted) regression as a model of the causal relation between the three variables:

$$\text{Log } FOODCON_i = 0.91 + 0.54 \log HINC_i - 0.43 \log AHSIZE_i +$$

$$\underset{(0.11)}{} \; \underset{(0.04)}{} \qquad \underset{(0.22)}{}$$

$$\underset{(0.08)}{0.17}\ (\log AHSIZEE_i)(\log HINC_i) + \underset{(0.09)}{0.14}\ (\log AHSIZE_i)^2 + u_i\ .\ i \in N.$$

Diagnostic tests based on this specification have failed to find any evidence of misspecification.

The preceding comments and examples illustrate some of the issues with which an econometrician must cope in crossection analyses (CRA). The same issues arise in time-series analyses (TIA) as well, but there they seem to be much more involved. In CRA the values of the variables we observe pertain to a given period of time. In TIA we have observations on the values that a set of variables assume in many different periods of time. In CRA we divide the variables into dependent and independent variables, and we insist that in the equations we estimate the error terms be stochastically orthogonal to the independent variables and homoscedastic. In TIA we also have dependent and independent variables and we insist that the error terms in the equations we estimate be orthogonal to the independent variables and homoscedastic. In addition, we insist that the error terms vary over time according to the laws of a purely random process, and that the parameters in each equation be independent of time. Finally, when we use the TIA model for prediction, we insist that the past values of the dependent variables do not affect the current values of the independent variables.

In searching for a final model that satisfies the strictures that I listed above (STR), different econometricians will proceed in different ways. For example, a Baysian like Edward Leamer will envision a family of alternative models and choose among them according their respective posterior odds (cf. Leamer, 1983, pp. 288-291). A non-Baysian like David Hendry will start by analyzing the empirical relevance of the most general, estimable, statistical model that he believes can be postulated initially - that is his GUM (cf, Hendry, 1995, p. 361). In his analysis, Hendry will use standard testing procedures to weed out statistically insignificant variables and to check whether the reduced equations satisfy the pertinent STR. The weeding out process may lead to different terminal model specifications. If it does, Hendry will choose as a final model one whose characteristics cannot be explained by any of the other terminal models.

In my view there are three aspects of Hendry's method that are relevant here. One aspect is interesting because it lays down a rule for rational choice in model selection. Specifically, Hendry insists that the GUM contains a vector of parameters, μ, whose values the empirical analysis is to determine. A proposed step in the reduction process is allowed only if it does not lead to a loss of information concerning the values of the components of μ.

A second aspect is interesting because it seems to exhibit an important lacuna in Hendry's method. In spite of Hendry's insistence that he takes theory into account in his statistical analyses, I have difficulties seeing where hypotheses such as the TH in E.26 play a role in Hendry's reduction process. To me the natural place in Hendry's analysis to introduce hypotheses like TH is at the end. Hendry's final model provides a 'best possible representation' of the DGP, and it is such models with which we ought to confront our theoretical hypotheses. That brings me to a third aspect aspect of Hendry's method that is particularly intriguing to me. The reduction process may have many terminal models. Among them there may be many that cannot be encompassed by the other terminal models. That suggests to me that we combine data confrontation of theoretical hypotheses and choice of terminal model in a decision-theoretic analysis of the terminal models. That can be done in different ways and I shall describe one of them below.[4]

V.2.3 **Statistical Decision Theory and Time-Series Analysis**

In (Stigum, 1967a) I developed a decision-theoretic approach to time-series analysis that can be applied in the left-out analysis of Hendry's terminal models. This is so even though my article concerns univariate time-series models only. With some obvious modifications my theoretical arguments can be applied in multiple time-series analyses. In fact, with the appropriate modifications my decision-theoretic time-series analysis will apply to all the cases we considered above since an econometrician in characterizing a pertinent DGP must describe the bahavior of independent as well as dependent variables. Unfortunately, the formalism in my approach is involved and, for the uninitiated, the theorems are not easy to comprehend. Therefore, I shall here only sketch the ideas that I tried to convey in my paper.

In statistical decision theory the econometrician is a gambler playing a game against nature, and the strategies of nature are identified with the unknown statistical parameters in the relevant problem. To describe nature's strategies in time-series analysis I need the following definition.

D.1 Let $(\Omega, \aleph, \wp)$ be a probability space. On this space a stochastic process is a family of random variables, $\{S^t x(\omega), t \in T\}$, where $x(\cdot)$ is measurable with respect to $\aleph$, and S is a single-valued shift operator that takes $\aleph$ into itself and preserves complementations and countable intersections of events. Also T is an index set. If $T = \{\ldots,-1,0,1,\ldots\}$, then S is assumed to have a single-valued inverse.

A time series is a partial realization of a stochastic process. Hence most statistical analyses of such series are conducted for the purpose of obtaining information concerning the structural properties of the underlying process. That involves studying characteristics of the shift operator, S, and a parameter, θ, that in some sense determines the properties of the probability measure $\wp(\cdot)$ on $(\Omega, \aleph)$. Consequently, we can adopt D2.

D.2 Let Y be a set of single-valued shift operators on $(\Omega, \aleph)$ and let $\{\wp_\theta : \theta \in \Phi\}$ be a family of probability measures on the same space. The set of pure strategies of nature is a set $\Im$ of the form $\Im = Y \times \Phi$.

This sounds much more remote than it is. So here is an example to fix our ideas.

E.28 Consider a stochastic process, $\{S^t x(\omega): t \in T\}$, and let $x(t,\omega) = S^t x(\omega)$, $t \in T$. Also, suppose that there exist constants, c_j, $j = 1,\ldots,m$, and a random process, $\{S_\eta{}^t \eta(\omega): t \in T\}$ such that, for all $\omega \in \Omega$ and all $t \in T$, $x(t,\omega) = \sum_{1 \le j \le m} c_j x(t-j,\omega) + \eta(t,\omega)$, where $\eta(t,\omega) = S_\eta{}^t \eta(\omega)$, $t \in T$. Also suppose that the $\eta(t,\cdot)$ constitute a normally distributed, purely random process that is orthogonal to the $x(t,\cdot)$ and has mean μ and variance σ^2. Then Y can be taken to be a compact subset of R^m and Φ can be chosen to be a compact subset of $R \times R_{++}$.

We shall only consider fixed-sample-size games. Thus a time series will be a vector $r \in R^{n+1}$, where $r = (r_t,\ldots,r_{t-n})$ for some fixed n and $t \in T$. The probability distribution of r is represented by a function, $q(\cdot|\cdot): R^{n+1} \times Y \times \Phi \to R_+$, where for each pair $(S,\theta) \in Y \times \Phi$, $q(\cdot|S,\theta)$ is a continuous density function on R^{n+1}, and for each $r \in R^{n+1}$, $q(r|\cdot)$ is a measurable function on $Y \times \Phi$. The measurable sets in $Y \times \Phi$ are determined by a metric whose definition varies with the problem we are studying.

We shall denote the set of policies of our econometrician by A. A randomized strategy on A is a function, $\varphi(\cdot|\cdot): \Re \times R^{n+1} \to [0,1]$, where $\Re$ is a σ-field of subsets of A. For each $r \in R^{n+1}$, $\varphi(\cdot|r)$ is a probability measure on $\Re$, and for each $\beta\in\Re$, $\varphi(\beta|\cdot)$ is measurable with respect to the Borel subsets of R^{n+1}. The set of randomized strategies I shall denote by Ψ.

I assume that our econometrician will choose a strategy that minimizes his expected risk. To characterize such strategies we much first describe the econometrician's risk function. For that purpose I let $L(\cdot):Y\times\Phi\times\Psi\to R$ be his loss function and I insist that, for each choice of strategy by nature, (S,θ), our econometrician's expected risk equals the value of the function,

$$\rho(S,\theta,\varphi) = I_{Rn+1}I_A\ L(S,\theta,a)d\varphi(a|r)q(r|S,\theta)dr.$$

A given $\varphi\in\Psi$ is *inadmissible* if and only if there is a $\varphi^*\in\Psi$ such that , for all $(S,\theta)\in Y\times\Phi$, $\rho(S,\theta,\varphi^*) \leq \rho(S,\theta,\varphi)$ with inequality for some pair (S,θ). Otherwise φ is *admissible*. For a given prior, $\xi(\cdot)$, on the subsets of $Y\times\Phi$, φ^* is a Bayes strategy if $I_{Y\times\Phi}\ \rho(S,\theta,\varphi^*)d\xi(S,\theta) \leq I_{Y\times\Phi}\ \rho(S,\theta,\varphi)d\xi(S,\theta)$ for all $\varphi\in\Psi$. Finally, a pair, (ξ^*,φ^*), constitutes a pair of 'Good' strategies for nature and the statistician if it satisfies the conditions, $I_{Y\times\Phi}\rho(S,\theta,\varphi^*)d\xi(S,\theta) \leq I_{Y\times\Phi}\rho(S,\theta,\varphi^*)d\xi^*(S,\theta) \leq I_{Y\times\Phi}\rho(S,\theta,\varphi)d\xi^*(S,\theta)$, for all $(Y,\theta,\varphi)\in Y\times\Phi\times\Psi$.

With that much said about time series and strategies I can state the first theorem. In the statement of the theorem, a class Δ of randomized strategies is said to be complete if and only if, for all $\varphi\notin\Delta,\exists\varphi^*\in\Delta$ such that, for all $(S,\theta)\in Y\times\Phi$, $\rho(S,\theta,\varphi^*)\leq \rho(S,\theta,\varphi)$ with inequality for some (S,θ).

SDT.1 Let $Y\times\Phi$ and A be the sets of actions open, respectively, to nature and the econometrician, and suppose that, for each pair, $(S,\theta)\in Y\times\Phi$, $q(\cdot|S,\theta)$ is a continuous density function on R^{n+1}. Finally, let $L(\cdot)$ be the loss function on $Y\times\Phi\times A$ and suppose that $Y\times\Phi$ and A are, respectfully, compact in the metrics,

1. $d((S_n,\theta_n)-(S_m,\theta_m)) = \sup_{a\in A} |L(S_n,\theta_n,a)-L(S_m,\theta_m)| +$
   $I_{Rn+1}e^{-|r|}\ |q(r|S_n,\theta_n)-q(r|S_m,\theta_m)|dr$, and
2. $d(a_n,a_m) = \sup_{(S,\theta)\in Y\times\Phi} |L(S,\theta,a_n)-L(S,\theta,a_m)|$.

Then both nature and the econometrician have good strategies, and the classes of admissible and Bayes strategies are complete.

Here is an example to show that the model in SDT.1 specializes to the standard statistical model of testing a simple null-hypothesis against a simple alternative hypothesis. The interested reader can find the missing arguments in the example on p. 220 in my paper.

E.29 Suppose that Y and A consist of two elements each, (S_1,S_2) and (a_1,a_2), and that θ is known. Also, suppose that $L(S_1,a_1) = L(S_2,a_2) = 0$, $L(S_1,a_2) = b$, and $L(S_2,a_1) = 1$, and that $q(\cdot|S_i)$, i=1,2, is a continuous density function on R^{n+1}. Then Y, A, and $q(\cdot|\cdot)$ satisfy the conditions of SDT1. Consequently, both nature and the econometrician have good strategies and the classes of admissible and Bayes strategies are complete.

The class of Bayes strategies is easy to characterize in this case. Let $(\alpha,1-\alpha)$ be a prior distribution on Y. The corresponding family of Bayes strategies, $\{\varphi_\alpha^\lambda(\cdot|r)\}0\leq\lambda\leq1$, are completely characterized a.e. with respect to $q(\cdot|S_i)$, i=1,2, by the rule

$$\varphi_\alpha^\lambda(a_1|r) = \begin{cases} 1 \text{ whenever } q(r|S_2)/q(r|S_1) < b\alpha/(1-\alpha) \\ \lambda \text{ whenever } q(r|S_2)/q(r|S_1) = b\alpha/(1-\alpha) \\ 0 \text{ otherwise} \end{cases}$$

The class of Bayes strategies is obtained by letting α vary over [0,1].

To obtain the class of admissible strategies we note that in the present case any admissible strategy must be Bayes against some prior measure on Y. Also, a Bayes strategy against a pair, $(\alpha,1-\alpha)$, with $\alpha > 0$ is admissible. Finally, when $\alpha = 0$ $(\alpha = 1)$,$\varphi_0^\lambda(a_1|r)$ $(\varphi_1^\lambda(a_1|r))$ is an admissible strategy. This description determines the class of admissible strategies a.e. with respect to $q(\cdot|S_i)$, i=1,2.

At last the econometrician's good strategies. Since $\rho(S_i,\cdot)$ is a continuous funtion of φ on Ψ, for each i=1,2, a good strategy, $\varphi^*(\cdot|\cdot)$, must satisfy the functional equation, $\rho(S_1, \varphi^*) = \rho(S_2,\varphi^*)$. In particular, if φ^* is non-randomized, if $C = \{r:\varphi^*(a_1|r) =1\}$, and if for i=1,2, $Q(B|S_i) = I_B\, q(r|S_i)dr$, then C must satisfy the equation, $bQ(C^c|S_1) = Q(C|S_2)$.

The next theorem concerns the problem of finding a good predictor of $x(t+1,\omega)$, given that r is the observed value of $(x(t,\omega),\ldots,x(t-n,\omega))$. For simplicity, I shall assume that θ is known.

SDT.2 Let $\lambda(\cdot):R\rightarrow R_+$ be continuous and such that $\lambda(x) = \lambda(|x|)$ and such that $\lim_{x\rightarrow\infty} [x^2/\lambda(x)] = 0$. Also, let $g(\cdot|\cdot):R\times R^{n+1}\times Y\rightarrow R_+$ be such that $g(x|\cdot)$ is measurable with respect to the pertinent family of subsets of $R^{n+1}\times Y$ and $g(\cdot|r,S)$ is continuous on R, and suppose that $g(\cdot|r,S)$ is the conditional density function of x(t+1) given that $(x(t),\ldots,x(t-n)) = r$. Finally, suppose that, for each $S\in Y$, the density function of r, $q(\cdot|S)$, is continuous on R^{n+1}, and that Y is separable in the metric

$$d(S_n,S_m) = I_{R^{n+1}}I_R\, e^{-|(y,r)|}|g(y|r,S_n)q(r,S_n)-g(y|r,S_m)q(r|S_m)|dydr,$$

And let the risk function of the econometrician be defined by

$$\rho(S,t) = I_{R^{n+1}}I_R\, [|x-t(r)|^2/(1+\lambda(x))]\, q(r|S)g(x|r,S)dxdr.$$

Then the econometrician has a good strategy, $t^*(r)$, and the class of admissible strategies is complete.

The assumptions of SDT.2 were not strong enough to establish the existence of a good strategy for nature and the completeness of the class of Bayes strategies. However, we can obtain a characterization of all Bayes strategies as follows. Let $d\xi(\cdot)$ be an a priori probability measure on Y; let $dq_\xi(x|r) = I_Y g(x|r,S)q(r|S)d\xi(S)dx \,/\, I_R I_Y g(x|r,S)q(r|S)d\xi(S)dx$; and let $q_\xi^*(r) = I_R I_Y g(x|r,S)q(r|S)d\xi(S)dx$. Then

$$\rho(\xi,t) = I_{R^{n+1}}I_R\, [|x-t(r)|^2 /(1+\lambda(x))]\, dq_\xi(x|r)q_\xi^*(r)dr'$$

which is greater than or equal $\rho(\xi,t_\xi)$, where

$$t_\xi(r) = E_{q\xi}((x/(1+\lambda(x))|r) \,/\, E_{q\xi}((1/(1+\lambda(x))|r).$$

Now, $\rho(\xi,t) = \rho(\xi,t_\xi)$ for some $t(\cdot)$ if and only if $t(\cdot) = t_\xi(\cdot)$ a.e. with respect to $q_\xi^*(\cdot)$. Hence $t_\xi(\cdot)$ is Bayes against $d\xi(\cdot)$. Since $d\xi(\cdot)$ was chosen arbitrarily, $t_\xi(\cdot)$ gives the general form of Bayes strategies up to the usual equivalence.

V.3 **Publication**

We have discussed various aspects of the choices that an econometrician faces in his search for a happy ending to his research project. In each particular case what the econometrician finds rational to do is determined by his pecuniary and technical resources, by his stock of tacit knowledge, and by socially constructed rules for valid arguments. Now we shall discuss the choices that the same econometrician faces when he decides on the best way to present his results to colleagues and other interested parties. The factors that determine rational choice in such situations are a varied lot. Those that underlie characteristic features of an applied econometrics paper are of special interest here.

The first decision that an applied econometrician must make in writing his report concerns its format. In deciding on a format he must consider all the good stories that his results allow him to tell. The contents of these stories and the way the stories are related depend on the results that he has obtained and on the economic theory that motivated his analysis. He must also consider ways in which he can combine the pertinent stories in the writing of interesting papers. Each combination of stories and outline of a paper single out a small set of journals that might be interested in publishing such a paper. The relevant set of journals varies with the chosen combination of stories. To finalize the choice of format for his report, our econometrician must weigh the advantages that each journal offers a contributor in the form of prestige, good referees, and chances of acceptance. The comments of good referees help in formalizing the final version of a paper. Prestige is important for promotion, salary increases, and the respect of colleagues. Good chances of publication are relevant since rejection will delay publication of the paper, and the lost time may be disadvantageous in a career perspective.

Suppose now that our econometrician has decided on a combination of stories for a paper and singled out a journal to which he will submit the paper. In writing his paper he is likely to adhere to a standard form of an applied econometrics paper. So when the paper is ready for submission, we expect that it will contain an introduction, sections that relate the theoretical motivation of the study and the characteristics of the data, a section that presents the results, and a conclusion with lists of notes and references.

Different methodologies have different ideas as to the contents of the various sections. Here is what I believe. The introduction should introduce the subject matter of the paper, tell in general terms how the main results of the paper are related to the works of others, and provide an outline of the paper. The sections on motivation and data ought to contain, among other things, a formal statement of the presuppositions of the study, a list of assertions to be confronted with data, and a thorough discussion of relevant characteristics of the given data. The section that presents the results ought to contain a sketch of the theory and data universes with a list of all the bridge principles that our econometrician has adopted. It should also contain a list of his results with a discussion of their statistical properties and their economic interpretation. Finally, the section ought to contain a congruent specification of the process that the econometrician believes generated his data. The conclusion of the paper should contain a summary of the paper and suggestions for further studies.

Judging from the looks of published applied econometrics papers, writing an introduction is a political act. It is required that the author tells how his results are related to the results of others. Who these others are, depends of course on the author's results. Unfortunately, it seems that it depends on many other matters as well. From one point of view it might be disadvantageous to refer to all the pertinent others. Journal space is limited, and too many references may throw doubts on the need to publish the results. Also, some of the others might not belong to the author's 'ingroup' and can, therefore, be safely ignored. From another point of view it might be advantageous to refer to the works of some colleagues even though their results are barely tangential to the results of the present author. The relevant colleagues might be prospective reviewers of the paper. They might also be powerful members of the author's department or of some department to which the author intends to establish a good relation. Finally, they might be colleagues whose works a knowledgeable econometrician is supposed to have read. Neil Ericson and John Irons' extraordinary study of the citations of R. Lucas' 1976 paper on nonautonomous econometric relations is a case in point. Of the 513 citations that Ericson and Irons examined, 327 mentioned Lucas' paper only in passing, 98 discussed Lucas' ideas in the context of the topic at hand without examining their empirical relevance, and only 88 attempted to assess the implications of Lucas' ideas (cf. Ericson and Irons, 1998).

The content of the sections on theoretical motivation and data will vary with the purpose of the theory-data confrontation to which the paper pertains. Even so, the econometrician in his comments must face squarely the fact that the references of most of his data belong to a socially constructed world of ideas that has little in common with the social world in which we live. In order that his empirical results have relevance he must take care to explicate the meaning of his theoretical variables. He must also delineate the characteristic features of our social reality to which his theory addresses itself. First when this is properly done, can he use the insight it brings to interpret the empirical results that he recants in a later section of the paper.

To be concluded.

Notes

1, The relevant translations with commentaries are (Foster and Humphries, 1951), (Hammond, 1902), (Lawsen-Tancred, 1986), (Ross, 1980), and parts of Jon Vetlesen's unpublished lecture notes on Aristotle at the University of Oslo. The page references in de Anima refer to pages in Hammond's book.

2. The idea of this example is taken from an article by Evans-Pritchard on "Levy-Bruhl's Theory of Primitive Mentality." I learned of it from reading Peter Winch's article on "Understanding a Primitive Society" (cf. Wilson, 1970, pp. 79-80).

3. J. Eichberger and D. Kelsey have considered gametheoretic situations in which the players' mixed strategies are taken to be superadditive probability distribuitions (cf. Eichberger and Kelsey, 1993).

4. There is a fourth interesting aspect of Hendry's method that I aught to mention here. Hendry and Hans Martin Krolzig have managed the extraordinary feat of constructing a software program, the PcGets, that will perform the whole reduction process automatically (cf. Hendry and Krolzig, 1999). In their simulation studies their program managed to zero in on the true DGP in 95% of their case studies.

References

1. Aasnes, Joergen, Erik Bjoern, and Terje Skjerpen. 1993. "Engel Functions, Panel Data, and Latent Variables." **Econometrica**, Vol. 61, pp. 1395-1422
2. Allais, Maurice. 1979. "The So-Called Allais Paradox and Rational Decisions under Uncertainty." In **Expected Utility Hypotheses and the Allais Paradox**. (ed) M. Allais and O. Hagen. Boston: Reidel
3. Anderson, Heather M. and Farshid Vahid. 1997. "On the Correspondence between Individual and Aggregate Food Consumption Functions: Evidence from the USA and the Netherlands." **Journal of Applied Econometrics**. Vol. 12, pp. 477-507
4. Arrow, Kenneth J. 1965. **Aspects of the Theory of Risk Bearing**. Helsinki: Academic Book Store
5. Asheim, Geir. 1999. “On the Epistemic Foundation of Backward Induction.” Working paper, the University of Oslo. Oslo
6. Bar-Hillel, Maya. 1973. "On the Subjective Probability of Compound Events." **Organizational Behavior and Human Performance**. Vol. 9, pp. 396-406
7. Brown, Harold I. 1990. **Rationality**. New York: Routledge
8. Burke, Kenneth. 1945. **A Grammar of Motives**. New York: Prentice-Hall, Inc.
9. Chateauneuf, Allain and J.Y. Jaffray. 1989. "Some characterizations of lower probabilities and other monotone capacities through the use of Möbius inversion." **Mathematical Social Science**. Vol. 17, pp. 263-283
10. Cohen, L. Jonathan. 1981. "Can human irrationality be experimentally demonstrated?" **The Behavioral and Brain Sciences**. Vol. 4, pp. 317-370
11. Cooper, Russel J. And Keith McLaren. 1996. "A System of Demand Equations Satisfying Effectively Global Regularity Conditions." **Review of Economics and Statistics.** Vol. 78, pp. 359-364
12. Cooper, Russel W., Douglas V. DeJong, Robert Forsythe, and Thomas W. Ross. 1991. "Cooperation without Reputation," Working paper University of Iowa
13. Davidson, Donald. 1982.
14. Davis, Douglas D. And Charles A. Holt.1993. **Experimental Economics**. Princeton University Press, Princeton

15. Deaton, Angus S. and John Muellbauer. 1980. "An Almost Ideal Dem.and System." **American Economic Review**, Vol. 70, pp. 312-326

16. Debreu, Gerard. 1959. **Theory of Value**. New York: Wiley

17. De Finetti, Bruno. 1964. "La Prevision: ses Lois Logiques, ses Sources Subjectives." In **Studies in Subjective Probability**, (ed) H.E. Kyburg, Jr. and H.E. Smokler. New York: Wiley

18. Dempster, A. P. 1967. "Upper and Lower Probabilities Induced by a Multivalued Mapping." **Annals of Mathematical Statistics**. Vol. 38

19. Eichberger, Jurgen and David Kelsey. 1993. "Non-additive Beliefs and Game Theory." Unpublished paper, The University of Melbourne, Australia

20. Engel, Ernst. 1857. "Die Produktions- und Consumtionsverhältnisse des Königsreichs Sachsen." **Zeitschrift des Statistische Büreaus des Königlich Sächsichen**

21. Engle, Rob. 1982. "Autoregressive Conditional Heteroscedasticity, with Estimates of the Variance of United Kingdom Inflations." Econometrica. Vol. 50, pp. 987-1007

22. Ericson, Neil R. and John S. Irons. 1995. "The Lucas Critique in Practice: Theory without Measurement." In Hoover, K. (ed) **Macroeconometrics: Developments, Tensions and Prospects.** Amsterdam: Kluwer Academic Publishers

23. Evans-Pritchard, E.E. 1934. "Levy-Bruhl's Theory of Primitive Mentality." **Bulletin of the Faculty of Arts**. University of Egypt

24. Feller, William. 1957. **An Introduction to Probability Theory and its Applications**. Vol. 1, 2d ed. New York: Wiley

25. Fisher, Frank M. 1968 "Embodied Technology and the Existence of Labour and Output Aggregates." **The Review of Economic Studies**. Vol. 35

26. Foster, Kenelm, and Silvester Humphries (Transl.). 1951. **Aristotle's De Anima in the Version of William of Moerbeke and the Commentary of ST. Thomas Aquinas**. New Haven: Yale University Press **Ministeriums des Innern**. Dresden

27. Fraumeni, Barbara. 1997. "The Measurement of Depreciation in the U.S. National Income and Product Accounts." **Survey of Current Business**. Vol. 77, pp. 7-23

28. Friedman, Milton. 1957. **A Theory of the Consumption Function**. Princeton: Princeton University Press

29. Friedman, Milton. 1968. "The Role of Monetary Policy." **The American Economic Review**, Vol. 58, pp. 1-17

30. Gilboa, Itzak. 1987. "Expected Utility with Purely Subjective Non-additive Probabilities." **Journal of Mathematical Economics**. Vol. 16

31. Gilligan, Carol. 1982. **In a Different Voice**. Cambridge: Harvard University Press

32. Greenland Sander, James M. Robins, and Judea Pearl. 1999. "Confounding and Collapsibility in Causal Inference." **Statistical** Science. Vol. 14, pp. 29-46

33. Hammerton, M. 1973. "A case of radical probability estimation." **Journal of Experimental Psychology**. Vol. 101, pp. 252-254

34. Hammond, William A. 1902. **Aristotle's Psychology - a Treatise on the Principal of Life**. London: Swan Sonnenschein and Co., LTM

35. Hempel, Carl G. 1962. "Rational Action." Proceedings and Addresses of the American Philosophical Association, 1961-1962. Vol. XXXV, pp.5-23

36. Hendry, David F. 1995. **Dynamic Econometrics**. Oxford: Oxford University Press

37. Jaffray, Jean-Yves. 1989. "Linear utility theory and belief functions." Operations Research Letters. Vol. 8, pp. 107-112

38. Knorr-Cetina, Karin. 1981. **The Manufacture of Knowledge**. Oxford: Pergamon Press

39. Krolzig, Hans-Martin and David F. Hendry. 1999. "Computer Automation of General-to-Specific Model Selection Procedures." Unpublished Paper, Nuffield College, Oxford

40. Kuhn, Thomas S. 1970. **The Structure of Scientific Revolutions**, Second Edition. Cicago: University of Chicago Press

41. Lakatos, Imre. 1978. **The Methodology of Scientific Research Programmes**, Vol I. Cambridge: Cambridge University Press

42. Laplace, Pierre S. 1951. **A Philosophical Essay on Probabilities**. New York: Dover

43. Lawson-Tancred, Hugh. 1986. **Aristotle - De Anima**. London: Penguin Books

44. Leamer, Edward E. 1983. "Model Choice and Specification Analysis." In D. Belsley, Z. Griliches, M. Intriligator, and P Schmidt (ed) **Handbook of Econometrics, Vol I**, Amsterdam: North Holland Company

45. Lucas, Robert E. 1972. ""Expectations and the Neutrality of Money." Journal of Economic Theory. Vo. 4, pp. 103-124

46. McCloskey, Donald. 1985. **The Rhetoric of Economics**. Madison: The University of Wisconsin Press
47. MacCrimmon, K.R. and S. Larsson. 1979. "Utility Theory: Axioms versus Paradoxes." In M. Allais and O. Hagen (ed) The Expected Utility Hypothesis and the Allais Paradox. Dordrecht: D. Riedel
48. McFadden, Daniel. 1978. "Duality of Production, Cost, and Profit Functions." In (ed) Melvyn Fuss and Daniel McFadden, **Production Economics: a Dual Approach to Theory and Applications**, Vol. 1, Amsterdam: North Holland Publishing Co.
49. Modigliani, Franco and Richard Brumberg. 1955. "Utility Analysis and the Consumption Function. An Interpretation of Cross-Section Data." In **Post-Keynesian Economics**. (ed) K.K. Kurihara. London: Allen and Unwin
50. Mosteller, Frederick C. And Phillip Nogee. 1951. "An Experimental Measurement of Utility." **The Journal of Political Economy**. Vol. 59
51. Pesaran, M. Hashem. 1987. **The Limits to Rational Expectations**. London: Basil Blackwell
52. Phelps, Edmund S. 1970. "The New Microeconomics in Employment and Inflation Theory." In E. Phelps (ed.) **Microeconomic Foundations of Employment and Inflation Theory**. New York: Norton
53. Phillips, A.W. 1958. "The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957." **Economica**. Vol. 25, pp. 283-294
54. Rawls, John. 1971. **A Theory of Justice**. Cambridge: Harvard University Press
55. Rawls, John. 1996. **Political Liberalism**. New York: Columbia University Press
56. Ross, David (Trans). 1925. **Aristotle: the Nicomachean Ethics**. Oxford: Oxford University Press
57. Samuelson, Paul A. 1950. **Foundations of Economic Analysis**. Cambridge: Harvard University Press
58. Savage, Leonard J. 1954. **The Foundations of Statistics**. New York: Wiley
59. Selten, Reinhard. 1978. “The Chain Store Paradox.” **Theory and Decision**. Vol. 9, pp. 127-159.
60. Shafer, Glenn. 1976. **A Mathematical Theory of Evidence**. Princeton: Princeton University Press

61. Sheffrin, Steven M. 1996. **Rational Expectations**, Second Edition. Cambridge: Cambridge University Press
62. Solow, Robert M. 1957. "Technical Change and the Aggregate Production Function**." The Review of Economics and Statistics**. Vol. , pp. 312-320.
63. Stigum, Bernt P. 1967a. "A Decision Theoretic Approach to Time Series Analysis." **Annals of the Institute of Statistical Mathematics**, Vol. 19, pp.207-243.
64. Stigum, Bernt P. 1967b. "On Certain Problems of Aggregation." **International Economic Review**. Vol. 8
65. Stigum, Bernt P. 1968. "On a Property of Concave Functions." **The Review of Economic Studies**. Vol. 35, pp. 413-416
66. Stigum, Bernt P. 1969. "Competitive Equilibria under Uncertainty." **The Quarterly Journal of Economics**." Vol. 83
67. Stigum, Bernt P. 1972. "Resource Allocation under Uncertainty." **International Economic Review**. Vol. 13
68. Stigum, Bernt P. 1990. **Toward a Formal Science of Economics**. Cambridge: MIT Press
69. Stigum, Bernt P. 1995. "Theory-Data Confrontations in Economics." **Dialogue**, Vol. XXXIV, pp. 581-604
70. Tversky, A. And D. Kahneman. 1974. "Judgement under uncertainty: Heuristics and biases." **Science**, Vol. 125, pp. 1124-31
71. von Neumann, John and Oscar Morgenstern. 1944**. Theory of Games and Economic Behavior**. Princeton: Princeton University Press
72. Wilson, Bryan R. (ed). 1970. **Rationality**. New York: Basil Blackwell
73. Winch, Peter. 1970. "Understanding a Primitive Society." In **Rationality**, (ed) Bryan R. Wilson. New York: Basil Blackwell