Asymmetric and Common Absorption of Shocks in Nonlinear Autoregressive Models*

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Abstract

A key feature of many nonlinear time series models is that they allow for the possibility that the model structure experiences changes, depending on for example the state of the economy or of the financial market. A common property of these models is that it generally is not possible to fully understand the structure of the model by considering the estimated values of the model parameters only. Put differently, it often is difficult to interpret a specific nonlinear model. To shed light on the characteristics of a nonlinear model it can then be useful to consider the effect of shocks on the future patterns of a time series variable. Most interest in such impulse response analysis has concentrated on measuring the persistence of shocks, or the *magnitude* of the (ultimate) effect of shocks. Interestingly, far less attention has been given to measuring the *speed* at which this final effect is attained, that is, how fast shocks are 'absorbed' by a time series. In this paper we develop and implement a framework that can be used to assess the absorption rate of shocks in nonlinear models. The current-depth-of-recession model of Beaudry and Koop (1993), the floor-and-ceiling model of Pesaran and Potter (1997) and a multivariate STAR model are used to illustrate the various concepts.

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1 Introduction

Nonlinear time series models are frequently considered in, for example, empirical macroeconomics and empirical finance to describe and forecast the relevant time series variables, see Granger and Teräsvirta (1993), Kuan and Liu (1995) and Franses and van Dijk (2000), among many others. Typical examples of such variables are GNP, industrial production and unemployment, all of which display pronounced business cycle fluctuations, and exchange rates and interest rates. A key feature of many nonlinear time series models is that they allow for the possibility that the model structure (lag length, parameters, variance) experiences changes, depending on the state of the economy (expansions or recessions) or of the financial market (for example, high or low volatility). Examples of often considered models are the threshold autoregressive [TAR] model, see Tong (1990), the smooth transition (auto)regression [ST(A)R] model, see Teräsvirta (1994, 1998), the Markov-Switching model put forward in Hamilton (1989), and the Artificial Neural Network [ANN] model advocated by Kuan and White (1994), among others.

A common property of many of these (univariate) nonlinear models (and this holds true even more so for their multivariate counterparts) is that it generally is not possible to completely grasp the implied properties of time series generated by the model by only considering (estimates of) the model parameters. Put differently, it is difficult to interpret a specific nonlinear model and to understand why it can or should be useful in a particular application. Therefore, to shed light on the characteristics of a nonlinear model it often is useful to consider the effect of shocks on the future patterns of a time series variable. Impulse response functions provide a convenient tool to measure such effects of shocks. Most interest in impulse response analysis has concentrated on measuring the persistence of shocks, indicated by the *magnitude* of the (ultimate) effect of shocks. Interestingly, far less attention has been given to measuring the *speed* at which this final effect is attained, that is, how fast shocks are 'absorbed' by a time series. Due to the properties of impulse responses in linear models, they can be used straightforwardly to gain insight in this rate of absorption of shocks as well, see, for example, Lütkepohl (1991) for a discussion of impulse response functions in linear models. However, impulse response analysis in nonlinear models is more complicated, as discussed at length in Koop, Pesaran and Potter (1996). The complications arise because in nonlinear models (1) the effect of a shock depends on the history of the time series up to the point where the shock occurs, (2) the effect of a shock need not be proportional to its size and (3) the effect of a shock depends on shocks occurring in periods between the moment at which the impulse occurs and the moment at which the response is measured. Because of these properties of impulse responses, assessing the absorption speed of shocks in nonlinear models also is more involved, as will become clear below. In this paper we develop and implement a framework that can be used to assess the absorption rate of shocks in nonlinear models. Among others, we demonstrate that our absorption measure can be used to address relevant questions such as

1. Are positive and negative shocks absorbed at the same speed?
2. Are shocks absorbed at the same speed by the different components of a multivariate time series?
3. Are shocks absorbed at the same speed by linear combinations of the components in a multivariate time series and by the individual components themselves?

Hence, together with familiar impulse response functions, our absorption measure allows one to obtain a more complete picture of the propagation mechanism of a nonlinear model as it can highlight interesting asymmetric or common properties of shocks to economic time series.

Finally it should be remarked that an alternative approach to absorption is considered by Lee and Pesaran (1993) and Pesaran and Shin (1996). They examine the time profile of the effect of shocks by means of so-called 'persistence profiles', defined as the difference between the conditional variances of n-step and $(n-1)$-step ahead forecasts, viewed as a function of n.

Our paper proceeds as follows. In Section 2, we briefly recapitulate the main aspects of impulse response analysis in nonlinear time series models and the Generalized Impulse Response Functions introduced by Koop *et al.* (1996). In Section 3, we develop our measure of the speed of the absorption of shocks. To facilitate the understanding of the concept of absorption, we concentrate on univariate models first. In this section we also demonstrate how to address the question whether positive and negative shocks are absorbed at different speeds. Empirical examples involving the current-depth-of-recession model of Beaudry and Koop (1993) and the floor-and-ceiling model of Pesaran and Potter (1997) are used to illustrate the various concepts. In Section 4, we generalize our absorption measure to multivariate models. Particular attention is given to the question whether shocks are absorbed at the same speed by the different components of a multivariate time series. We also outline how to obtain the absorption speed for a linear combination of the components of a multivariate time series. An empirical example involving a trivariate nonlinear STAR model for income, consumption and investment is used for illustration. Finally, Section 5 contains some concluding remarks.

2 Preliminaries

Consider the nonlinear multivariate time series model

$$Y_t = F(Y_{t-1}, \ldots, Y_{t-p}; \theta) + V_t, \tag{1}$$

where Y_t is a $(k \times 1)$ random vector, $F(\cdot)$ is a known function which depends on the $(q \times 1)$ parameter vector θ, V_t is a $(k \times 1)$ vector of random disturbances with $\mathrm{E}[V_t|\Omega_{t-1}] = 0$ and $\mathrm{E}[V_t V_t'|\Omega_{t-1}] = H(Y_{t-1}, \ldots, Y_{t-p}; \xi)$, where the $(k \times k)$ conditional covariance matrix $H(Y_{t-1}, \ldots, Y_{t-p}; \xi) \equiv H_t = \{H_{t,ij}, i,j = 1, \ldots, k\}$ depends on the $(r \times 1)$ parameter vector ξ.

Throughout, we use upper-case letters to denote random variables and lower-case letters to denote realizations of those random variables. For example, y_t and v_t are realizations of Y_t and V_t, respectively. The 'history' or information set at $t-1$, which is used to forecast future values of Y_t, is denoted as Ω_{t-1}, with corresponding realizations denoted as ω_{t-1}. Because the nonlinear model (1) is Markov of order p, it suffices to take $\Omega_{t-1} = \{Y_{t-1}, \ldots, Y_{t-p}\}$.

2.1 Impulse response functions

Impulse response functions are meant to provide a measure of the response of Y_{t+n} to a shock or impulse v_t at time t. The impulse response measure which is commonly used in the analysis of linear models is defined as the difference between two realizations of Y_{t+n} which start from identical histories ω_{t-1}. In one realization, the process is hit by a shock of size v_t at time t, while in the other realization no shock occurs at time t. All shocks in intermediate periods between t and $t+n$ are set equal to zero in both realizations. Hence, the traditional impulse response function $[TI]$ is given by

$$\begin{aligned} TI_Y(n, v_t, \omega_{t-1}) = {} & \mathrm{E}[Y_{t+n}|V_t = v_t, V_{t+1} = \ldots = V_{t+n} = 0, \omega_{t-1}] - \\ & \mathrm{E}[Y_{t+n}|V_t = 0, V_{t+1} = \ldots = V_{t+n} = 0, \omega_{t-1}], \end{aligned} \tag{2}$$

for $n = 0, 1, 2, \ldots$. The second conditional expectation usually is called the benchmark profile.

This traditional impulse response function has some characteristic properties in case the model is linear. First, it is *symmetric* in the sense that a shock of $-v_t$ has exactly the opposite effect as a shock of size $+v_t$. Furthermore, it might be called *linear* as the impulse response is proportional to the size of the shock. Finally, the impulse response is *history independent* as it does not depend on the particular history ω_{t-1}. For example, in the univariate AR(1) model $Y_t = \phi Y_{t-1} + V_t$, it follows easily that $TI_Y(n, v_t, \omega_{t-1}) = \phi^n v_t$, which clearly demonstrates the aforementioned properties of the impulse response function.

These properties do not carry over to nonlinear models. In nonlinear models, the impact of a shock depends on the sign and the size of the shock, as well as on the history of the process. Furthermore, if the effect of a shock on the time series $n > 1$ periods ahead is to be analyzed, the assumption that no shocks occur in intermediate periods might give rise to quite misleading inference concerning the propagation mechanism of the model, see Pesaran and Potter (1997) for an example.

The Generalized Impulse Response Function [GI], introduced by Koop *et al.* (1996), provides a natural solution to the problems involved in defining impulse responses in nonlinear models. The GI for a specific shock v_t and history ω_{t-1} is defined as

$$GI_Y(n, v_t, \omega_{t-1}) = \mathrm{E}[Y_{t+n}|V_t = v_t, \omega_{t-1}] - \mathrm{E}[Y_{t+n}|\omega_{t-1}], \tag{3}$$

for $n = 0, 1, 2, \ldots$. In the GI, the expectations of Y_{t+n} are conditioned only on the history and/or on the shock at time t. Put differently, the problem of dealing with shocks occurring in intermediate time periods is dealt with by averaging them out. Given this choice, the natural benchmark profile for the impulse response is the expectation of Y_{t+n} conditional only on the history of the process ω_{t-1}. Thus, in the benchmark profile the current shock is averaged out as well. It is straightforward to show that for linear models the GI in (3) is equivalent to the traditional impulse response in (2).

The GI as defined in (3) is a function of v_t and ω_{t-1}, which are realizations of the random variables V_t and Ω_{t-1}. Koop *et al.* (1996) stress that hence $GI_Y(n, v_t, \omega_{t-1})$ itself is a realization of a random variable given by

$$GI_Y(n, V_t, \Omega_{t-1}) = \mathrm{E}[Y_{t+n}|V_t, \Omega_{t-1}] - \mathrm{E}[Y_{t+n}|\Omega_{t-1}]. \tag{4}$$

It is useful to note that $GI_Y(n, v_t, \omega_{t-1})$ can still be interpreted as a random variable if parameter uncertainty is taken into account, as in Koop (1996).

Using this interpretation of the GI as a random variable, various conditional versions can be defined which are of potential interest. For example, one might consider a particular history ω_{t-1} and treat the GI as a random variable in terms of V_t only, that is,

$$GI_Y(n, V_t, \omega_{t-1}) = \mathrm{E}[Y_{t+n}|V_t, \omega_{t-1}] - \mathrm{E}[Y_{t+n}|\omega_{t-1}]. \tag{5}$$

Alternatively, one could reverse the role of the shock and the history by fixing the shock at $V_t = v_t$ and consider the GI as a random variable in terms of the history Ω_{t-1}. In general, one might compute the GI conditional on subsets A and B of shocks and histories respectively, that is, $GI_Y(n, A, B)$. For example, one might condition on all histories such that $Y_{t-1} \leq 0$ and consider only negative shocks.

Finally, note that as for nonlinear models analytic expressions for the conditional expectations involved in the GI in (4) usually are not available, stochastic simulation should be used to obtain estimates of the impulse response measures. See Koop *et al.* (1996) for a detailed description of the relevant techniques.

The two aspects of impulse responses that appear to be of interest are (1) the final response to an impulse, and (2) the speed at which this final response is approached. Traditionally, most attention has been given to the first aspect, usually referred to as *persistence*. In the present paper we focus on the second aspect, which we call *absorption rate*. Before we proceed to discuss how this absorption rate can be measured in the next section, we summarize how persistence of shocks can be assessed by means of the GI. This section then closes with some remarks on how to determine whether positive and negative shocks have asymmetric effects.

2.2 Measuring persistence of shocks

A shock v_t is said to be *transient* at history ω_{t-1} if in the long run the shock does not affect the pattern of the time series, that is, if $GI_Y(n, v_t, \omega_{t-1})$ becomes equal to 0 as the horizon n goes to infinity. If this is not the case, the shock is said to be persistent. The final impulse response for a specific shock and history can be obtained as

$$GI_Y^{\infty}(v_t, \omega_{t-1}) = \lim_{n \to \infty} GI_Y(n, v_t, \omega_{t-1}), \tag{6}$$

if this limit exists. In practice, the final impulse response $GI_Y^{\infty}(v_t, \omega_{t-1})$ can be estimated by $GI_Y(m, v_t, \omega_{t-1})$ for certain large m.

Potter (1995a) and Koop *et al.* (1996) suggest that the dispersion of the distribution of $GI_Y(n, V_t, \Omega_{t-1})$ at finite horizons can be interpreted as a measure of persistence of shocks. It is intuitively clear that if a time series process is stationary and ergodic, the effect of all shocks eventually becomes zero for all possible histories of the process. Hence, $GI_Y^\infty(v_t, \omega_{t-1})$ in (6) is equal to zero for all choices of v_t and ω_{t-1}. Alternatively, the distribution of $GI_Y(n, V_t, \Omega_{t-1})$ collapses to a spike at 0 as $n \to \infty$. By contrast, for nonstationary time series the dispersion of the distribution of $GI_Y(n, V_t, \Omega_{t-1})$ is positive for all n. Conditional versions of the GI are particularly suited to assess the persistence of shocks. For example, one might compare the dispersion of the distributions of GIs conditional on positive and negative shocks to determine whether negative shocks are more persistent than positive, or vice versa. A potential problem with this approach is that no unambiguous measure of dispersion exists, although the notion of second-order stochastic dominance might be useful in this context, see Potter (2000).

2.3 Measuring asymmetric impulse response

One possible use of the GI is to assess the significance of asymmetric effects of positive and negative shocks. Potter (1994) defines a measure of asymmetric response to a particular shock $V_t = v_t$ given a particular history ω_{t-1} as the sum of the GI for this particular shock and the GI for the shock of the same magnitude but with opposite sign, that is,

$$ASY_Y(n, v_t, \omega_{t-1}) = GI_Y(n, v_t, \omega_{t-1}) + GI_Y(n, -v_t, \omega_{t-1}). \tag{7}$$

By taking into account parameter uncertainty as an additional source of randomness, $ASY_Y(n, v_t, \omega_{t-1})$ can still be interpreted as a random variable. Potter (1995b) uses a straightforward simulation procedure to assess whether the asymmetry measure is significantly different from zero or not.

Alternatively, one could consider the distribution of the random asymmetry measure

$$ASY_Y(n, V_t^+, \Omega_{t-1}) = GI_Y(n, V_t^+, \Omega_{t-1}) + GI_Y(n, -V_t^+, \Omega_{t-1}) \tag{8}$$

where $V_t^+ = \{v_t | v_t > 0\}$ indicates the set of all possible positive shocks. If positive and negative shocks have exactly the same effect (with opposite sign), $ASY_Y(n, V_t^+, \Omega_{t-1})$ should be equal to zero almost surely. More generally, we say that shocks have a symmetric effect (on average) when $ASY_Y(n, V_t^+, \Omega_{t-1})$ has a symmetric distribution with mean equal to zero. The dispersion of this distribution might be interpreted as a measure of the asymmetry in the effects of positive and negative shocks.

3 Absorption of shocks in univariate models

Irrespective of whether shocks are persistent of not, it should be of interest to assess how fast innovations are absorbed, that is, how fast the GI approaches the final response $GI_Y^{\infty}(v_t, \omega_{t-1})$. In this section we discuss how this absorption rate can be measured.

3.1 Definition of absorption

Suppose for the moment that Y_t is a univariate time series. Define the indicator function

$$I_Y(\pi, n, v_t, \omega_{t-1}) \equiv I[|GI_Y(n, v_t, \omega_{t-1}) - GI_Y^{\infty}(v_t, \omega_{t-1})| \leq \pi |v_t - GI_Y^{\infty}(v_t, \omega_{t-1})|]$$

for certain π such that $0 \leq \pi \leq 1$, where $I[A] = 1$ if the event A occurs and 0 otherwise, and where it is assumed that the limit defining $GI_Y^{\infty}(v_t, \omega_{t-1})$ in (6) exists. In words, the function $I_Y(\pi, n, v_t, \omega_{t-1})$ is equal to 1 if the absolute difference between the GI at horizon n and the eventual response to the shock v_t, as given by $GI_Y^{\infty}(v_t, \omega_{t-1})$, is less than or equal to a fraction π of the absolute difference between the shock v_t, which is equal to the initial impact of the shock or the GI at horizon 0, and the eventual response. Put differently, $I_Y(\pi, n, v_t, \omega_{t-1}) = 1$ if at least a fraction $1 - \pi$ of the initial effect of v_t has been absorbed after n periods. Notice that for a random walk, $GI_Y(n, v_t, \omega_{t-1}) = v_t$ for all $n \geq 0$, so that $I_Y(\pi, n, v_t, \omega_t) = 1$ in all cases.

The 'π-life' or 'π-absorption time' of v_t can now be defined as

$$N_Y(\pi, v_t, \omega_{t-1}) = \sum_{n=0}^{\infty} \left(1 - \prod_{m=n}^{\infty} I_Y(\pi, m, v_t, \omega_{t-1})\right). \quad (9)$$

In words, $N_Y(\pi, v_t, \omega_{t-1})$ is the minimum horizon beyond which the difference between the impulse responses *at all larger horizons* and the eventual response is less than or equal to a fraction π of the difference between the initial impact and the eventual response. That is, $N_Y(\pi, v_t, \omega_{t-1}) = m$ if $I_Y(\pi, n, v_t, \omega_{t-1}) = 1$ for all $n \geq m$ and $I_Y(\pi, m - 1, v_t, \omega_{t-1}) = 0$. The reason for *not* defining $N_Y(\pi, v_t, \omega_{t-1})$ as the smallest horizon for which $I_Y(\pi, n, v_t, \omega_{t-1}) = 1$ is that the GI need not approach the limit $GI_Y^{\infty}(v_t, \omega_{t-1})$ monotonically.

Just like the shock- and history-specific GI in (3) can be regarded as a realization of the random variable $GI_Y(n, V_t, \Omega_{t-1})$ in (4), the π-absorption time $N_Y(\pi, v_t, \omega_{t-1})$ in (9) can be regarded as a realization of the random variable

$$N_Y(\pi, V_t, \Omega_{t-1}) = \sum_{n=0}^{\infty} \left(1 - \prod_{m=n}^{\infty} I_Y(\pi, m, V_t, \Omega_{t-1})\right), \quad (10)$$

where the random indicator function $I_Y(\pi, n, V_t, \Omega_{t-1})$ is defined as

$$I_Y(\pi, n, V_t, \Omega_{t-1}) \equiv I[|GI_Y(n, V_t, \Omega_{t-1}) - GI_Y^{\infty}(V_t, \Omega_{t-1})| \leq \pi|V_t - GI_Y^{\infty}(V_t, \Omega_{t-1})|].$$

Conditional versions $N_Y(\pi, A, B)$ for particular subsets A and B of shocks and histories respectively can be defined in a straightforward manner.

As an example of the π-absorption measure, consider again the linear AR(1) model $Y_t = \phi Y_{t-1} + V_t$ with $|\phi| < 1$. It then follows that $GI_Y(n, v_t, \omega_{t-1}) = \phi^n v_t$, and $GI_Y^{\infty}(v_t, \omega_{t-1}) = 0$. Thus, $I_Y(\pi, n, v_t, \omega_{t-1}) = I[|\phi^n v_t| \leq \pi|v_t|]$, which is equal to 1 if $|\phi^n| = |\phi|^n \leq \pi$, or $n \geq \ln(\pi)/\ln(|\phi|)$. From (9) it then follows that $N_Y(\pi, v_t, \omega_{t-1}) = \ln(\pi)/\ln(|\phi|)$. Thus, for linear models, the π-absorption time for $\pi = 0.50$ corresponds to the usual measure of the half-life of shocks. Observe that $N_Y(\pi, v_t, \omega_{t-1})$ increases as ϕ approaches 1, whereas the π-absorption time is 0 for a random walk. This illustrates that models with persistent shocks may display faster absorption than models with transient shocks. Finally, note that $N_Y(\pi, v_t, \omega_{t-1})$ is independent of v_t and ω_{t-1} in this case. Hence, the dispersion of the distribution of $N_Y(\pi, V_t, \Omega_{t-1})$ might be interpreted as a rough measure of the 'degree of nonlinearity' of a particular model.

3.2 Measuring asymmetric absorption

Possible asymmetry in the absorption of positive and negative shocks can be examined in a way similar to asymmetry in impulse responses, as discussed in Section 2.3. For a specific shock v_t and history ω_{t-1}, a measure of asymmetric absorption can be defined as the difference in π-absorption times of v_t and $-v_t$, that is,

$$ASYN_Y(\pi, v_t, \omega_{t-1}) = N_Y(\pi, v_t, \omega_{t-1}) - N_Y(\pi, -v_t, \omega_{t-1}). \tag{11}$$

If v_t has symmetric absorption speed at ω_{t-1}, $ASYN_Y(\pi, v_t, \omega_{t-1}) = 0$ for all values of π.

Note that symmetry in $GI_Y(n, v_t, \omega_{t-1})$, that is, $ASY_Y(n, v_t, \omega_{t-1}) = 0$ for all $n \geq 0$ in (7), implies symmetry in the absorption speed, that is, $ASYN_Y(\pi, v_t, \omega_{t-1}) = 0$ for all $\pi \in (0, 1)$. Interestingly, the reverse does not hold, that is, a shock can have symmetric absorption speed but an asymmetric impulse response. Also, $ASYN_Y(\pi, v_t, \omega_{t-1}) \neq 0$ for certain $\pi \in (0, 1)$ implies that $ASY_Y(n, v_t, \omega_{t-1}) \neq 0$ for certain $n \geq 0$, whereas the reverse does not hold. This again indicates the added value of the absorption measure.

As before, the asymmetry measure in (11) can be regarded as a realization of the random variable

$$ASYN_Y(\pi, V_t^+, \Omega_{t-1}) = N_Y(\pi, V_t^+, \Omega_{t-1}) - N_Y(\pi, -V_t^+, \Omega_{t-1}), \tag{12}$$

where V_t^+ is defined just below (8). If positive and negative shocks have symmetric effects, in the sense that they are absorbed at the same speed on average, $ASYN_Y(\pi, V_t^+, \Omega_{t-1})$ should have a distribution with mean equal to zero. Obviously, the asymmetry measure can also be defined for subsets A and B of shocks and histories.

By taking into account parameter uncertainty, one can examine whether a specific shock v_t has symmetric absorption rate at ω_{t-1} by examining whether $ASYN_Y(\pi, v_t, \omega_{t-1})$ is significantly different from zero. To assess whether the absorption of shocks in the set A for the set of histories B is symmetric on average, it is necessary to test whether the mean of the distribution of $ASYN_Y(\pi, A^+, B)$ is equal to zero. This is complicated by the fact that the different realizations $ASYN_Y(\pi, v_t, \omega_{t-1})$ which are used to estimate this distribution are not independent across histories ω_{t-1}. Hence, the standard error for the mean of $ASYN_Y(\pi, A^+, B)$ is not equal to $\sigma_{ASYN_Y(\pi,A^+,B)}/\sqrt{n_{AB}}$, where $\sigma_{ASYN_Y(\pi,A^+,B)}$ is the standard deviation of $ASYN_Y(\pi, A^+, B)$ and n_{AB} is the number of combinations of shocks v_t and histories ω_{t-1} for which $ASYN_Y(\pi, v_t, \omega_{t-1})$ is computed. Note however that the $ASYN_Y(\pi, v_t, \omega_{t-1})$ are independent across shocks v_t. Therefore, as a conservative standard error for the mean of $ASYN_Y(\pi, A^+, B)$ we suggest to use $\sigma_{ASYN_Y(\pi,A^+,B)}/\sqrt{n_A}$, where n_A is the number of shocks v_t for which $ASYN_Y(\pi, v_t, \omega_{t-1})$ is computed.

Alternatively, the asymmetry of the distribution of $ASYN_Y(\pi, A^+, B)$ can be assessed by confidence regions. Following Hyndman (1995), we consider three different $100\cdot(1-\alpha)\%$ confidence regions:

1. An interval symmetric around the mean of the distribution

$$S_\alpha = (\hat{\mu}_{ASYN_Y(\pi,A^+,B)} - w, \;\; \hat{\mu}_{ASYN_Y(\pi,A^+,B)} + w),$$

   where $\hat{\mu}_{ASYN_Y(\pi,A^+,B)}$ is the mean of the asymmetry measure $ASYN_Y(\pi, A^+, B)$ and w is such that $P(ASYN_Y(\pi, A^+, B) \in S_\alpha) = 1 - \alpha$.

2. The interval between the $\alpha/2$ and $(1 - \alpha/2)$ quantiles of the distribution, denoted $q_{\alpha/2}$ and $q_{1-\alpha/2}$, respectively,

$$Q_\alpha = (q_{\alpha/2}, q_{1-\alpha/2}).$$

3. The highest-density region $[HDR]$

$$HDR_\alpha = \{ASYN_Y(\pi, A^+, B) | g(ASYN_Y(\pi, A^+, B)) \geq g_\alpha\}, \qquad (13)$$

where $g(\cdot)$ is the density of the argument and g_α is such that $P(ASYN_Y(\pi, A^+, B) \in HDR_\alpha) = 1 - \alpha$.

For symmetric and unimodal distributions, these three regions are identical. For asymmetric or multimodal distributions they are not, see Hyndman (1995) for discussion. In the applications below, we report α^*, which is the minimum value of $\alpha \in (0,1)$ such that 0 would not be included in the relevant confidence region. Note that the three confidence regions all provide different information. The interval symmetric around the mean indicates the position of 0 relative to the mean of the distribution. The interval with equal quantiles in the tail indicates whether 0 is located in the tails or in the central part of the distribution. Finally, the HDR indicates the probability that the asymmetry measure is equal to 0.

3.3 Example A: the current-depth-of-recession model

As a first example, we consider the model of Beaudry and Koop (1993), which includes the gap between the current value of output and its historical maximum value as an additional variable in a linear autoregressive model for the growth rate. Define the current depth of recession [CDR] as

$$CDR_t = Y_t - \max_{j \geq 0} Y_{t-j}, \tag{14}$$

where Y_t denotes the logarithm of output. Note that CDR_t has a negative value when current output is below its historical maximum, and is equal to 0 if current output is at its historical maximum. The current-depth-of-recession model for output growth then is given by

$$\phi(L)\Delta Y_t = \phi_0 + (\theta(L) - 1)CDR_t + V_t, \tag{15}$$

where $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$ and $\theta(L) = 1 + \theta_1 L + \cdots + \theta_q L^q$ are lag polynomials of orders p and q, respectively, with the lag operator defined as $L^m Y_t = Y_{t-m}$ for all m and $\Delta = 1 - L$ is the first-difference operator. A difference with the original model of Beaudry and Koop (1993) is that we allow the variance of the shock to be different in recessions ($CDR_{t-1} < 0$) and expansions ($CDR_{t-1} = 0$), as the disturbance V_t is assumed to have conditional mean equal to zero and conditional variance given by

$$\mathrm{E}[V_t^2|\Omega_{t-1}] \equiv H_t = \sigma_R^2 I[CDR_{t-1} < 0] + \sigma_E^2 I[CDR_{t-1} = 0]. \tag{16}$$

As we use similar data, we follow Beaudry and Koop (1993) and set $p = 2$ and $q = 1$ in (15), that is, we consider the model

$$\Delta Y_t = \phi_0 + \phi_1 \Delta Y_{t-1} + \phi_2 \Delta Y_{t-2} + \theta_1 CDR_{t-1} + V_t.$$

We use quarterly observations on seasonally adjusted real US GNP, from 1947:1-1995:2. The series is taken from Citibase. Parameter estimates are obtained by iterative weighted least squares as $\hat{\phi}_0 = 0.178$, $\hat{\phi}_1 = 0.432$, $\hat{\phi}_2 = 0.199$, $\hat{\theta}_1 = -0.328$, which are similar to the estimates obtained by Beaudry and Koop (1993) for US GDP over the sample 1947:1-1989:4. The residual standard deviations in the two regimes are estimated to be $\hat{\sigma}_R = 1.090$ and $\hat{\sigma}_E = 0.845$. The CDR_{t-1} variable takes a negative value in 50 of the 191 quarters in the effective estimation sample (1947:4-1995:2).

We compute impulse responses $GI_{\Delta Y}(n, v_t, \omega_{t-1})$ for all 191 histories in the sample, for values of the normalized shock equal to $v_t/\sqrt{h_t} = \pm 3, \pm 2.9, \ldots, \pm 0.1, 0$, where h_t denotes a realization of H_t in (16). Note that in this case the relevant history consists of the growth rate in the two previous periods and the lagged CDR variable, that is $\Omega_{t-1} = \{\Delta Y_{t-1}, \Delta Y_{t-2}, CDR_{t-1}\}$. GIs are computed for horizons $n = 0, 1, \ldots, N$ with $N = 20$, using the algorithm outlined in Koop *et al.* (1996), using $R = 10000$ replications to average out the effect of shocks occurring in intermediate periods. The shocks in intermediate periods are sampled from a normal distribution. Impulse responses for the log level of GNP are obtained by accumulating the impulse responses for the growth rate, that is $GI_Y(n, v_t, \omega_{t-1}) = \sum_{i=0}^{n} GI_{\Delta Y}(i, v_t, \omega_{t-1})$. Figure 1 shows distributions of $GI_Y(n, A, B)$ at horizons $n = 0, 4, 8$ and 20, where A is taken to be the set of either all, negative or positive shocks, and B is the set of all histories or all histories for which CDR_{t-1} is either negative or zero. The latter two are labeled recession and expansion, respectively. These and all subsequent distributions are obtained with a standard Nadaraya-Watson kernel estimator, using $\phi(v_t/\sqrt{h_t})$ as weight for $GI_Y(n, v_t, \omega_{t-1})$, where $\phi(z)$ denotes the standard normal probability distribution. The reason for using this weighting scheme is that the standardized shocks $v_t/\sqrt{h_t}$ then effectively are sampled from a discretized normal distribution and the resulting distribution of $GI_Y(n, V_t, \Omega_{t-1})$ should resemble a normal distribution if the effect of shocks is symmetric and proportional to their magnitude (as is the case in linear models).

Figure 1 shows that in both regimes, the final impulse response appears to be larger for positive shocks. This is confirmed by the distributions of the asymmetry measure

$ASY_Y(n, V_t^+, B)$ shown in panels (j)-(l) of Figure 1. Table 1 contains summary statistics for these distributions at horizon $n = 20$, as well as for $ASY_Y(n, A^+, B)$, where A is taken to be the set of small ($0 < |V_t| \leq 1$), medium ($1 < |V_t| \leq 2$) or large ($2 < |V_t| \leq 3$) shocks. The mean of $ASY_Y(n, V_t^+, B)$ is seen to be close to zero in all three cases, suggesting that, on average, shocks have symmetric effects. Distinguishing between different magnitudes of shocks shows that small negative shocks have larger effects than positive ones and vice versa for medium and large shocks. The means of $ASY_Y(n, A^+, B)$ which are larger than two times the conservative standard error $\sigma_{ASY_Y(\pi, A^+, B)}/\sqrt{n_A}$ in absolute value are marked with an asterisk. It appears that the asymmetry is significant for all sets of shocks and histories considered. This is confirmed by the values of α^* reported in the final three rows for the different confidence regions. Note that the main conclusion of Beaudry and Koop (1993) is that positive shocks are more persistent than negative ones. The results in Table 1 suggest that this depends on the magnitude of the shock.

Truncating the summations in (9) at $N = 20$ and using $GI_Y(N, v_t, \omega_{t-1})$ as an estimate of the final impulse response $GI_Y^\infty(v_t, \omega_{t-1})$, we compute π-absorption times $N_Y(\pi, v_t, \omega_{t-1})$ and asymmetry measures $ASYN_Y(\pi, v_t, \omega_{t-1})$ for $\pi = 0.50, 0.40, \ldots, 0.10$. Table 2 reports the means of $N_Y(\pi, A, B)$, while Table 3 contains summary statistics for the distribution of $ASYN_Y(\pi, A^+, B)$, where A and B are defined above. To save space, Table 3 only reports results for $\pi = 0.50$ and 0.10. Results for other values of π are available on request.

From Table 2 it is seen that large shocks occurring in a recession are absorbed faster than small shocks, which in turn are absorbed faster than medium-sized shocks. By contrast, this ordering of average absorption times is reversed during expansions. Absorption of small and medium-sized shocks during recessions occurs much slower than in expansions, whereas absorption of large shocks occurs at roughly the same speed. The mean asymmetry measures in Table 3 show that in both regimes negative shocks are absorbed faster when they are small and slower when they are medium-sized or large. The fact that the overall mean of the asymmetry measure is positive is caused by the weighting scheme that we use, which gives (much) larger weight to small shocks. Based on the conservative standard error $\sigma_{ASYN_Y(\pi, A^+, B)}/\sqrt{n_A}$ the hypothesis that the mean of $ASYN_Y(\pi, A^+, B)$ does not differ significantly from zero can be rejected only for large shocks at $\pi = 0.50$ and 0.10, and for small and medium shocks occurring during expansions at $\pi = 0.10$. The values of α^* for HDR-regions, symmetric intervals around the mean and equal quantile intervals confirm that the distribution of $ASYN_Y(\pi, A^+, B)$ is most asymmetric for large shocks, particularly those occurring during expansions.

Figures 2 and 3 show distributions of $N_Y(\pi, A, B)$ for $\pi = 0.50$ and 0.10, respectively, with A the set of all, negative or positive shocks. Distributions of $ASYN_Y(\pi, A^+, B)$ are shown in Figures 4 and 5 for the same values of π, with A the set of all, small, medium or large shocks. From the last two figures, it is seen that even though the distribution can have all kinds of highly asymmetric shapes, quite a large probability is attached to 0, especially for small and medium shocks. This explains the large values for α^* based on the HDR (and, to a lesser extent, the equal-quantile interval), as reported in Table 3.

Based on these results we conclude that this current-depth-of-recession model generates data that seems to have only a modest degree of nonlinearity. Whether this is due to the model or the data can perhaps be learned from looking at the properties of an alternative, more elaborate, nonlinear model for the same data. This is done in the next section.

3.4 Example B: the floor-and-ceiling model

As a second example, we consider the floor-and-ceiling model of Pesaran and Potter (1997), which extends the current-depth-of-recession model discussed above by including an 'overheating variable' as additional regressor in a linear autoregressive model for the growth rate. Define the indicators F_t, C_t for the floor and ceiling regimes recursively as

$$F_t = \begin{cases} I[\Delta Y_t < r_F] & \text{if } F_{t-1} = 0, \\ I[CDR_{t-1} + \Delta Y_t < r_F] & \text{if } F_{t-1} = 1, \end{cases} \tag{17}$$

$$C_t = I[F_t = 0] I[\Delta Y_t > r_C] I[\Delta Y_{t-1} > r_C], \tag{18}$$

where the current-depth-of-recession variable now is defined as

$$CDR_t = \begin{cases} (\Delta Y_t - r_F) F_t & \text{if } F_{t-1} = 0, \\ (CDR_{t-1} - \Delta Y_t) F_t & \text{if } F_{t-1} = 1, \end{cases} \tag{19}$$

and the overheating variable is given by

$$OH_t = C_t (OH_{t-1} + \Delta Y_t - r_c). \tag{20}$$

Note that (19) with (17) is identical to (14) in case the floor threshold $r_F = 0$. The floor-and-ceiling model for output growth then is given by

$$\phi(L)\Delta Y_t = \phi_0 + \theta_1 CDR_{t-1} + \theta_2 OH_{t-1} + V_t, \tag{21}$$

where $\mathrm{E}[V_t|\Omega_{t-1}] = 0$ and the conditional variance of V_t is given by

$$\mathrm{E}[V_t^2|\Omega_{t-1}] \equiv H_t = \sigma_F^2 F_{t-1} + \sigma_{COR}^2 COR_{t-1} + \sigma_C^2 C_{t-1},$$

where the indicator for the corridor regime is defined as

$$COR_t = I[F_t + C_t = 0],$$

see Pesaran and Potter (1997) for an extensive discussion and motivation of this model. Following Pesaran and Potter (1997), we set $p = 2$ in (21) and estimate the model with iterative weighted least squares, using a grid search over the floor and ceiling thresholds r_F and r_C. Again we use quarterly observations on seasonally adjusted real US GNP, from 1947:1-1995:2. The parameter estimates are given by $\hat{\phi}_0 = 0.206$, $\hat{\phi}_1 = 0.441$, $\hat{\phi}_2 = 0.283$, $\hat{\theta}_1 = -0.540$, $\hat{\theta}_2 = -0.055$, $\hat{\sigma}_F = 1.337$, $\hat{\sigma}_{COR} = 0.890$, $\hat{\sigma}_C = 0.717$, $\hat{r}_F = -0.716$, and $\hat{r}_C = 0.531$. Similar estimates are obtained by Pesaran and Potter (1997) for US GDP over the sample 1954:1-1992:4. In the effective estimation sample, 24, 77 and 90 observations are located in the floor, corridor and ceiling regimes, respectively.

We compute impulse responses $GI_{\Delta Y}(n, v_t, \omega_{t-1})$ for all 191 histories in the sample, for values of the normalized shock equal to $v_t/\sqrt{h_t} = \pm 3, \pm 2.9, \ldots, \pm 0.1, 0$. GIs are computed for horizons $n = 0, 1, \ldots, N$ with $N = 20$ with $R = 10000$ replications.

Figure 6 shows distributions of impulse responses for the log level of GNP $GI_Y(n, A, B)$, where B is the set of all histories in a particular regime. In all three regimes, the final impulse response appears to be larger for positive shocks. This is confirmed by the distributions of the asymmetry measure $ASY_Y(n, V_t^+, B)$ shown in panels (j)-(l) of Figure 6. Table 4 contains summary statistics for these distributions at horizon $n = 20$, as well as for $ASY_Y(n, A^+, B)$, where A again is taken to be the set of small ($0 < V_t \leq 1$), medium ($1 < V_t \leq 2$) or large ($2 < V_t \leq 3$) shocks. The mean of $ASY_Y(n, V_t^+, B)$ is seen to be close to zero in all three cases, thus suggesting that on average shocks have symmetric effects. Distinguishing between different magnitudes of shocks shows that small negative shocks have larger effects than small positive ones and vice versa for medium and large shocks. Comparing the mean of $ASY_Y(n, A^+, B)$ with the conservative standard error $\sigma_{ASY_Y(\pi, A^+, B)}/\sqrt{n_A}$, it appears that the asymmetry is significant in the floor and corridor regimes for all magnitudes of shocks, and only for large shocks in the ceiling regime. The values of α^* reported in the final three rows for the different confidence regions suggest that the asymmetry is most pronounced for large shocks occurring in the floor and corridor regimes. This is in contrast with Pesaran and Potter (1997), who find that negative shocks are more persistent than positive ones on average. We do confirm their finding that shocks are more persistent in the corridor regime, although the difference with especially the ceiling regime is not all that large.

Again truncating the summations in (9) at $N = 20$ and using $GI_Y(N, v_t, \omega_{t-1})$ as an estimate of the final impulse response $GI_Y^{\infty}(v_t, \omega_{t-1})$, we compute π-absorption times $N_Y(\pi, v_t, \omega_{t-1})$ and asymmetry measures $ASYN_Y(\pi, v_t, \omega_{t-1})$ for $\pi = 0.50, 0.40, \ldots, 0.10$. Table 5 reports the means of $N_Y(\pi, A, B)$, while Table 6 contains summary statistics for the distribution of $ASYN_Y(\pi, A^+, B)$ for $\pi = 0.50$ and 0.10, where A and B are defined as above.

Comparing the columns headed 'A' in Table 5 shows that the ranking of the absorption speed in the different regimes depends on the value of π. For $\pi = 0.50$ and 0.40, shocks are absorbed fastest in the corridor regime, followed by the ceiling and floor regimes. For $\pi = 0.30$ and 0.20, absorption is still fastest in the corridor regime but now the absorption speed in the floor regime is higher than in the ceiling regime. For $\pi = 0.10$, absorption is fastest in the floor regime, followed by the corridor and ceiling regimes. Hence, one can conclude that absorption of shocks in the floor regime is slow initially, but accelerates during the second half of the 'lifetime of shocks'. Comparing the mean absorption speeds for the different subsets of shocks shows that this effect is present for all magnitudes of shocks, although it is more pronounced for small and large shocks.

The columns headed 'A' in Table 6 show that the mean absorption time of positive shocks is larger than that for negative shocks in the corridor and ceiling regimes, whereas the opposite holds in the floor regime. Focusing on the subsets of shocks, it is seen that positive small shocks are absorbed faster in the floor regime, and vice versa in the corridor and ceiling regimes. Negative medium-sized shocks are absorbed faster in the floor regime, and vice versa in the ceiling regime. Note that in the floor regime there is a 'reversal', in the sense that positive large shocks are absorbed faster for larger values of π, while they are absorbed slower for smaller values than π. A similar reversal occurs for medium-sized and large shocks in the corridor regime. In the ceiling regime, large positive shocks are absorbed faster for all values of π considered. Based on the standard error $\sigma_{ASYN_Y(\pi, A^+, B)} / \sqrt{n_A}$, the mean absorption time is different from zero for all shocks in the corridor regime and for large shocks occurring in the ceiling regime at $\pi = 0.50$, and for medium and large shocks in the floor regime and for small shocks in the corridor regime at $\pi = 0.10$.

Figures 7 and 8 show distributions of $N_Y(\pi, A, B)$ for $\pi = 0.50$ and 0.10, respectively. Distributions of $ASYN_Y(\pi, A^+, B)$ are shown in Figures 9 and 10. Comparing panels (j) and (k) in Figure 9 helps to understand the differences that occur in the values of α^* for large shocks in the floor and corridor regimes at $\pi = 0.50$. In both cases, the probability that $ASYN_Y(\pi, A^+, B) = 0$ is rather small, hence the small value of α^* based on the HDR. In the corridor regime, most probability mass is concentrated close to the mean of -3.13, which explains the small values of α^* based on the symmetric interval around the mean and the equal-quantile interval. By contrast, in the floor regime the probability that $ASYN_Y(\pi, A^+, B)$ is positive is quite large. Hence, when a symmetric interval around the mean of -0.71 is constructed, 0 will be included already for small confidence levels. A similar reasoning holds for the equal-quantile interval.

Upon comparing the two univariate nonlinear models for US GNP, while relying on the empirical results for the persistence and absorption of shocks, we conclude that both models perform equally good (or bad), in the sense that one model is not outperforming the other by extracting more nonlinearity (if there is any) from the data.

4 Absorption of shocks in multivariate models

The absorption rate can also be used to investigate the properties of multivariate nonlinear models. In this section, we first define the multivariate extension of the univariate absorption measure used so far. Next, we discuss how to measure common absorption, which we then illustrate for a trivariate STAR model.

4.1 Definition of absorption in multivariate models

Extending the concept of π-absorption times to multivariate models is fairly straightforward. Following Pesaran and Shin (1998), we restrict attention to the generalized impulse response of the effect of a shock in the j-th equation only, while integrating out the effects of shocks to the other equations. In this case we have

$$GI_Y(n, v_{jt}, \omega_{t-1}) = E[Y_{t+n}|V_{jt} = v_{jt}, \omega_{t-1}] - E[Y_{t+n}|\omega_{t-1}]. \tag{22}$$

The immediate effect of the shock is given by the impulse response at horizon $n = 0$, which is equal to $GI_Y(0, v_{jt}, \omega_{t-1}) = \mathrm{E}[V_t|V_{jt} = v_{jt}, \omega_{t-1}]$. In case V_t is conditionally normally distributed with covariance matrix h_t, that is, conditional upon the history ω_{t-1}, it can be shown that

$$\mathrm{E}[V_t|V_{jt} = v_{jt}, \omega_{t-1}] = (h_{t,1j}, h_{t,2j}, \ldots, h_{t,kj})' h_{t,jj}^{-1} v_{jt} = h_t e_j h_{t,jj}^{-1} v_{jt},$$

where e_j is a $(k \times 1)$ vector with unity as its j-th element and zeros elsewhere, see Pesaran and Shin (1998). Thus, the indicator function $I_{Y_i}(\pi, n, v_{jt}, \omega_{t-1})$ now should be defined as

$$I_{Y_i}(\pi, n, v_{jt}, \omega_{t-1}) = \\ I[|GI_{Y_i}(n, v_{jt}, \omega_{t-1}) - GI^{\infty}_{Y_i}(v_{jt}, \omega_{t-1})| \leq \pi |h_{t,ij} h^{-1}_{t,jj} v_{jt} - GI^{\infty}_{Y}(v_{jt}, \omega_{t-1})|].$$

The 'π-life' or 'π-absorption time' of v_{jt} for Y_i then can be defined as

$$N_{Y_i}(\pi, v_{jt}, \omega_{t-1}) = \sum_{n=0}^{\infty} \left(1 - \prod_{m=n}^{\infty} I_{Y_i}(\pi, m, v_{jt}, \omega_{t-1}) \right). \tag{23}$$

As in the univariate case, $N_{Y_i}(\pi, v_{jt}, \omega_{t-1})$ can be regarded as a realization of the random variable

$$N_{Y_i}(\pi, V_{jt}, \Omega_{t-1}) = \sum_{n=0}^{\infty} \left(1 - \prod_{m=n}^{\infty} I_{Y_i}(\pi, m, V_{jt}, \Omega_{t-1}) \right), \tag{24}$$

where the random indicator function $I_{Y_i}(\pi, m, V_{jt}, \Omega_{t-1})$ is obviously defined. Similarly, one can define the asymmetry measure

$$ASYN_{Y_i}(\pi, V^+_{jt}, \Omega_{t-1}) = N_{Y_i}(\pi, V^+_{jt}, \Omega_{t-1}) - N_{Y_i}(\pi, -V^+_{jt}, \Omega_{t-1}), \tag{25}$$

where $V^+_{jt} = \{V_{jt} | V_{jt} > 0\}$, which can be used to assess whether positive and negative shocks are absorbed at different speeds.

4.2 Measuring common absorption

In multivariate models, an additional question of interest is whether shocks are absorbed at the same speed by different variables in the system. Define the random variable $CN_{Y_i,Y_l}(\pi, V_{jt}, \Omega_{t-1})$ as the difference of the π-absorption times of Y_i and Y_l, that is

$$CN_{Y_i,Y_l}(\pi, V_{jt}, \Omega_{t-1}) = N_{Y_i}(\pi, V_{jt}, \Omega_{t-1}) - N_{Y_l}(\pi, V_{jt}, \Omega_{t-1}). \tag{26}$$

If shocks V_{jt} are absorbed at the same speed by Y_i and Y_l on average, $CN_{Y_i,Y_l}(\pi, V_{jt}, \Omega_{t-1})$ should have a distribution with mean equal to zero.

Alternatively, one may ask whether there exists a linear combination $\beta' Y$, for certain $(k \times 1)$ vector β, for which the effects of shocks die out faster than for the component series Y_i, $i = 1, \ldots, k$. If so, this linear combination can be viewed as a more stable variable as shocks last shorter. From the definition of the GI given in (4) and elementary properties of the conditional expectations operator it follows that

$$GI_{\beta' Y}(n, V_t, \Omega_{t-1}) = \beta' GI_Y(n, V_t, \Omega_{t-1}). \tag{27}$$

Hence, the GI for a linear combination of the elements in Y_t can be obtained directly as the same linear combination of the GI of Y_t. Note that such a simple relationship does not exist between the π-absorption times of a linear combination and the absorption times of the elements of Y_t. That is, in general

$$N_{\beta' Y}(\pi, V_{jt}, \Omega_{t-1}) \neq \beta' N_Y(\pi, V_{jt}, \Omega_{t-1}),$$

where $N_Y(\pi, V_{jt}, \Omega_{t-1}) = (N_{Y_1}(\pi, V_{jt}, \Omega_{t-1}), \dots, N_{Y_k}(\pi, V_{jt}, \Omega_{t-1}))'$. It is however straightforward to define the π-absorption time for $\beta' Y_t$ as

$$N_{\beta' Y}(\pi, V_{jt}, \Omega_{t-1}) = \sum_{n=0}^{\infty} \left(1 - \prod_{m=n}^{\infty} I_{\beta' Y}(\pi, m, V_{jt}, \Omega_{t-1}) \right),$$

where the indicator function $I_{\beta' Y_t}(\pi, m, V_{jt}, \Omega_{t-1})$ is defined as

$$\begin{aligned} &I_{\beta' Y}(\pi, n, v_{jt}, \omega_{t-1}) = \\ &\qquad I[|\beta'(GI_Y(n, v_{jt}, \omega_{t-1}) - GI_Y^{\infty}(v_{jt}, \omega_{t-1}))| < \pi |\beta'(h_t e_j h_{t,jj}^{-1} v_{jt} - GI_Y^{\infty}(v_t, \omega_{t-1}))|]. \end{aligned}$$

From this definition it should be clear that $N_{\beta' Y}(\pi, V_{jt}, \Omega_{t-1}) \neq \beta' N_Y(\pi, V_{jt}, \Omega_{t-1})$, as $|\beta' x| \neq \beta' |x|$ in general.

Consequently, an alternative common absorption measure $CAN_{Y_i, Y_l}(\pi, V_{jt}, \Omega_{t-1})$ can be defined as the difference of the π-absorption times of Y_i and $\beta' Y$, that is

$$CAN_{Y_i, \beta' Y}(\pi, V_{jt}, \Omega_{t-1}) = N_{Y_i}(\pi, V_{jt}, \Omega_{t-1}) - N_{\beta' Y}(\pi, V_{jt}, \Omega_{t-1}), \qquad i = 1, \dots, k. \quad (28)$$

If shocks V_{jt} are not absorbed at a different speed by the linear combination $\beta' Y$ than by the individual series Y_i on average, $CAN_{Y_i, \beta' Y}(\pi, V_{jt}, \Omega_{t-1})$ should have a distribution with mean equal to zero for all $i = 1, \dots, k$.

4.3 Example C: A STAR model for income, consumption and investment

For illustration, we consider the smooth transition vector error-correction model [STVECM] for US income, consumption and investment of Anderson and Vahid (1998). The data are quarterly, covering the period 1951:1-1992:4. Let $Y_t = (X_t, C_t, I_t)'$ denote the vector consisting of log transformed per-capita income, consumption and investment, and $Z_t = (X_t - C_t, X_t - I_t)'$ the vector consisting of the 'great ratios'. A STVECM then is given by

$$\begin{aligned} \Delta Y_t = \Phi_0 + \Xi Z_{t-1} + \Phi_1 \Delta Y_{t-1} + \cdots + \Phi_p \Delta Y_{t-p} \\ + (\Theta_0 + \Psi Z_{t-1} + \Theta_1 \Delta Y_{t-1} + \cdots + \Theta_p \Delta Y_{t-p}) F(S_t; \gamma, c) + V_t, \quad (29) \end{aligned}$$

where Φ_i and Θ_i, $i = 1, \dots, p$, are (3×3) matrices, Ξ and Ψ are (3×2) matrices, and $F(S_t; \gamma, c)$ is the logistic function

$$F(S_t; \gamma, c) = (1 + \exp\{-\gamma(S_t - c)\})^{-1}, \qquad \gamma > 0. \tag{30}$$

The parameter c in (30) can be interpreted as the threshold between the two regimes corresponding to $F(S_t; \gamma, c) = 0$ and $F(S_t; \gamma, c) = 1$, in the sense that the logistic function changes monotonically from 0 to 1 as the transition variable S_t increases, while $F(c; \gamma, c) = 0.5$. The parameter γ determines the smoothness of the change in the value of the logistic function and, thus, the smoothness of the transition from one regime to the other.

Based on a set of linearity tests, Anderson and Vahid (1998) select the growth rate in investment lagged one quarter as the transition variable, that is, $S_t = \Delta I_{t-1}$. Furthermore, they consider a model with so-called common nonlinearity. In general, the k-dimensional time series Y_t is said to contain s common nonlinear components if there exist $k - s$ linear combinations $\alpha_i' Y_t$, $i = 1, \dots, k - s$, whose conditional expectations are linear in the past of Y_t. For example, in the STVECM in (29), the existence of two common nonlinear components means that there exists a (3×1) vector α such that

$$\alpha'(\Theta_0 + \Psi Z_{t-1} + \Theta_1 \Delta Y_{t-1} + \cdots + \Theta_p \Delta Y_{t-p}) F(S_t; \gamma, c) = 0, \tag{31}$$

for all Z_{t-1}, $\Delta Y_{t-1}, \dots, \Delta Y_{t-p}$ and S_t. Anderson and Vahid (1998) develop test statistics for the existence of common STAR-type nonlinearity based upon canonical correlations.

Anderson and Vahid (1998) find evidence for a single common nonlinear component in the STVECM for income, consumption and investment. This implies that (29) can be rewritten as

$$\begin{aligned} \Delta Y_t = {} & \Phi_0 + \Xi Z_{t-1} + \Phi_1 \Delta Y_{t-1} + \cdots + \Phi_p \Delta Y_{t-p} \\ & + \alpha^*(\theta_0 + \psi Z_{t-1} + \theta_1' \Delta Y_{t-1} + \cdots + \theta_p' \Delta Y_{t-p}) F(S_t; \gamma, c) + V_t, \end{aligned} \tag{32}$$

where α^* and θ_i, $i = 1, \dots, p$, are (3×1) vectors, θ_0 is a scalar, ψ is a (2×1) vector.

The STVECM with common nonlinearity (32) is estimated with nonlinear least squares using the complete sample period, where $p = 1$ and some additional parameter constraints are imposed to obtain a parsimonious model (that is, $\Xi_{21} = 0$, $\Phi_{1,13} = 0$, $\Phi_{1,22} = 0$, $\Phi_{1,32} = 0$, $\Phi_{1,33} = 0$, $\psi_1 = 0$, $\psi_2 = -\Xi_{22}$, $\theta_{1,1} = -\Phi_{1,21}$, $\theta_{1,2} = 0$, and $\theta_{1,3} = -\Phi_{1,23}$, where $A_{l,ij}$ denotes the (i,j)-th element of the matrix A_l). This leaves 18 parameters to be estimated in total. For the parameters in the transition function (30) with $S_t = \Delta I_{t-1}$ we obtain estimates $\hat{\gamma} = 5.11$ and $\hat{c} = -0.73$. This implies that for 51 of the 176 observations in the effective estimation sample, the value of the transition function is smaller than 0.5, while the transition of $F(\Delta I_{t-1}; \gamma, c)$ is rather smooth and occurs as ΔI_{t-1} changes from about -4 to 2 percent.

We compute generalized impulse responses $GI_{\Delta Y_i}(n, v_{jt}, \omega_{t-1})$ as given in (22) for all 176 histories in the sample, for values of the normalized shock equal to $v_{jt}/\sqrt{h_t} = \pm 3, \pm 2.8, \ldots, \pm 0.2, 0$. GIs are computed for horizons $n = 0, 1, \ldots, N$ with $N = 40$ with $R = 2500$ replications. We also obtain impulse responses for the great ratios, according to (27) with $\beta = (1, -1, 0)$ and $(1, 0, -1)$, respectively. Figures 11-13 show distributions of impulse responses $GI_{Y_i}(n, V_{jt}, B)$ at horizons $n = 0, 4, 8, 20$ and 40 for the log levels of income, consumption and investment for shocks occurring in either of the three variables. The set B consists of all histories or those histories for which the value of the transition function $F(\Delta I_{t-1}; \gamma, c)$ is either larger or smaller than 0.5. The latter two are referred to as recession and expansion, respectively. Clearly, shocks have persistent effects on the individual variables in the system. However, shocks are transient for the great ratios, as the distributions of their impulse responses quickly collapse to a spike at zero as the horizon n increases. Therefore these results are not shown here. There appears to be little asymmetry in the GI for positive and negative shocks, as the distributions in Figures 11-13 seem quite symmetric.

Figures 14 and 15 show distributions of absorption times $N_{Y_i}(\pi, V_{jt}, \Omega_{t-1})$ defined in (24) for $\pi = 0.50$ and 0.10. Tables 7 and 8 contain means of the absorption times $N_{Y_i}(\pi, A, B)$ and asymmetry measure $ASYN_{Y_i}(\pi, A^+, B)$ for choices of A and B defined earlier. The mean absorption times in Table 7 suggest that on average shocks are absorbed at approximately the same speed in recessions and expansions. The mean asymmetry measures in Table 8 however suggest that absorption can be very asymmetric and, furthermore, that the asymmetry can be very different depending on the regime. This holds especially for medium and large shocks, which show positive asymmetry during recessions and negative asymmetry during expansions. Based on the conservative standard error $\sigma_{ASYN_{Y_i}(\pi, A^+, B)}/\sqrt{n_A}$, the asymmetry is significant in a limited number of cases only.

Shocks in income are absorbed fastest by income, followed by investment, followed by consumption. Shocks in consumption are absorbed fastest by investment, followed by consumption, followed by income. Finally, shocks in investment are absorbed fastest by investment, followed by income, followed by consumption. The differences in absorption times are largest for shocks to income, and smallest for shocks to investment.

Note that the absorption times for the great ratios are, generally, not smaller than the absorption times for the individual variables. In fact, in most cases they are larger. Also, the absorption time of $X - C$ resembles that of I, while the absorption time of $X - I$ resembles that of C. This effect is observed in particular for shocks to income. This finding might be explained by the fact that $\exp(X) \approx \exp(C) + \exp(I)$.

In quite a few cases the distribution of absorption times is bi-modal - see, for example, panel (b) of Figure 14 (absorption of shocks to income by consumption). This also leads to bi-modality in the distribution of the common absorption measure $CN_{Y_i,Y_l}(\pi, V_{jt}, \Omega_{t-1})$ as defined in (26). The latter distributions are shown in Figures 16 and 17.

Tables 9, 10 and 11 contain summary statistics for the distribution of $CN_{Y_i,Y_l}(\pi, A, B)$ in case of a shock to income, consumption and investment, respectively. As expected, common absorption is never rejected for shocks to investment (except for medium-sized shocks during expansions for $\pi = 0.50$, $Y_i = X$ and $Y_l = I$), more so for shocks to consumption, and quite often for shocks to income, especially for $\pi = 0.10$. Hence, assuming the validity of the nonlinear model, it seems that most nonlinearity in this trivariate system is due to the income variable.

5 Concluding remarks

In this paper we proposed a new tool which can be used to examine the properties of univariate and multivariate nonlinear models. This tool, which we called the absorption rate, can be viewed as complementary to the familiar impulse response function, as both consider certain aspects of the propagation of shocks. The absorption rate can be used to examine whether the speed of the propagation of different types of shocks, such as large and small shocks, positive and negative shocks, and shocks in various regimes, follows the same or different patterns. In multivariate models, the absorption rate can also reveal whether the effects of shocks last longer on certain variables than on others or not. Hence, the absorption rate can help to interpret a possibly complicated nonlinear model, with potentially a large number of parameters.

In a sense, the absorption rate is informative for the degree of nonlinearity a particular model is picking up from the data. If all kinds of shocks have similar effects on the future path of a time series variable, the nonlinear model can be said to have linear properties, even though parameters for the nonlinear component are highly significant. Such a finding can imply that, either, there is not enough nonlinearity in the data, or the model is not capturing the nonlinear features adequately.

The above leads to the suggestion that the absorption rate can provide useful prior information as to how successful a particular nonlinear model will be when it comes to out-of-sample forecasting. With respect to our illustrations on US GNP, we found only little evidence for asymmetry in the absorption rate of different types of shocks in the different regimes in the current-depth-of-recession model and the floor-and-ceiling model. Hence, it may not come as a surprise that linear models tend to beat these nonlinear models in terms of forecasting.

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Table 1: Asymmetry measures for impulse responses in current-depth-of-recession model

	Unconditional				Recession				Expansion			
	A	S	M	L	A	S	M	L	A	S	M	L
Mean	0.01	-0.34^*	0.61^*	2.25^*	0.02	-0.50^*	0.93^*	3.19^*	0.01	-0.28^*	0.50^*	1.92^*
St.dev.	0.73	0.26	0.53	0.90	1.02	0.34	0.65	0.88	0.60	0.19	0.43	0.64
Skewness	2.11	-0.66	1.07	0.66	1.80	0.05	0.74	0.13	2.07	-0.23	0.71	0.05
HDR_{α^*}	0.61	0.19	0.48	0.00	0.57	0.15	0.21	0.00	0.57	0.16	0.55	0.00
S_{α^*}	0.98	0.16	0.22	0.02	0.99	0.17	0.15	0.00	0.98	0.16	0.26	0.00
Q_{α^*}	0.81	0.15	0.20	0.00	0.82	0.17	0.08	0.00	0.77	0.13	0.23	0.00

Summary statistics for asymmetry measure $ASY_Y(\pi, A^+, B)$ in current-depth-of-recession model. Entries in the row labelled Mean which are larger than two times $\sigma_{ASY_Y(\pi,A^+,B)}/\sqrt{n_A}$ are marked with an asterisk, where $\sigma_{ASY_Y(\pi,A^+,B)}$ is the standard deviation of $ASY_Y(\pi, A^+, B)$ and n_A is the number of shocks v_t for which $ASY_Y(\pi, v_t, \omega_{t-1})$ is computed. Entries in rows labelled Z_{α^*} represent the minimum value of $\alpha \in (0,1)$ such that 0 would not be included in the relevant confidence region Z_α, $Z = HDR$, S and Q. The different sets of shocks are defined as A(ll)= $\{V_t\}$, S(mall)= $\{V_t | 1 \geq |V_t/\sqrt{H_t}| > 0\}$, M(edium)= $\{V_t | 2 \geq |V_t/\sqrt{H_t}| > 1\}$, L(arge)= $\{V_t | 3 \geq |V_t/\sqrt{H_t}| > 2\}$.

Table 2: Absorption times in current-depth-of-recession model

	Unconditional				Recession				Expansion			
π	A	S	M	L	A	S	M	L	A	S	M	L
0.50	3.05	3.03	3.00	3.67	4.27	4.13	4.74	3.70	2.61	2.64	2.39	3.66
0.40	3.46	3.48	3.33	3.96	4.74	4.63	5.15	4.05	3.00	3.07	2.68	3.93
0.30	4.01	4.02	3.89	4.53	5.26	5.11	5.76	4.54	3.56	3.63	3.23	4.53
0.20	4.69	4.73	4.54	5.17	5.96	5.83	6.45	5.12	4.24	4.33	3.87	5.18
0.10	6.01	6.05	5.88	6.26	7.11	6.93	7.69	6.28	5.62	5.73	5.25	6.25

Mean of $N_Y(\pi, A, B)$ in current-depth-of-recession model. The different sets of shocks are defined as A(ll)= $\{V_t\}$, S(mall)= $\{V_t | 1 \geq |V_t/\sqrt{H_t}| > 0\}$, M(edium)= $\{V_t | 2 \geq |V_t/\sqrt{H_t}| > 1\}$, L(arge)= $\{V_t | 3 \geq |V_t/\sqrt{H_t}| > 2\}$.

Table 3: Asymmetry measures for absorption times in current-depth-of-recession model

	Unconditional				Recession				Expansion			
	A	S	M	L	A	S	M	L	A	S	M	L
$\pi = 0.50$												
Mean	0.35	1.11	−1.12	−3.16*	0.07	0.90	−1.69	−2.72*	0.45	1.19	−0.92	−3.32*
St.dev.	2.89	2.51	2.87	2.87	3.24	2.73	3.66	2.02	2.75	2.42	2.50	3.10
Skewness	−0.06	1.04	−1.20	−0.12	−0.23	0.49	−0.48	1.31	0.08	1.35	−1.65	−0.17
HDR_{α^*}	1.00	1.00	1.00	0.20	1.00	1.00	1.00	0.10	1.00	1.00	1.00	0.30
S_{α^*}	1.00	0.85	0.98	0.36	1.00	0.86	0.94	0.19	1.00	0.91	1.00	0.42
Q_{α^*}	1.00	1.00	1.00	0.56	1.00	0.93	1.00	0.34	1.00	1.00	1.00	0.64
$\pi = 0.10$												
Mean	0.31	1.45	−2.07	−3.93*	0.23	0.71	−0.64	−2.43*	0.33	1.72*	−2.58*	−4.47*
St.dev.	3.26	2.61	3.03	3.05	2.88	2.58	3.23	2.62	3.39	2.57	2.79	3.01
Skewness	−0.25	0.51	−0.44	−0.26	−0.32	−0.10	−0.35	0.24	−0.23	0.78	−0.80	−0.34
HDR_{α^*}	1.00	0.82	1.00	0.19	1.00	1.00	1.00	0.49	1.00	0.51	1.00	0.02
S_{α^*}	1.00	0.76	0.63	0.23	1.00	0.81	0.89	0.62	1.00	0.62	0.46	0.19
Q_{α^*}	1.00	0.83	0.82	0.33	1.00	0.99	1.00	0.68	1.00	0.77	0.68	0.21

Summary statistics for asymmetry measure $ASYN_Y(\pi, A^+, B)$ in current-depth-of-recession model. Entries in rows labelled Mean which are larger than two times $\sigma_{ASYN_Y(\pi,A^+,B)}/\sqrt{n_A}$ are marked with an asterisk, where $\sigma_{ASYN_Y(\pi,A^+,B)}$ is the standard deviation of $ASYN_Y(\pi, A^+, B)$ and n_A is the number of shocks v_t for which $ASYN_Y(\pi, v_t, \omega_{t-1})$ is computed. Entries in rows labelled Z_{α^*} represent the minimum value of $\alpha \in (0,1)$ such that 0 would not be included in the relevant confidence region Z_α, $Z = HDR$, S and Q. The different sets of shocks are defined as A(ll)= $\{V_t\}$, S(mall)= $\{V_t | 1 \geq |V_t/\sqrt{H_t}| > 0\}$, M(edium)= $\{V_t | 2 \geq |V_t/\sqrt{H_t}| > 1\}$, L(arge)= $\{V_t | 3 \geq |V_t/\sqrt{H_t}| > 2\}$.

Table 4: Asymmetry measures for impulse responses in floor-and-ceiling model

	floor regime				corridor regime				ceiling regime			
	A	S	M	L	A	S	M	L	A	S	M	L
Mean	0.02	−0.65*	1.19*	4.04*	0.01	−0.47*	0.83*	3.10*	0.00	−0.14	0.22	1.12*
St.dev.	1.31	0.46	0.87	1.14	1.06	0.34	1.08	0.75	0.45	0.27	0.39	0.89
Skewness	1.76	−0.27	0.74	0.29	1.76	0.22	0.13	−0.61	2.16	0.32	1.20	0.11
HDR_{α^*}	0.63	0.18	0.31	0.00	0.48	0.12	0.74	0.02	0.67	0.42	0.98	0.57
S_{α^*}	0.98	0.17	0.18	0.00	1.00	0.13	0.67	0.01	1.00	0.55	0.54	0.20
Q_{α^*}	0.85	0.17	0.13	0.00	0.65	0.10	0.75	0.02	0.86	0.55	0.68	0.19

Summary statistics for asymmetry measure $ASY_Y(\pi, A^+, B)$ in floor-and-ceiling model. Entries in the row labelled Mean which are larger than two times $\sigma_{ASY_Y(\pi,A^+,B)}/\sqrt{n_A}$ are marked with an asterisk, where $\sigma_{ASY_Y(\pi,A^+,B)}$ is the standard deviation of $ASY_Y(\pi, A^+, B)$ and n_A is the number of shocks v_t for which $ASY_Y(\pi, v_t, \omega_{t-1})$ is computed. Entries in rows labelled Z_{α^*} represent the minimum value of $\alpha \in (0,1)$ such that 0 would not be included in the relevant confidence region Z_α, $Z = HDR$, S and Q. The different sets of shocks are defined as A(ll)= $\{V_t\}$, S(mall)= $\{V_t | 1 \geq |V_t/\sqrt{H_t}| > 0\}$, M(edium)= $\{V_t | 2 \geq |V_t/\sqrt{H_t}| > 1\}$, L(arge)= $\{V_t | 3 \geq |V_t/\sqrt{H_t}| > 2\}$.

Table 5: Absorption times in floor-and-ceiling model

	floor regime				corridor regime				ceiling regime			
π	A	S	M	L	A	S	M	L	A	S	M	L
0.50	4.41	4.22	5.06	3.56	3.35	3.61	2.69	3.26	3.91	4.20	3.31	2.89
0.40	4.94	4.73	5.61	4.12	3.90	4.14	3.30	3.62	4.70	4.85	4.44	3.76
0.30	5.49	5.24	6.24	4.74	4.53	4.75	4.00	4.05	5.55	5.58	5.60	4.68
0.20	6.26	6.01	6.95	6.00	5.83	5.81	5.96	5.28	6.60	6.47	7.02	6.26
0.10	7.66	7.46	8.23	7.37	8.30	8.45	8.04	7.52	8.69	8.77	8.61	7.91

Mean of $N_Y(\pi, A, B)$ in floor-and-ceiling model. The different sets of shocks are defined as A(ll)= $\{V_t\}$, S(mall)= $\{V_t | 1 \geq |V_t/\sqrt{H_t}| > 0\}$, M(edium)= $\{V_t | 2 \geq |V_t/\sqrt{H_t}| > 1\}$, L(arge)= $\{V_t | 3 \geq |V_t/\sqrt{H_t}| > 2\}$.

Table 6: Asymmetry measures for absorption times in floor-and-ceiling model

	floor regime				corridor regime				ceiling regime			
	A	S	M	L	A	S	M	L	A	S	M	L
$\pi = 0.50$												
Mean	−0.44	−1.16	1.50	−0.71	1.56*	3.01*	−1.58*	−3.13*	0.41	1.60	−2.30	−2.48*
St.dev.	3.62	3.24	3.94	2.99	3.66	3.20	2.19	1.54	4.39	4.11	3.86	2.73
Skewness	−0.39	−0.99	−0.13	0.74	0.65	1.32	0.30	−3.79	−0.01	−0.01	−0.90	0.60
HDR_{α^*}	0.84	0.79	0.40	0.00	0.57	0.19	0.53	0.04	1.00	0.68	1.00	0.59
S_{α^*}	1.00	0.64	0.40	0.97	0.74	0.31	0.59	0.04	1.00	0.73	0.78	0.51
Q_{α^*}	1.00	0.83	0.54	0.83	0.87	0.47	0.65	0.04	1.00	0.83	0.97	0.71
$\pi = 0.10$												
Mean	−0.75	−2.51	3.43*	2.26*	1.62*	2.29*	0.03	0.32	1.13	2.40	−1.79	−1.51
St.dev.	5.07	4.40	4.23	3.01	3.37	3.57	2.12	2.67	5.08	5.19	3.43	2.81
Skewness	−0.01	−0.01	−0.40	−0.05	1.31	1.26	0.85	−2.19	0.40	0.24	−0.63	0.67
HDR_{α^*}	1.00	1.00	0.41	0.09	1.00	1.00	1.00	0.31	1.00	1.00	1.00	0.51
S_{α^*}	0.89	0.56	0.44	0.26	0.73	0.71	1.00	1.00	0.84	0.70	0.69	0.51
Q_{α^*}	1.00	0.65	0.34	0.16	1.00	0.92	1.00	0.62	1.00	0.76	0.82	0.55

Summary statistics for asymmetry measure $ASYN_Y(\pi, A^+, B)$ in floor-and-ceiling model. Entries in rows labelled Mean which are larger than two times $\sigma_{ASYN_Y(\pi,A^+,B)}/\sqrt{n_A}$ are marked with an asterisk, where $\sigma_{ASYN_Y(\pi,A^+,B)}$ is the standard deviation of $ASYN_Y(\pi, A^+, B)$ and n_A is the number of shocks v_t for which $ASYN_Y(\pi, v_t, \omega_{t-1})$ is computed. Entries in rows labelled Z_{α^*} represent the minimum value of $\alpha \in (0,1)$ such that 0 would not be included in the relevant confidence region Z_α, $Z = HDR$, S and Q. The different sets of shocks are defined as A(ll)= $\{V_t\}$, S(mall)= $\{V_t | 1 \geq |V_t/\sqrt{H_t}| > 0\}$, M(edium)= $\{V_t | 2 \geq |V_t/\sqrt{H_t}| > 1\}$, L(arge)= $\{V_t | 3 \geq |V_t/\sqrt{H_t}| > 2\}$.

Table 7: Absorption times in STVECM for income, consumption and investment

Y_i	π	Unconditional				Recession				Expansion			
		A	S	M	L	A	S	M	L	A	S	M	L
Shock to income													
X	0.50	9.00	9.07	8.84	8.63	9.04	9.16	8.79	8.47	8.98	9.03	8.86	8.70
	0.10	12.97	13.21	12.40	12.22	13.35	13.60	12.79	12.54	12.81	13.05	12.24	12.08
C	0.50	16.26	14.68	20.23	19.06	14.82	12.85	19.80	17.94	16.87	15.45	20.41	19.53
	0.10	29.11	27.95	31.98	31.39	28.42	26.94	32.07	31.28	29.40	28.38	31.94	31.43
I	0.50	8.90	8.98	8.72	8.55	8.91	9.00	8.73	8.49	8.90	8.98	8.72	8.57
	0.10	20.56	20.43	20.90	20.72	20.40	20.25	20.79	20.52	20.63	20.50	20.95	20.80
$X-C$	0.50	8.80	8.77	8.88	8.78	8.74	8.71	8.82	8.78	8.82	8.79	8.91	8.78
	0.10	15.98	16.05	15.78	16.02	16.42	16.47	16.26	16.50	15.79	15.87	15.58	15.82
$X-I$	0.50	11.69	12.07	10.77	10.76	10.75	11.29	9.46	9.39	12.08	12.40	11.32	11.34
	0.10	26.38	26.36	26.40	26.47	26.48	26.46	26.53	26.58	26.34	26.33	26.35	26.42
Shock to consumption													
X	0.50	15.23	14.91	16.13	15.08	14.31	14.09	14.98	13.85	15.61	15.25	16.61	15.60
	0.10	22.89	22.76	23.30	22.55	21.59	21.65	21.57	20.61	23.43	23.22	24.03	23.37
C	0.50	12.21	12.36	11.93	11.15	11.24	11.43	10.86	10.21	12.62	12.75	12.38	11.55
	0.10	19.03	19.70	17.63	15.94	17.54	18.32	15.80	14.57	19.65	20.27	18.39	16.52
I	0.50	10.55	10.62	10.40	10.24	10.48	10.54	10.35	10.18	10.58	10.66	10.43	10.27
	0.10	15.47	15.78	14.73	14.45	14.63	14.95	13.86	13.66	15.82	16.13	15.10	14.79
$X-C$	0.50	16.91	15.67	19.89	20.12	16.13	15.03	18.76	19.01	17.24	15.94	20.37	20.58
	0.10	27.03	26.13	29.21	29.00	27.14	26.33	29.10	28.94	26.98	26.05	29.26	29.03
$X-I$	0.50	9.85	9.89	9.77	9.65	9.78	9.83	9.68	9.59	9.88	9.92	9.81	9.67
	0.10	14.08	14.46	13.18	13.04	13.99	14.27	13.35	13.18	14.12	14.55	13.10	12.98
Shock to investment													
X	0.50	9.46	9.77	8.79	8.15	9.40	9.85	8.38	7.81	9.48	9.74	8.96	8.29
	0.10	14.56	15.18	13.26	11.77	14.28	14.99	12.65	11.86	14.68	15.26	13.51	11.73
C	0.50	9.41	9.62	9.09	7.51	8.90	9.33	8.03	6.65	9.62	9.75	9.53	7.87
	0.10	15.10	15.51	14.39	12.08	14.08	14.58	13.07	11.59	15.53	15.90	14.94	12.29
I	0.50	8.44	8.51	8.32	7.82	8.28	8.41	8.04	7.54	8.51	8.56	8.44	7.94
	0.10	12.74	13.02	12.11	11.60	12.18	12.40	11.69	11.35	12.97	13.28	12.29	11.71
$X-C$	0.50	9.08	9.16	8.90	8.80	8.87	8.92	8.76	8.71	9.17	9.26	8.96	8.83
	0.10	14.51	15.26	12.79	11.97	14.48	15.23	12.71	12.44	14.52	15.27	12.82	11.78
$X-I$	0.50	8.16	8.21	8.09	7.65	8.05	8.17	7.84	7.39	8.20	8.23	8.19	7.76
	0.10	12.47	12.71	11.93	11.76	12.04	12.31	11.42	11.15	12.65	12.87	12.14	12.02

Mean of $N_{Y_i}(\pi, A, B)$ in STVECM for income, consumption and investment. The column headed Y_i contains the (linear combination of) variable(s) for which the impulse response is measured. The different sets of shocks are defined as A(ll)= $\{V_{jt}\}$, S(mall)= $\{V_{jt} | 1 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 0\}$, M(edium)= $\{V_{jt} | 2 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 1\}$, and L(arge)= $\{V_{jt} | 3 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 2\}$. The recession and expansion regimes contain all histories for which the value of the transition function $F(S_t; \hat{\gamma}, \hat{c})$ is smaller and larger than 0.5, respectively.

Table 8: Asymmetry measure for absorption times in STVECM for income, consumption and investment

		Unconditional				Recession				Expansion			
Y_i	π	A	S	M	L	A	S	M	L	A	S	M	L
Shock to income													
X	0.50	1.66*	2.38	−0.03	−0.39	2.20*	2.73	0.93*	0.82	1.44	2.24	−0.43	−0.90*
	0.10	1.75	2.63	−0.33	−0.81*	1.98	2.69	0.32	−0.06	1.65	2.61	−0.60	−1.12*
C	0.50	2.28	3.61	−0.40	−4.58	4.49	2.29	9.60	11.43*	1.36	4.17	−4.60	−11.29*
	0.10	0.02	0.41	−0.68	−2.53	0.75	−0.66	3.99	5.24	−0.28	0.86	−2.64	−5.79*
I	0.50	1.48	2.13	−0.06	−0.33	1.72*	2.13	0.74*	0.71	1.37	2.13	−0.40	−0.77*
	0.10	2.06*	2.87	0.14	−0.22	2.66*	3.40	0.89*	0.73	1.81*	2.65	−0.17	−0.62
$X-C$	0.50	0.27	0.40	−0.02	−0.11	0.25	0.24	0.25	0.41	0.29	0.47	−0.13	−0.33
	0.10	−1.80*	−2.42*	−0.35	−0.03	−1.90*	−2.13*	−1.34*	−1.41*	−1.76*	−2.54*	0.06	0.55
$X-I$	0.50	2.43	2.52	2.21	2.29	1.84	2.37	0.60	0.26	2.68	2.58	2.89	3.14
	0.10	−0.60	−0.83	−0.07	0.13	−0.60	−0.65	−0.48	−0.48	−0.61	−0.91	0.10	0.38
Shock to consumption													
X	0.50	0.81	1.50	−0.62	−2.57	2.90	2.31	4.26	4.76	−0.07	1.15	−2.66	−5.64
	0.10	1.86	2.48	0.51	−0.54	4.63*	4.01	5.96	7.37*	0.70	1.84	−1.77	−3.85
C	0.50	1.75	2.78	−0.70	−1.07	3.08	3.28	2.49	3.46*	1.19	2.57	−2.03	−2.98
	0.10	2.57	4.31	−1.48	−2.80	4.42	5.16	2.61	2.75	1.79	3.95	−3.19	−5.13
I	0.50	0.39	0.65	−0.22	−0.38	0.65	0.66	0.59	0.81	0.28	0.64	−0.56	−0.88*
	0.10	1.61	2.03	0.63	0.41	1.19	1.51	0.41	0.46	1.79	2.24	0.72	0.39
$X-C$	0.50	0.78	1.20	−0.22	−0.36	2.65	2.33	3.31	4.08	−0.01	0.72	−1.71	−2.22
	0.10	0.10	0.70	−1.28	−1.80	1.72	1.65	1.83	2.30	−0.57	0.31	−2.59	−3.51
$X-I$	0.50	0.20	0.33	−0.09	−0.22	0.45	0.43	0.48	0.65	0.09	0.28	−0.33	−0.58*
	0.10	0.56	0.72	0.20	0.11	0.54	0.62	0.34	0.40	0.57	0.76	0.14	−0.01
Shock to investment													
X	0.50	1.51	2.08	0.08	0.50	1.12	0.76	1.95*	2.24*	1.67	2.63	−0.71	−0.22
	0.10	2.56	3.76	−0.38	−0.04	1.95	1.86	2.32	1.25	2.81	4.55	−1.51	−0.58
C	0.50	1.96	2.74	−0.08	1.15	2.20	1.30	4.34*	4.56*	1.86	3.35	−1.93	−0.28
	0.10	2.59	3.56	0.15	1.01	2.26	1.36	4.59*	3.32*	2.73	4.48	−1.71	0.03
I	0.50	0.85	1.15	0.12	0.35	0.51	0.09	1.53*	1.57*	1.00	1.59	−0.48	−0.16
	0.10	1.50	1.95	0.37	0.68	0.83	0.67	1.25	1.05	1.78*	2.49	0.00	0.52
$X-C$	0.50	0.38	0.50	0.11	0.15	0.01	−0.15	0.39	0.57*	0.54	0.77	−0.00	−0.02
	0.10	1.00	1.71	−0.73	−0.54	1.44	1.89	0.49	−0.61	0.82	1.63	−1.24	−0.51
$X-I$	0.50	0.72	0.97	0.07	0.29	0.42	0.08	1.24*	1.24*	0.84	1.35	−0.42	−0.11
	0.10	0.81	0.74	0.94	1.33	1.08	1.09	1.04	1.05	0.70	0.59	0.90	1.44

Mean of $ASYN_{Y_i}(\pi, A^+, B)$ in STVECM for income, consumption and investment. The column headed Y_i contains the variable (or linear combination of variables) for which the impulse response is measured. Entries which are larger than two times $\sigma_{ASYN_{Y_i}(\pi,A^+,B)}/\sqrt{n_A}$ are marked with an asterisk, where $\sigma_{ASYN_{Y_i}(\pi,A^+,B)}$ is the standard deviation of $ASYN_{Y_i}(\pi, A^+, B)$ and n_A is the number of shocks v_{jt} for which $ASYN_{Y_i}(\pi, v_{jt}, \omega_{t-1})$ is computed. The different sets of shocks are defined as A(ll)= $\{V_{jt}\}$, S(mall)= $\{V_{jt} | 1 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 0\}$, M(edium)= $\{V_{jt} | 2 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 1\}$, and L(arge)= $\{V_{jt} | 3 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 2\}$. The recession and expansion regimes contain all histories for which the value of the transition function $F(S_t; \hat{\gamma}, \hat{c})$ is smaller and larger than 0.5, respectively.

Table 9: Common absorption measure in STVECM for income, consumption and investment, shock to income equation

	Unconditional				Recession				Expansion			
	A	S	M	L	A	S	M	L	A	S	M	L
$\pi = 0.50, Y_i = X, Y_l = C$												
Mean	-6.45^*	-5.62	-11.39^*	-10.43^*	-5.05^*	-3.69	-11.01^*	-9.47^*	-7.05^*	-6.42	-11.54^*	-10.83^*
St.dev.	9.60	9.17	8.60	8.88	9.64	8.78	9.27	9.30	9.52	9.21	8.30	8.66
Skewness	-0.32	-0.50	-0.09	-0.14	-0.47	-0.66	0.00	-0.08	-0.26	-0.44	-0.16	-0.19
HDR_{α^*}	0.73	0.48	0.62	0.32	0.63	0.56	0.31	0.43	0.69	0.45	0.59	0.26
S_{α^*}	0.73	0.79	0.34	0.42	0.88	0.99	0.40	0.52	0.63	0.72	0.30	0.38
Q_{α^*}	0.91	1.00	0.46	0.57	1.00	0.88	0.59	0.71	0.84	0.92	0.41	0.51
$\pi = 0.10, Y_i = X, Y_l = C$												
Mean	-14.59^*	-14.74^*	-19.58^*	-19.17^*	-13.69^*	-13.35^*	-19.28^*	-18.73^*	-14.97^*	-15.32^*	-19.71^*	-19.35^*
St.dev.	8.98	7.80	4.28	4.27	9.20	8.25	4.51	4.57	8.86	7.52	4.17	4.12
Skewness	1.35	1.39	-0.18	-0.34	1.54	1.73	-0.01	-0.22	1.26	1.19	-0.28	-0.44
HDR_{α^*}	0.09	0.03	0.00	0.00	0.14	0.02	0.00	0.00	0.06	0.04	0.00	0.00
S_{α^*}	0.09	0.05	0.00	0.00	0.10	0.08	0.00	0.00	0.09	0.05	0.00	0.00
Q_{α^*}	0.19	0.11	0.00	0.00	0.20	0.14	0.00	0.00	0.18	0.09	0.00	0.00
$\pi = 0.50, Y_i = X, Y_l = I$												
Mean	-0.38	0.09	0.12	0.08	-0.35	0.17	0.06	-0.02	-0.40	0.05	0.14	0.13
St.dev.	2.93	1.52	0.39	0.32	2.96	1.77	0.40	0.26	2.92	1.40	0.39	0.34
Skewness	-3.97	1.98	1.02	1.65	-3.60	5.10	0.51	-0.70	-4.14	-0.76	1.29	2.07
HDR_{α^*}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
S_{α^*}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Q_{α^*}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\pi = 0.10, Y_i = X, Y_l = I$												
Mean	-7.38^*	-7.21^*	-8.50^*	-8.50^*	-6.92^*	-6.66^*	-8.00^*	-7.98^*	-7.58^*	-7.45^*	-8.72^*	-8.72^*
St.dev.	3.78	3.56	1.05	1.00	3.92	3.93	1.08	0.97	3.69	3.37	0.96	0.94
Skewness	1.83	2.59	0.67	0.59	1.92	2.42	0.64	0.18	1.79	2.67	0.63	0.84
HDR_{α^*}	0.05	0.04	0.00	0.00	0.04	0.03	0.00	0.00	0.08	0.04	0.00	0.00
S_{α^*}	0.07	0.07	0.00	0.00	0.09	0.10	0.00	0.00	0.06	0.06	0.00	0.00
Q_{α^*}	0.12	0.15	0.00	0.00	0.15	0.20	0.00	0.00	0.11	0.12	0.00	0.00
$\pi = 0.50, Y_i = C, Y_l = I$												
Mean	6.07^*	5.70	11.50^*	10.51^*	4.69	3.85	11.07^*	9.45^*	6.65^*	6.48	11.69^*	10.95^*
St.dev.	10.23	9.22	8.66	8.93	10.14	8.62	9.40	9.37	10.21	9.34	8.33	8.71
Skewness	0.13	0.50	0.06	0.11	0.31	0.79	-0.02	0.07	0.06	0.39	0.12	0.16
HDR_{α^*}	0.74	0.69	0.47	0.27	0.65	0.57	0.26	0.29	0.66	0.60	0.59	0.24
S_{α^*}	0.75	0.80	0.32	0.40	0.92	0.98	0.41	0.52	0.69	0.73	0.30	0.38
Q_{α^*}	0.94	1.00	0.46	0.57	1.00	0.92	0.59	0.71	0.87	0.92	0.41	0.51
$\pi = 0.10, Y_i = C, Y_l = I$												
Mean	7.20^*	7.53^*	11.08^*	10.66^*	6.77^*	6.69^*	11.28^*	10.75^*	7.39^*	7.88^*	10.99^*	10.63^*
St.dev.	7.43	5.94	4.14	4.21	7.62	6.41	4.15	4.11	7.34	5.70	4.13	4.25
Skewness	-1.10	-0.30	0.24	0.32	-0.98	-0.44	0.07	0.13	-1.15	-0.17	0.31	0.39
HDR_{α^*}	0.24	0.33	0.00	0.00	0.28	0.38	0.00	0.00	0.19	0.17	0.00	0.00
S_{α^*}	0.30	0.21	0.00	0.00	0.34	0.28	0.00	0.00	0.30	0.21	0.00	0.00
Q_{α^*}	0.31	0.24	0.00	0.00	0.35	0.31	0.00	0.00	0.29	0.20	0.00	0.00

Summary statistics for common absorption measure $CN_{Y_i,Y_l}(\pi, A, B)$ in STVECM for income, consumption and investment. Entries in rows labelled Mean which are larger than two times $\sigma_{CN_{Y_i,Y_l}(\pi,A,B)}/\sqrt{n_A}$ are marked with an asterisk, where $\sigma_{CN_{Y_i,Y_l}(\pi,A,B)}$ is the standard deviation of $CN_{Y_i,Y_l}(\pi, A, B)$ and n_A is the number of shocks v_{jt} for which $CN_{Y_i,Y_l}(\pi, v_{jt}, \omega_{t-1})$ is computed. The entries in rows labelled Z_{α^*} represent the minimum value of $\alpha \in (0, 1)$ such that 0 would not be included in the relevant confidence region Z_α for the distribution of the common absorption measure $CN_{Y_i,Y_l}(\pi, A, B)$ with $Z = HDR$, S and Q. The different sets of shocks are defined as A(ll)= $\{V_{jt}\}$, S(mall)= $\{V_{jt} | 1 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 0\}$, M(edium)= $\{V_{jt} | 2 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 1\}$, and L(arge)= $\{V_{jt} | 3 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 2\}$. The recession and expansion regimes contain all histories for which the value of the transition function $F(S_t; \hat{\gamma}, \hat{c})$ is smaller and larger than 0.5, respectively.

Table 10: Common absorption measure in STVECM for income, consumption and investment, shock to consumption equation

	Unconditional				Recession				Expansion			
	A	S	M	L	A	S	M	L	A	S	M	L
$\pi = 0.50, Y_i = X, Y_l = C$												
Mean	2.72	2.55	4.19	3.93	2.83	2.67	4.12	3.64*	2.68	2.50	4.22	4.05
St.dev.	7.37	7.25	5.42	4.95	7.07	6.83	4.69	3.94	7.49	7.42	5.71	5.31
Skewness	−0.78	−0.75	0.07	0.25	−1.00	−1.01	0.95	1.24	−0.70	−0.66	−0.14	0.04
HDR_{α^*}	0.62	0.63	0.42	0.17	0.49	0.48	0.35	0.16	0.63	0.64	0.37	0.12
S_{α^*}	0.53	0.52	0.30	0.30	0.44	0.42	0.25	0.21	0.56	0.58	0.33	0.28
Q_{α^*}	0.44	0.49	0.19	0.09	0.37	0.38	0.14	0.07	0.47	0.53	0.21	0.10
$\pi = 0.10, Y_i = X, Y_l = C$												
Mean	3.31	3.06	5.67	6.61*	3.61	3.33	5.77*	6.04*	3.19	2.95	5.63	6.85*
St.dev.	8.80	8.67	6.91	5.97	8.00	7.92	5.88	5.22	9.11	8.97	7.29	6.24
Skewness	−0.66	−0.49	−0.78	−1.08	−0.70	−0.54	−0.04	−0.33	−0.64	−0.46	−0.93	−1.29
HDR_{α^*}	0.71	0.86	0.46	0.37	0.87	0.89	0.71	0.69	0.65	0.68	0.33	0.26
S_{α^*}	0.77	0.75	0.32	0.19	0.64	0.68	0.29	0.19	0.80	0.82	0.33	0.19
Q_{α^*}	0.63	0.67	0.41	0.29	0.57	0.62	0.36	0.30	0.65	0.69	0.43	0.29
$\pi = 0.50, Y_i = X, Y_l = I$												
Mean	3.85*	4.29	5.72*	4.84	2.98	3.55	4.63	3.67	4.21*	4.60	6.18*	5.33*
St.dev.	6.93	6.13	5.70	5.46	7.03	5.86	5.60	4.81	6.86	6.22	5.68	5.65
Skewness	−0.22	0.45	0.86	1.18	−0.41	0.56	1.30	1.76	−0.13	0.41	0.71	0.99
HDR_{α^*}	0.69	0.67	0.50	0.43	0.68	0.66	0.45	0.40	0.69	0.68	0.15	0.39
S_{α^*}	0.53	0.47	0.25	0.34	0.51	0.47	0.37	0.39	0.50	0.43	0.18	0.30
Q_{α^*}	0.43	0.40	0.13	0.16	0.51	0.46	0.26	0.30	0.40	0.37	0.07	0.11
$\pi = 0.10, Y_i = X, Y_l = I$												
Mean	6.25*	6.97*	8.57*	8.10*	5.86*	6.69*	7.70*	6.95*	6.42*	7.09*	8.93*	8.58*
St.dev.	7.94	6.99	5.17	4.99	8.14	7.12	6.04	5.54	7.84	6.94	4.71	4.66
Skewness	−0.86	−0.41	0.46	0.52	−0.77	−0.34	0.29	0.18	−0.90	−0.43	0.76	0.92
HDR_{α^*}	1.00	1.00	0.81	0.79	1.00	1.00	1.00	1.00	0.40	0.61	0.17	0.07
S_{α^*}	0.37	0.31	0.14	0.14	0.52	0.39	0.31	0.36	0.32	0.25	0.08	0.05
Q_{α^*}	0.39	0.32	0.18	0.20	0.58	0.48	0.48	0.57	0.31	0.25	0.05	0.04
$\pi = 0.50, Y_i = C, Y_l = I$												
Mean	1.12	1.74	1.53	0.91	0.15	0.88	0.51	0.03	1.53	2.10	1.96	1.28
St.dev.	6.10	5.48	4.45	3.82	5.66	5.06	3.37	3.03	6.23	5.61	4.77	4.04
Skewness	0.98	2.59	3.76	4.95	0.79	3.21	4.44	4.53	1.02	2.40	3.55	4.98
HDR_{α^*}	1.00	1.00	0.76	1.00	1.00	0.80	0.77	0.74	1.00	1.00	1.00	1.00
S_{α^*}	0.69	0.65	0.53	0.79	1.00	0.88	0.77	1.00	0.62	0.61	0.53	0.63
Q_{α^*}	1.00	1.00	0.91	1.00	1.00	1.00	1.00	1.00	0.97	0.94	0.86	1.00
$\pi = 0.10, Y_i = X, Y_l = I$												
Mean	2.94	3.91	2.90	1.49	2.24	3.37	1.94	0.90	3.23	4.14	3.30	1.73
St.dev.	7.63	7.29	6.63	5.43	7.07	7.09	4.48	3.11	7.84	7.36	7.31	6.14
Skewness	0.73	1.16	1.63	1.82	0.97	1.42	2.89	3.56	0.64	1.06	1.35	1.51
HDR_{α^*}	1.00	1.00	0.74	0.73	1.00	1.00	0.72	0.63	1.00	0.80	0.75	1.00
S_{α^*}	0.65	0.62	0.59	0.60	0.67	0.68	0.52	0.63	0.63	0.58	0.61	0.62
Q_{α^*}	0.84	0.77	0.81	0.93	0.96	0.94	0.83	0.89	0.79	0.70	0.81	0.94

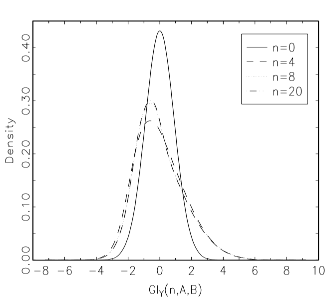
Summary statistics for common absorption measure $CN_{Y_i,Y_l}(\pi, A, B)$ in STVECM for income, consumption and investment. Entries in rows labelled Mean which are larger than two times $\sigma_{CN_{Y_i,Y_l}(\pi,A,B)}/\sqrt{n_A}$ are marked with an asterisk, where $\sigma_{CN_{Y_i,Y_l}(\pi,A,B)}$ is the standard deviation of $CN_{Y_i,Y_l}(\pi, A, B)$ and n_A is the number of shocks v_{jt} for which $CN_{Y_i,Y_l}(\pi, v_{jt}, \omega_{t-1})$ is computed. The entries in rows labelled Z_{α^*} represent the minimum value of $\alpha \in (0,1)$ such that 0 would not be included in the relevant confidence region Z_α for the distribution of the common absorption measure $CN_{Y_i,Y_l}(\pi, A, B)$ with $Z = HDR$, S and Q. The different sets of shocks are defined as A(ll)= $\{V_{jt}\}$, S(mall)= $\{V_{jt}|1 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 0\}$, M(edium)= $\{V_{jt}|2 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 1\}$, and L(arge)= $\{V_{jt}|3 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 2\}$. The recession and expansion regimes contain all histories for which the value of the transition function $F(S_t; \hat{\gamma}, \hat{c})$ is smaller and larger than 0.5, respectively.

Table 11: Common absorption measure in STVECM for income, consumption and investment, shock to investment equation

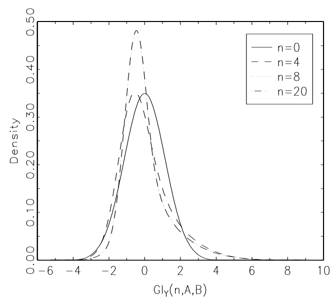
	Unconditional				Recession				Expansion			
	A	S	M	L	A	S	M	L	A	S	M	L
$\pi = 0.50, Y_i = X, Y_l = C$												
Mean	0.32	0.15	−0.30	0.65	0.55	0.52	0.36	1.16	0.22	−0.01	−0.58	0.43
St.dev.	4.51	4.70	2.47	1.02	4.46	4.91	1.92	1.47	4.53	4.59	2.62	0.65
Skewness	−0.39	−0.77	−4.78	1.52	−0.50	−0.47	−0.90	0.80	−0.34	−0.93	−5.40	0.21
HDR_{α^*}	1.00	1.00	1.00	1.00	1.00	0.81	0.73	1.00	1.00	1.00	1.00	1.00
S_{α^*}	1.00	1.00	1.00	0.67	0.83	0.81	1.00	0.64	1.00	1.00	0.77	1.00
Q_{α^*}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.91	1.00	1.00	1.00	1.00
$\pi = 0.10, Y_i = X, Y_l = C$												
Mean	−0.47	−0.33	−1.13	−0.31	0.01	0.41	−0.41	0.27	−0.68	−0.65	−1.43	−0.55
St.dev.	5.46	6.09	2.82	2.02	5.57	6.22	2.22	1.75	5.40	6.00	2.99	2.08
Skewness	0.10	−0.04	−2.20	−1.81	−0.11	−0.18	−1.62	−2.23	0.19	0.01	−2.22	−1.71
HDR_{α^*}	1.00	1.00	1.00	1.00	0.80	0.80	1.00	1.00	1.00	1.00	1.00	1.00
S_{α^*}	1.00	1.00	0.75	1.00	1.00	1.00	1.00	1.00	0.88	0.90	0.75	0.92
Q_{α^*}	1.00	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00
$\pi = 0.50, Y_i = X, Y_l = I$												
Mean	0.48	1.25	0.47	0.33	0.43	1.44	0.35	0.27	0.49	1.18	0.52*	0.35
St.dev.	3.44	2.82	0.56	0.51	4.09	3.19	0.54	0.56	3.12	2.64	0.55	0.48
Skewness	−0.43	3.48	1.46	0.31	−0.79	3.26	−0.01	−0.03	−0.05	3.54	2.09	0.59
HDR_{α^*}	1.00	0.56	1.00	1.00	1.00	1.00	1.00	1.00	0.57	0.54	0.50	1.00
S_{α^*}	1.00	0.52	1.00	1.00	1.00	0.54	1.00	1.00	1.00	0.51	0.50	1.00
Q_{α^*}	1.00	0.89	1.00	1.00	1.00	0.90	1.00	1.00	1.00	0.88	0.99	1.00
$\pi = 0.10, Y_i = X, Y_l = I$												
Mean	1.22	2.16	1.14	0.17	1.40	2.59	0.96	0.51	1.14	1.98	1.22	0.02
St.dev.	4.64	4.33	2.42	1.67	5.01	4.54	1.78	0.96	4.48	4.22	2.64	1.87
Skewness	−0.16	0.91	1.14	−1.03	−0.07	1.36	2.54	4.01	−0.23	0.66	0.87	−1.10
HDR_{α^*}	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
S_{α^*}	0.75	0.68	0.70	1.00	0.73	0.67	0.66	0.63	0.76	0.71	0.73	1.00
Q_{α^*}	1.00	0.86	1.00	1.00	1.00	0.86	1.00	1.00	1.00	0.86	1.00	1.00
$\pi = 0.50, Y_i = C, Y_l = I$												
Mean	0.16	1.11	0.77	−0.32	−0.12	0.92	−0.01	−0.89	0.27	1.19	1.09	−0.08
St.dev.	4.62	3.91	2.66	1.24	4.48	3.96	2.23	1.81	4.67	3.89	2.75	0.78
Skewness	0.41	3.01	4.15	−1.49	0.57	2.89	0.51	−0.73	0.35	3.07	5.07	−0.38
HDR_{α^*}	1.00	1.00	0.69	1.00	0.85	0.84	0.55	0.69	1.00	0.77	1.00	1.00
S_{α^*}	1.00	0.68	0.69	1.00	1.00	0.89	1.00	0.88	1.00	0.64	0.57	1.00
Q_{α^*}	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97	1.00
$\pi = 0.10, Y_i = C, Y_l = I$												
Mean	1.69	2.49	2.27	0.48	1.39	2.18	1.37	0.24	1.82	2.62	2.65	0.58
St.dev.	6.04	6.15	3.94	2.13	5.66	6.06	3.06	2.02	6.19	6.19	4.20	2.17
Skewness	0.92	1.50	2.09	1.58	1.35	1.75	1.40	2.05	0.77	1.40	2.10	1.42
HDR_{α^*}	1.00	1.00	1.00	1.00	1.00	0.81	1.00	1.00	1.00	1.00	1.00	1.00
S_{α^*}	0.77	0.73	0.74	1.00	0.76	0.70	0.76	1.00	0.79	0.72	0.70	0.88
Q_{α^*}	1.00	0.95	0.96	1.00	1.00	0.97	1.00	1.00	1.00	0.94	0.90	1.00

Summary statistics for common absorption measure $CN_{Y_i,Y_l}(\pi, A, B)$ in STVECM for income, consumption and investment. Entries in rows labelled Mean which are larger than two times $\sigma_{CN_{Y_i,Y_l}(\pi,A,B)}/\sqrt{n_A}$ are marked with an asterisk, where $\sigma_{CN_{Y_i,Y_l}(\pi,A,B)}$ is the standard deviation of $CN_{Y_i,Y_l}(\pi, A, B)$ and n_A is the number of shocks v_{jt} for which $CN_{Y_i,Y_l}(\pi, v_{jt}, \omega_{t-1})$ is computed. The entries in rows labelled Z_{α^*} represent the minimum value of $\alpha \in (0, 1)$ such that 0 would not be included in the relevant confidence region Z_α for the distribution of the common absorption measure $CN_{Y_i,Y_l}(\pi, A, B)$ with $Z = HDR$, S and Q. The different sets of shocks are defined as A(ll)= $\{V_{jt}\}$, S(mall)= $\{V_{jt}|1 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 0\}$, M(edium)= $\{V_{jt}|2 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 1\}$, and L(arge)= $\{V_{jt}|3 \geq |V_{jt}/\sqrt{H_{t,jj}}| > 2\}$. The recession and expansion regimes contain all histories for which the value of the transition function $F(S_t; \hat{\gamma}, \hat{c})$ is smaller and larger than 0.5, respectively.

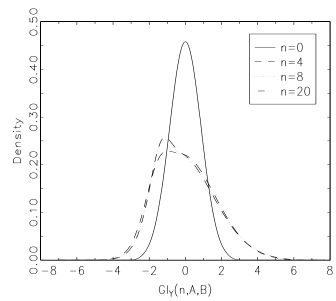
Figure 1: Impulse response functions and asymmetry measures in current-depth-of-recession model



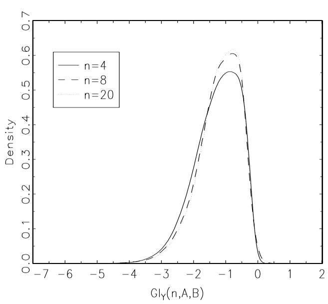
(a) A, unconditional



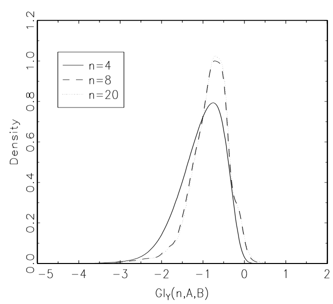
(b) A, recession



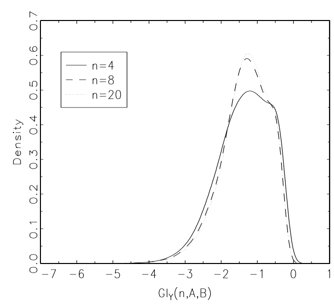
(c) A, expansion



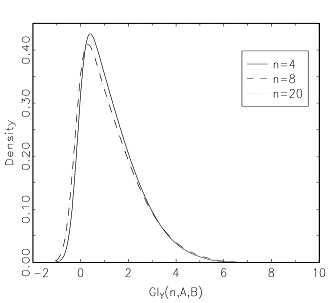
(d) N, unconditional



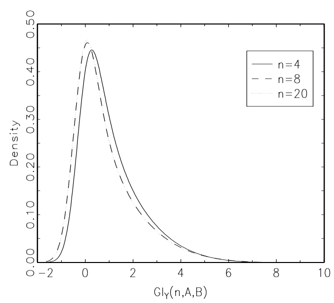
(e) N, recession



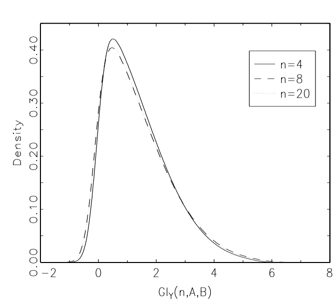
(f) N, expansion



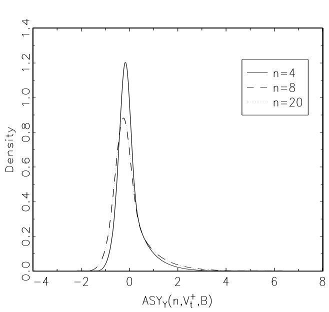
(g) P, unconditional



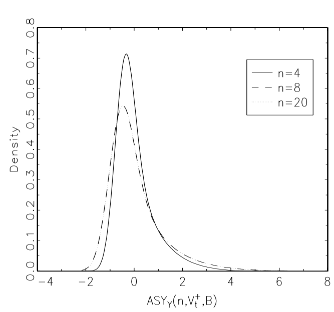
(h) P, recession



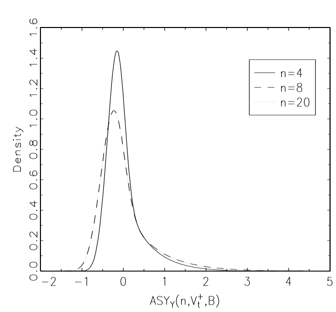
(i) P, expansion



(j) unconditional



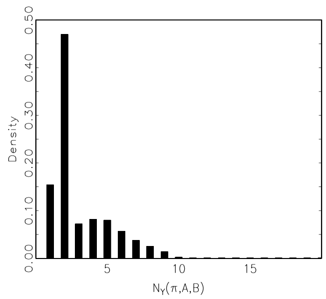
(k) recession



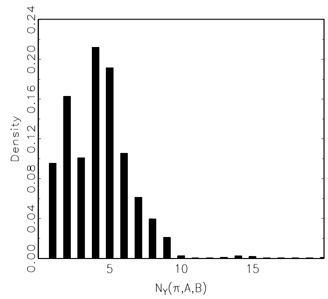
(l) expansion

Note: Distribution of impulse response functions and asymmetry measure in current-depth-of-recession model. The different sets of shocks are defined as A(ll)= $\{V_t\}$, N(egative)= $\{V_t | V_t < 0\}$, and P(ositive)= $\{V_t | V_t > 0\}$.

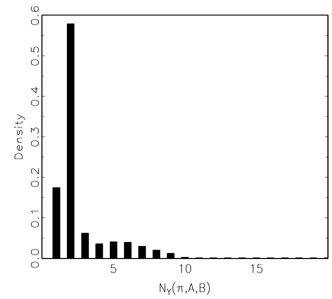
Figure 2: Absorption times in current-depth-of-recession model, $\pi = 0.50$



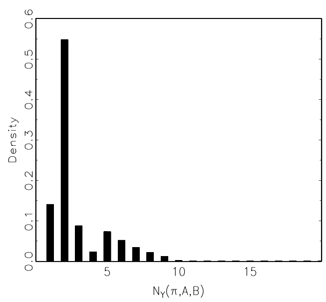
(a) A, unconditional



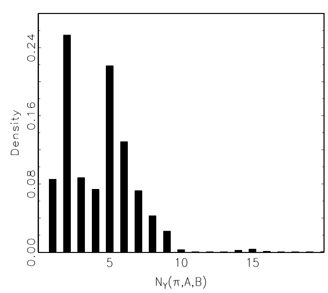
(b) A, recession



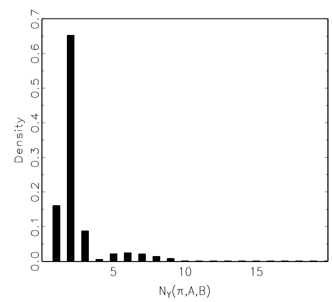
(c) A, expansion



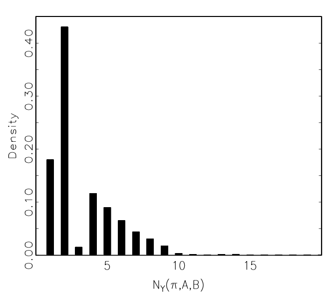
(d) N, unconditional



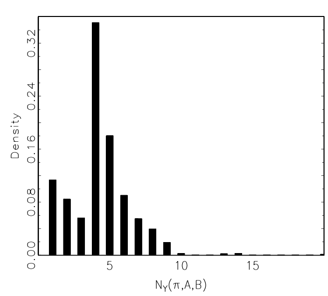
(e) N, recession



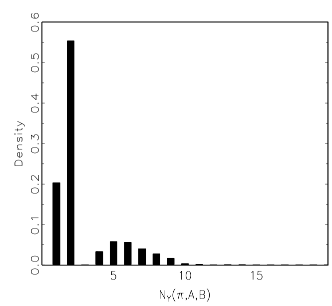
(f) N, expansion



(g) P, unconditional



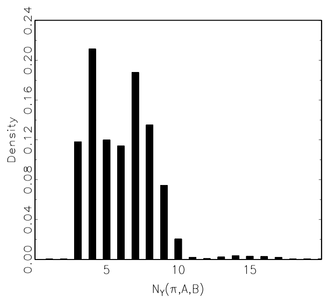
(h) P, recession



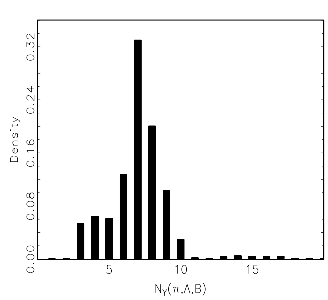
(i) P, expansion

Note: Distribution of absorption times in current-depth-of-recession model. The different sets of shocks are defined as A(ll)= $\{V_t\}$, N(egative)= $\{V_t | V_t < 0\}$, and P(ositive)= $\{V_t | V_t > 0\}$.

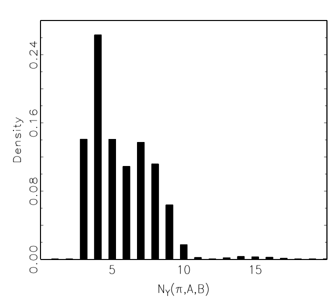
Figure 3: Absorption times in current-depth-of-recession model, $\pi = 0.10$



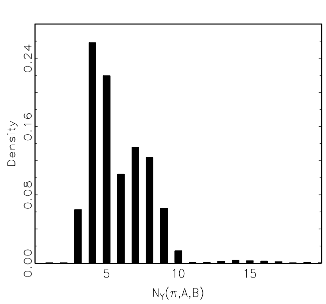
(a) A, unconditional



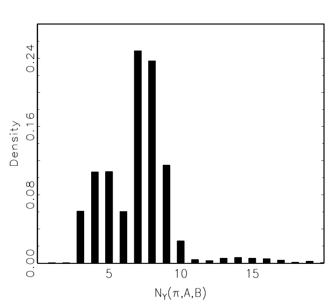
(b) A, recession



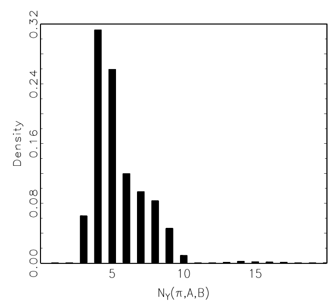
(c) A, expansion



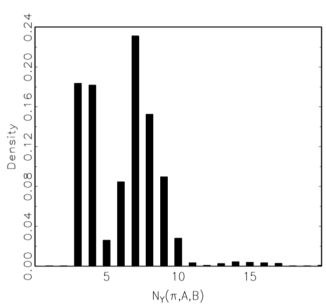
(d) N, unconditional



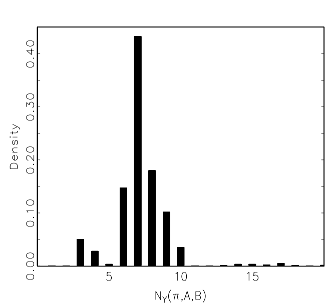
(e) N, recession



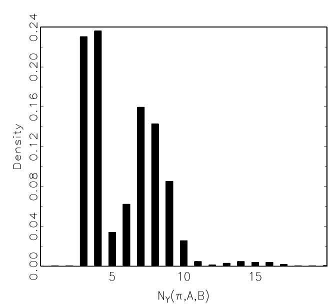
(f) N, expansion



(g) P, unconditional



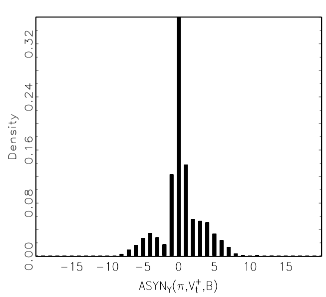
(h) P, recession



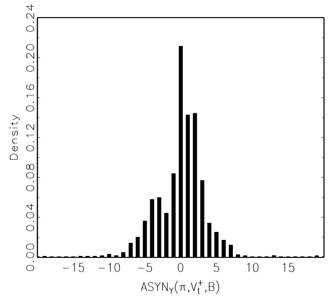
(i) P, expansion

Note: Distribution of absorption times in current-depth-of-recession model. The different sets of shocks are defined as A(ll)= $\{V_t\}$, N(egative)= $\{V_t|V_t < 0\}$, and P(ositive)= $\{V_t|V_t > 0\}$.

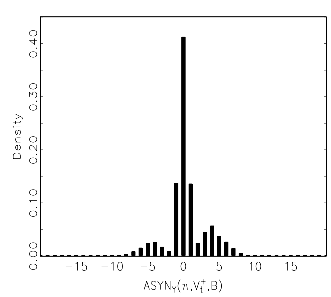
Figure 4: Asymmetry measures for absorption times in current-depth-of-recession model, $\pi = 0.50$



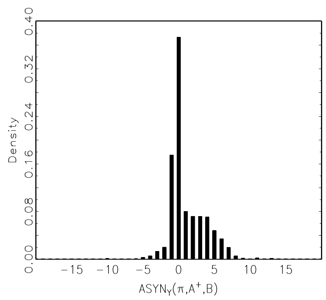
(a) A, unconditional



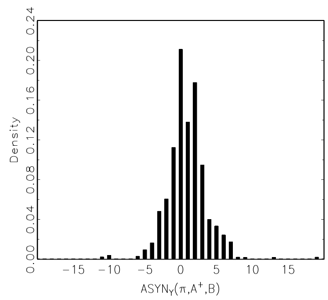
(b) A, recession



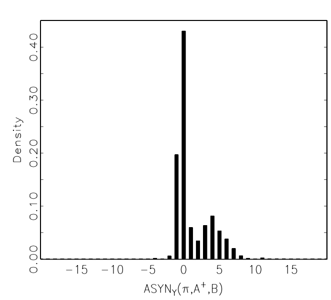
(c) A, expansion



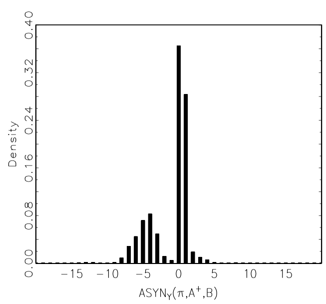
(d) S, unconditional



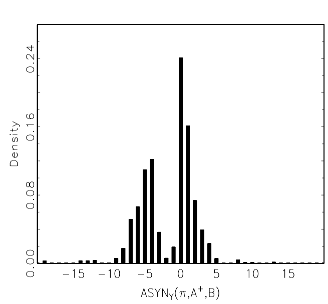
(e) S, recession



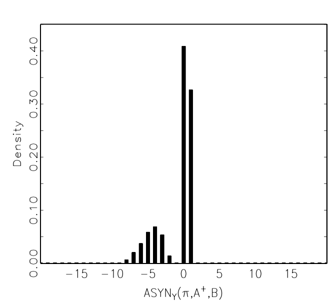
(f) S, expansion



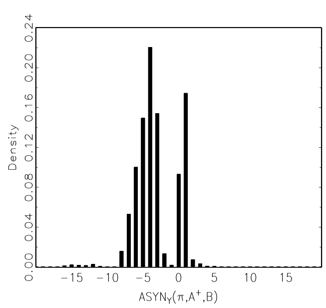
(g) M, unconditional



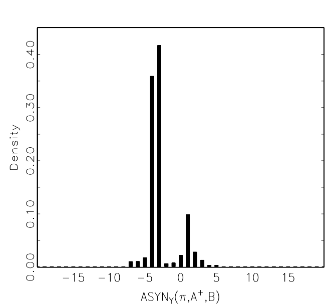
(h) M, recession



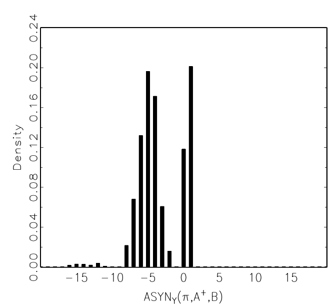
(i) M, expansion



(j) L, unconditional



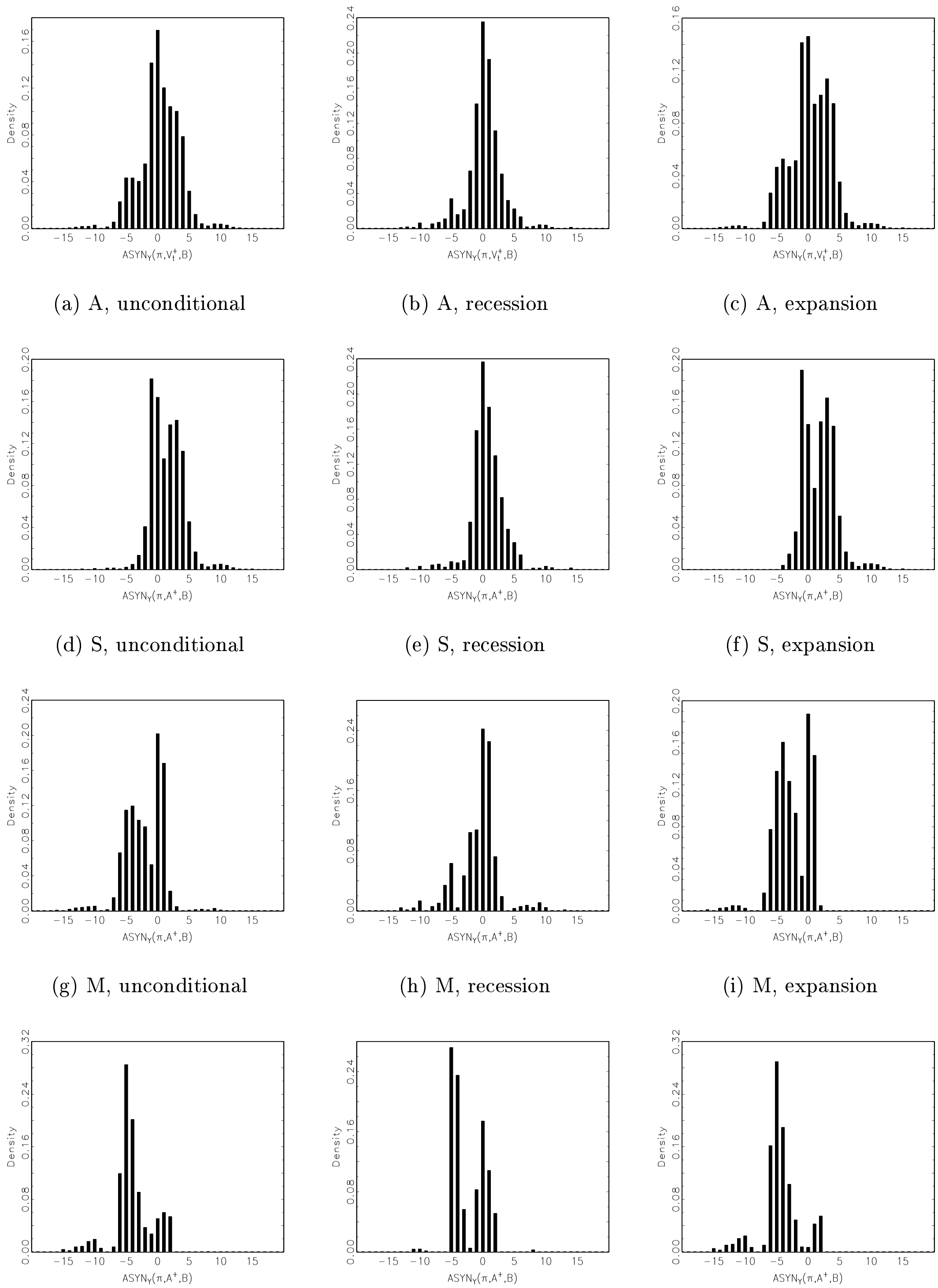
(k) L, recession



(l) L, expansion

Note: Distribution of asymmetry measures for absorption times in current-depth-of-recession model. The different sets of shocks are defined as A(ll)= $\{V_t\}$, S(mall)= $\{V_t | 1 \geq |V_t/\sqrt{H_t}| > 0\}$, M(edium)= $\{V_t | 2 \geq |V_t/\sqrt{H_t}| > 1\}$, L(arge)= $\{V_t | 3 \geq |V_t/\sqrt{H_t}| > 2\}$.

Figure 5: Asymmetry measures for absorption times in current-depth-of-recession model, $\pi = 0.10$



(a) A, unconditional (b) A, recession (c) A, expansion

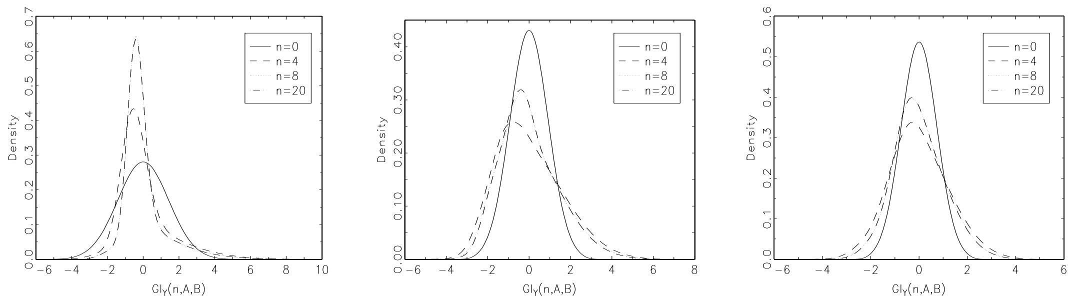
(d) S, unconditional (e) S, recession (f) S, expansion

(g) M, unconditional (h) M, recession (i) M, expansion

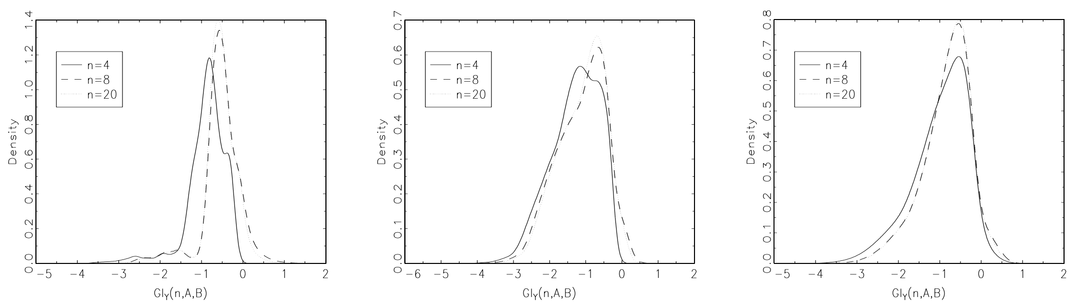
(j) L, unconditional (k) L, recession (l) L, expansion

Note: Distribution of asymmetry measures for absorption times in current-depth-of-recession model. The different sets of shocks are defined as A(ll)= $\{V_t\}$, S(mall)= $\{V_t|1 \geq |V_t/\sqrt{H_t}| > 0\}$, M(edium)= $\{V_t|2 \geq |V_t/\sqrt{H_t}| > 1\}$, L(arge)= $\{V_t|3 \geq |V_t/\sqrt{H_t}| > 2\}$.

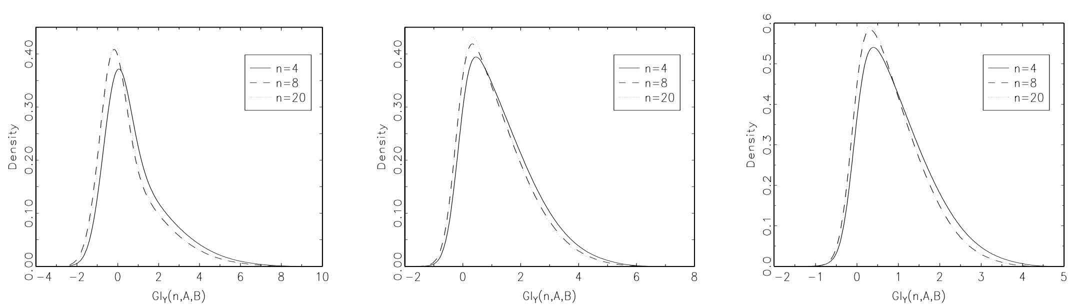
Figure 6: Impulse response functions and asymmetry measures in floor-and-ceiling model



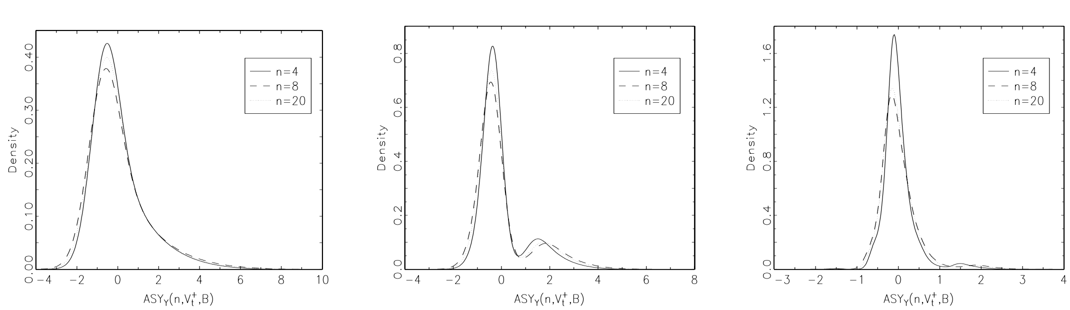
(a) A, floor (b) A, corridor (c) A, ceiling



(d) N, floor (e) N, corridor (f) N, ceiling



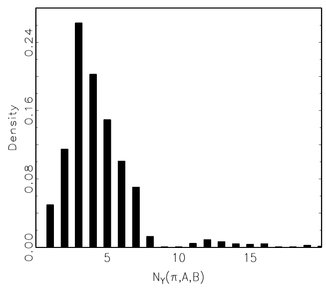
(g) P, floor (h) P, corridor (i) P, ceiling



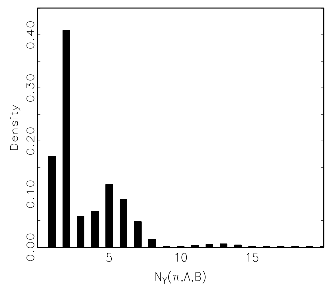
(j) floor (k) corridor (l) ceiling

Note: Distribution of impulse response functions in floor-and-ceiling model. The different sets of shocks are defined as A(ll)= $\{V_t\}$, N(egative)= $\{V_t|V_t < 0\}$, and P(ositive)= $\{V_t|V_t > 0\}$.

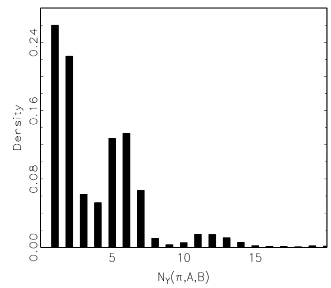
Figure 7: Absorption times in floor-and-ceiling model, $\pi = 0.50$



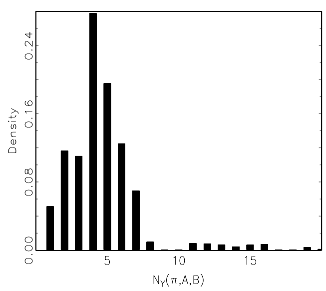
(a) A, floor



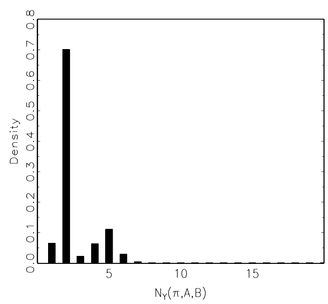
(b) A, corridor



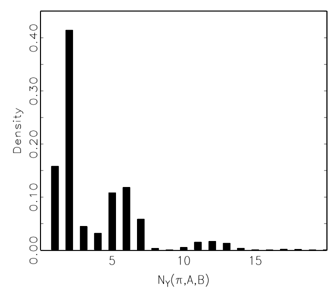
(c) A, ceiling



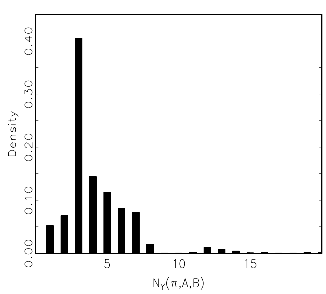
(d) N, floor



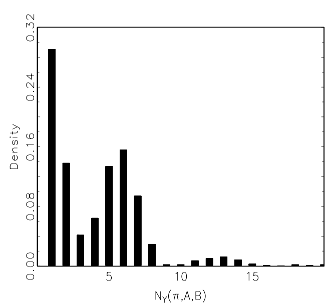
(e) N, corridor



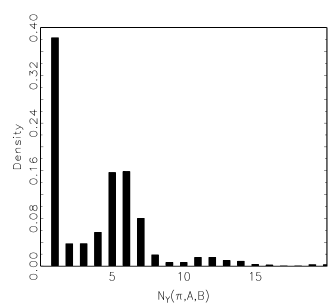
(f) N, ceiling



(g) P, floor



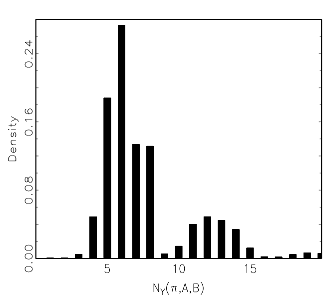
(h) P, corridor



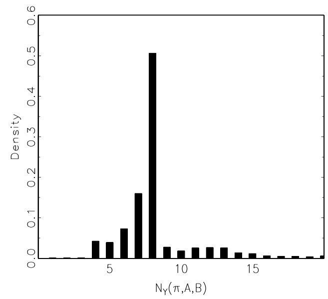
(i) P, ceiling

Note: Distribution of absorption times in floor-and-ceiling model. The different sets of shocks are defined as A(ll)= $\{V_t\}$, N(egative)= $\{V_t | V_t < 0\}$, and P(ositive)= $\{V_t | V_t > 0\}$.

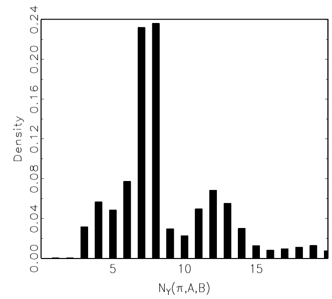
Figure 8: Absorption times in floor-and-ceiling model, $\pi = 0.10$



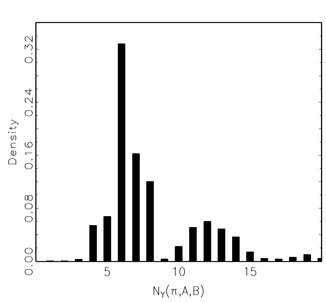
(a) A, floor



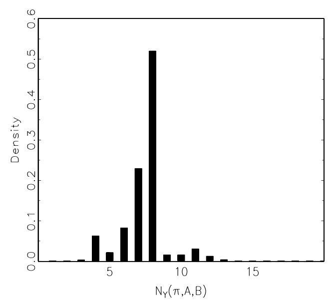
(b) A, corridor



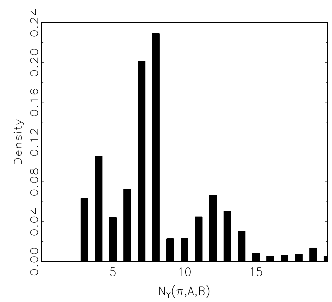
(c) A, ceiling



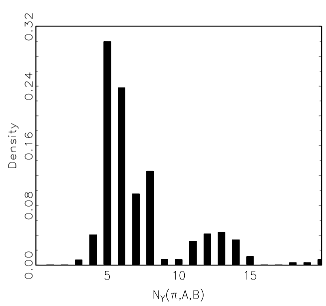
(d) N, floor



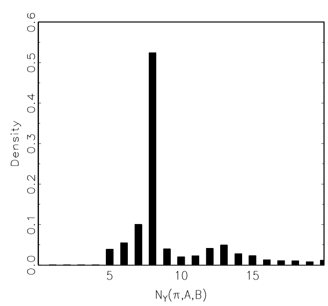
(e) N, corridor



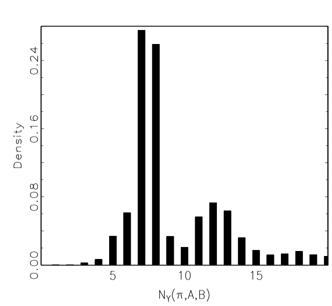
(f) N, ceiling



(g) P, floor



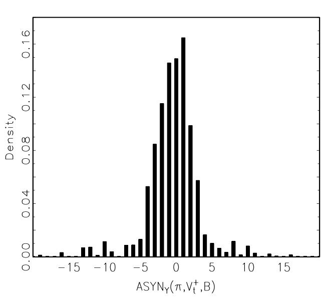
(h) P, corridor



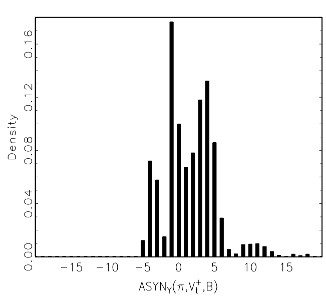
(i) P, ceiling

Note: Distribution of absorption times in floor-and-ceiling model. The different sets of shocks are defined as A(ll)= $\{V_t\}$, N(egative)= $\{V_t | V_t < 0\}$, and P(ositive)= $\{V_t | V_t > 0\}$.

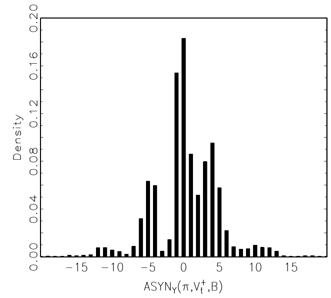
Figure 9: Asymmetry measures for absorption times in floor-and-ceiling model, $\pi = 0.50$



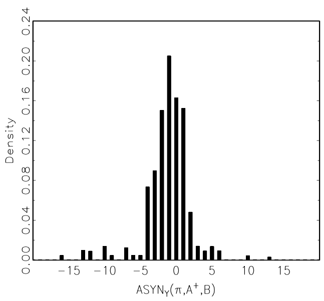
(a) A, floor



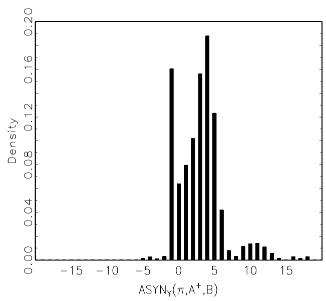
(b) A, corridor



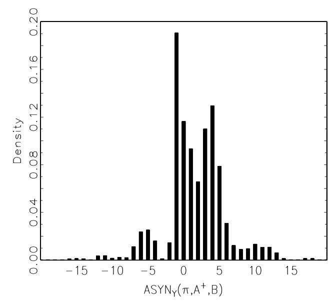
(c) A, ceiling



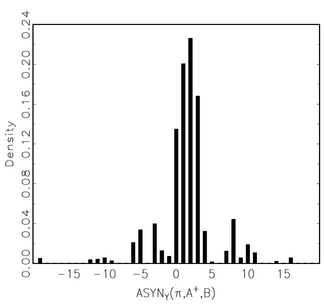
(d) S, floor



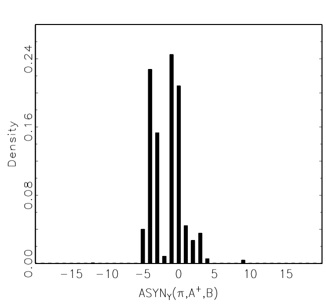
(e) S, corridor



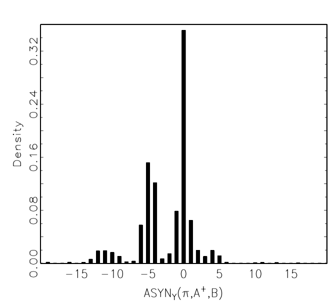
(f) S, ceiling



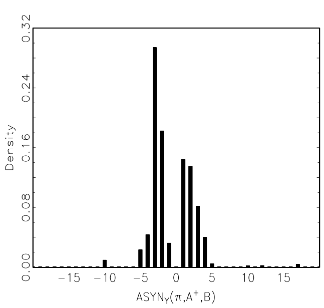
(g) M, floor



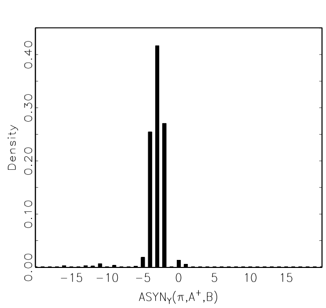
(h) M, corridor



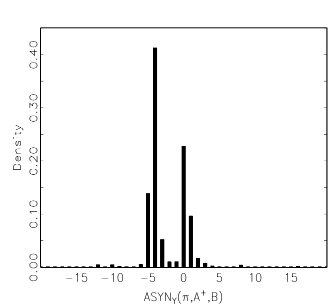
(i) M, ceiling



(j) L, floor



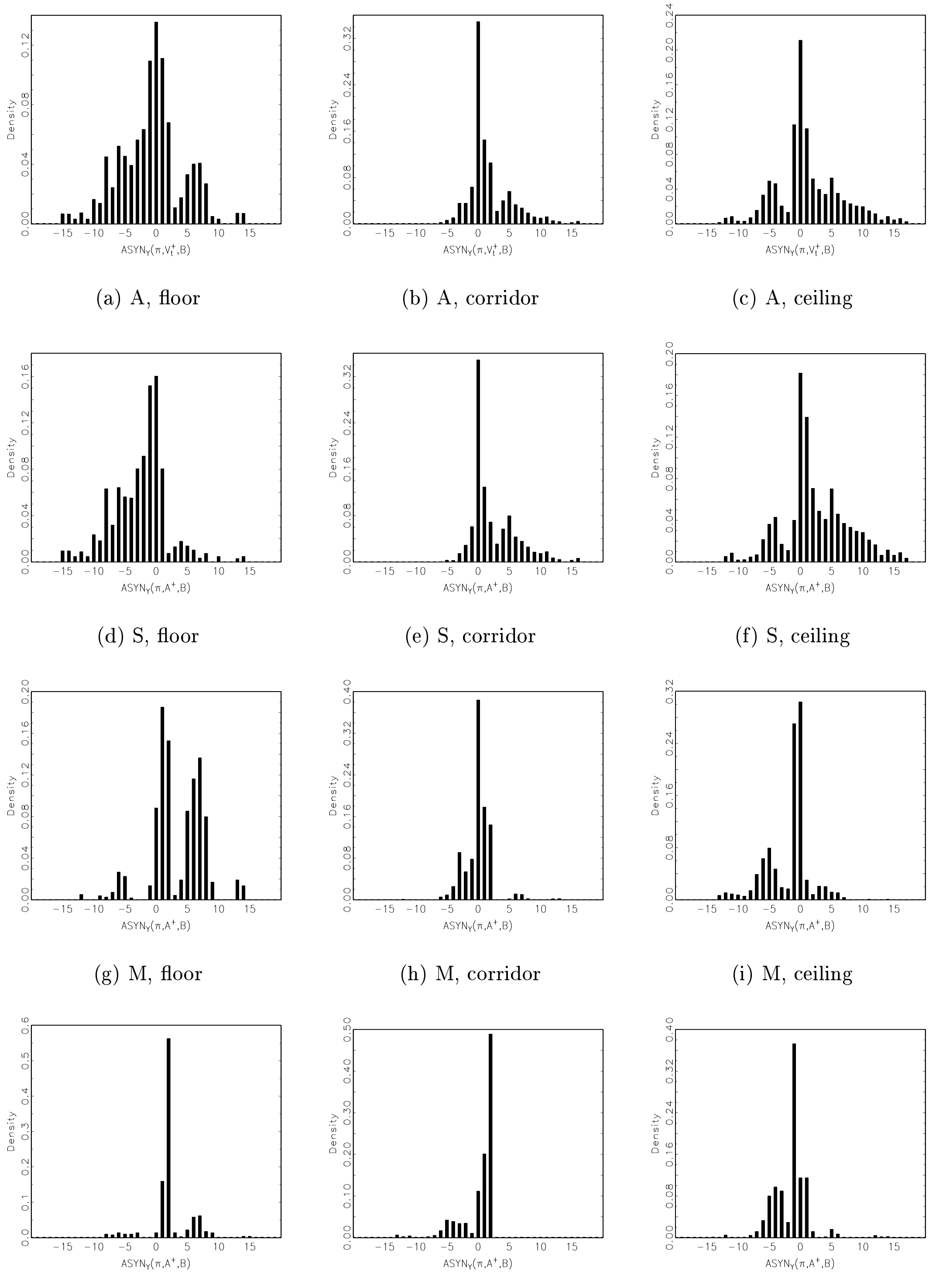
(k) L, corridor



(l) L, ceiling

Note: Distribution of asymmetry measures for absorption times in floor-and-ceiling model. The different sets of shocks are defined as A(ll)= $\{V_t\}$, S(mall)= $\{V_t | 1 \geq |V_t/\sqrt{H_t}| > 0\}$, M(edium)= $\{V_t | 2 \geq |V_t/\sqrt{H_t}| > 1\}$, L(arge)= $\{V_t | 3 \geq |V_t/\sqrt{H_t}| > 2\}$.

Figure 10: Asymmetry measures for absorption times in floor-and-ceiling model, $\pi = 0.10$



(a) A, floor (b) A, corridor (c) A, ceiling

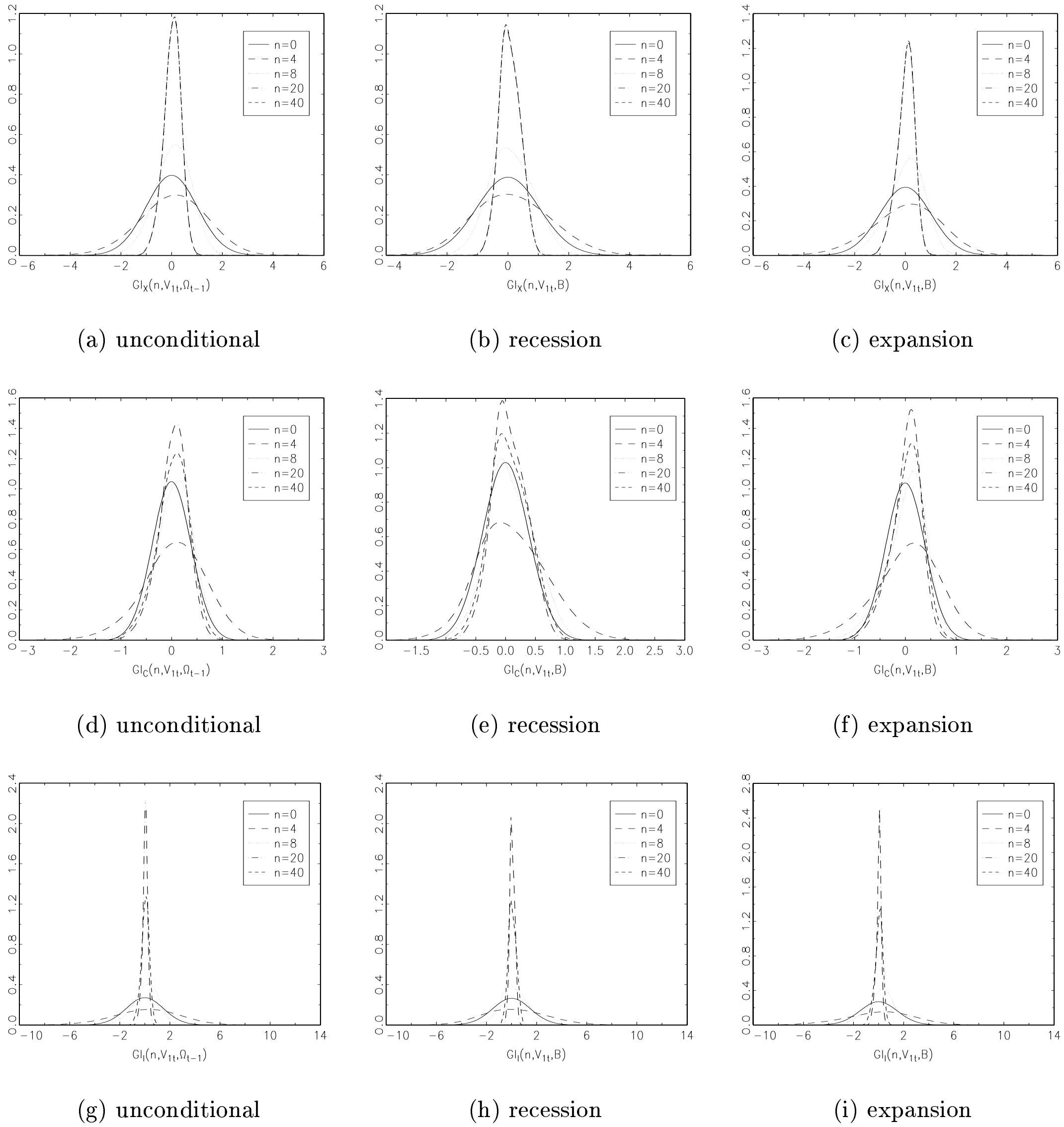
(d) S, floor (e) S, corridor (f) S, ceiling

(g) M, floor (h) M, corridor (i) M, ceiling

(j) L, floor (k) L, corridor (l) L, ceiling

Note: Distribution of asymmetry measures for absorption times in floor-and-ceiling model. The different sets of shocks are defined as A(ll)= $\{V_t\}$, S(mall)= $\{V_t|1 \geq |V_t/\sqrt{H_t}| > 0\}$, M(edium)= $\{V_t|2 \geq |V_t/\sqrt{H_t}| > 1\}$, L(arge)= $\{V_t|3 \geq |V_t/\sqrt{H_t}| > 2\}$.

Figure 11: Impulse response functions in STVECM, income shock



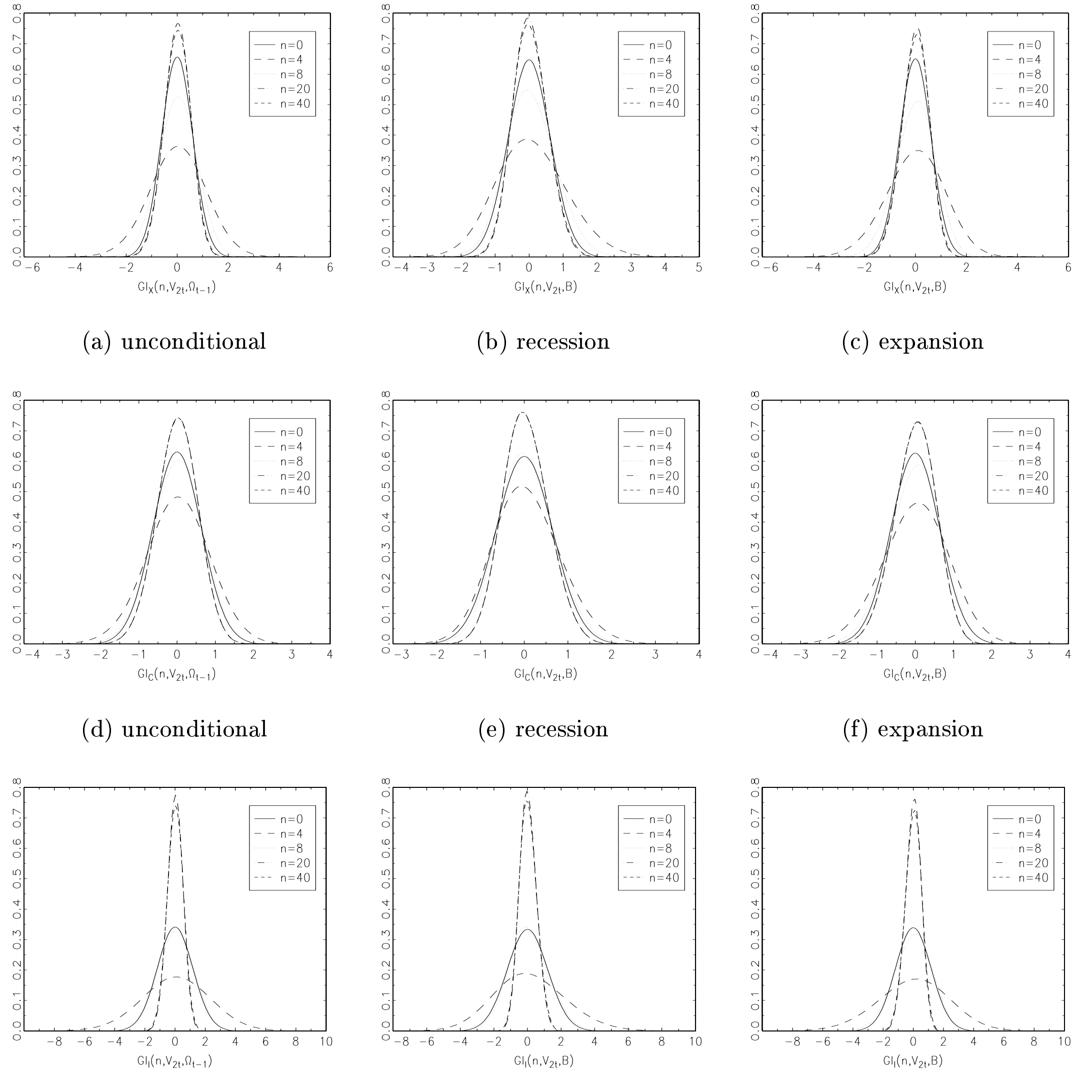
(a) unconditional (b) recession (c) expansion

(d) unconditional (e) recession (f) expansion

(g) unconditional (h) recession (i) expansion

Note: Distribution of impulse response functions for STVECM model for income, consumption and investment with common nonlinear component, for shock given to income equation.

Figure 12: Impulse response functions in STVECM, consumption shock



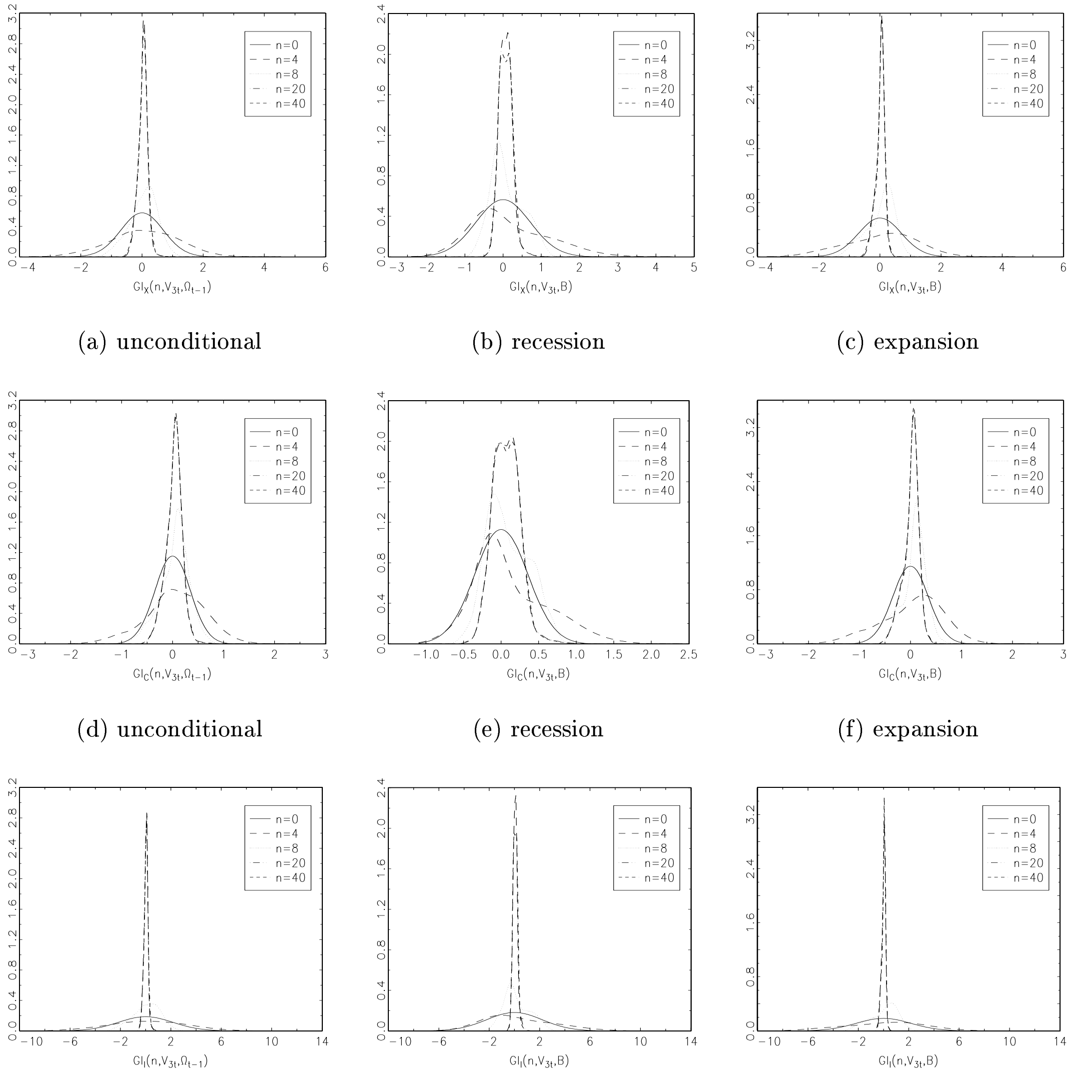
(a) unconditional (b) recession (c) expansion

(d) unconditional (e) recession (f) expansion

(g) unconditional (h) recession (i) expansion

Note: Distribution of impulse response functions for STVECM model for income, consumption and investment with common nonlinear component, for shock given to consumption equation.

Figure 13: Impulse response functions in STVECM, investment shock



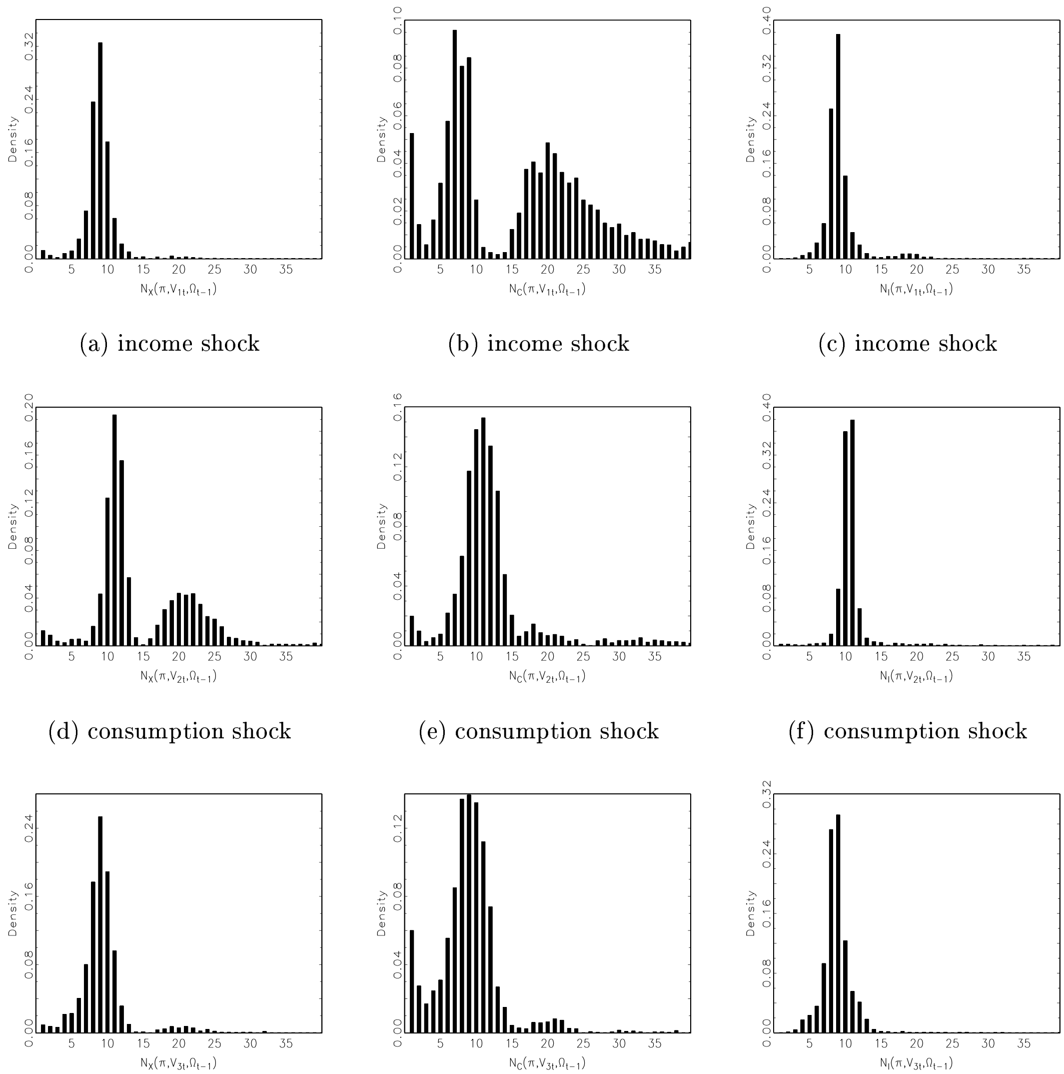
(a) unconditional (b) recession (c) expansion

(d) unconditional (e) recession (f) expansion

(g) unconditional (h) recession (i) expansion

Note: Distribution of impulse response functions for STVECM model for income, consumption and investment with common nonlinear component, for shock given to investment equation.

Figure 14: Absorption times in STVECM, $\pi = 0.50$



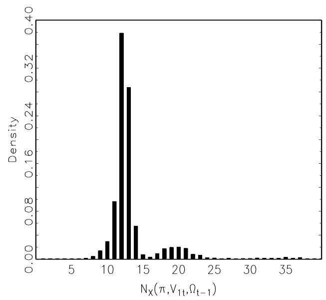
(a) income shock (b) income shock (c) income shock

(d) consumption shock (e) consumption shock (f) consumption shock

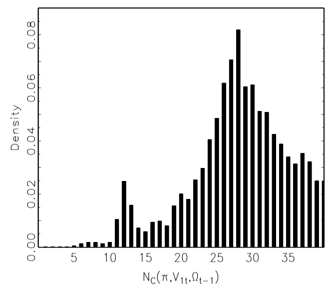
(g) investment shock (h) investment shock (i) investment shock

Note: Distribution of absorption times for STVECM for income, consumption and investment with common nonlinear component.

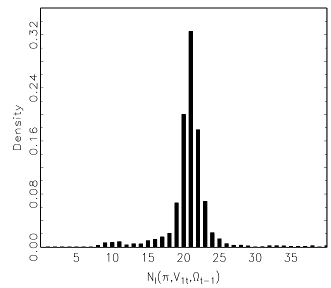
Figure 15: Absorption times in STVECM, $\pi = 0.10$



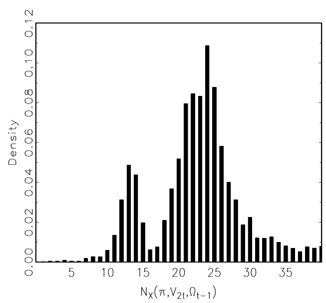
(a) income shock



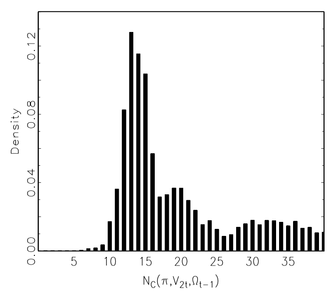
(b) income shock



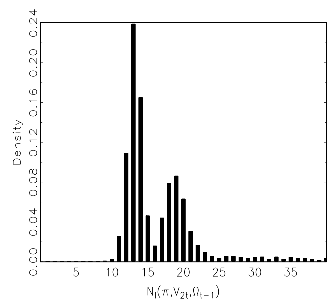
(c) income shock



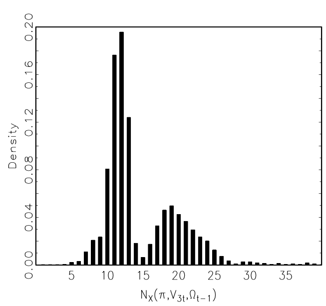
(d) consumption shock



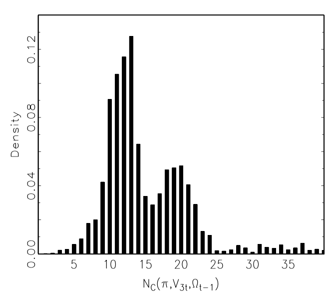
(e) consumption shock



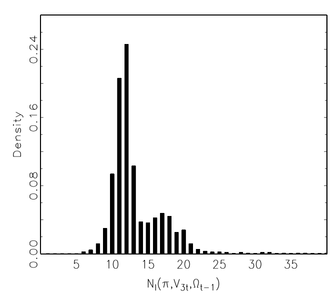
(f) consumption shock



(g) investment shock



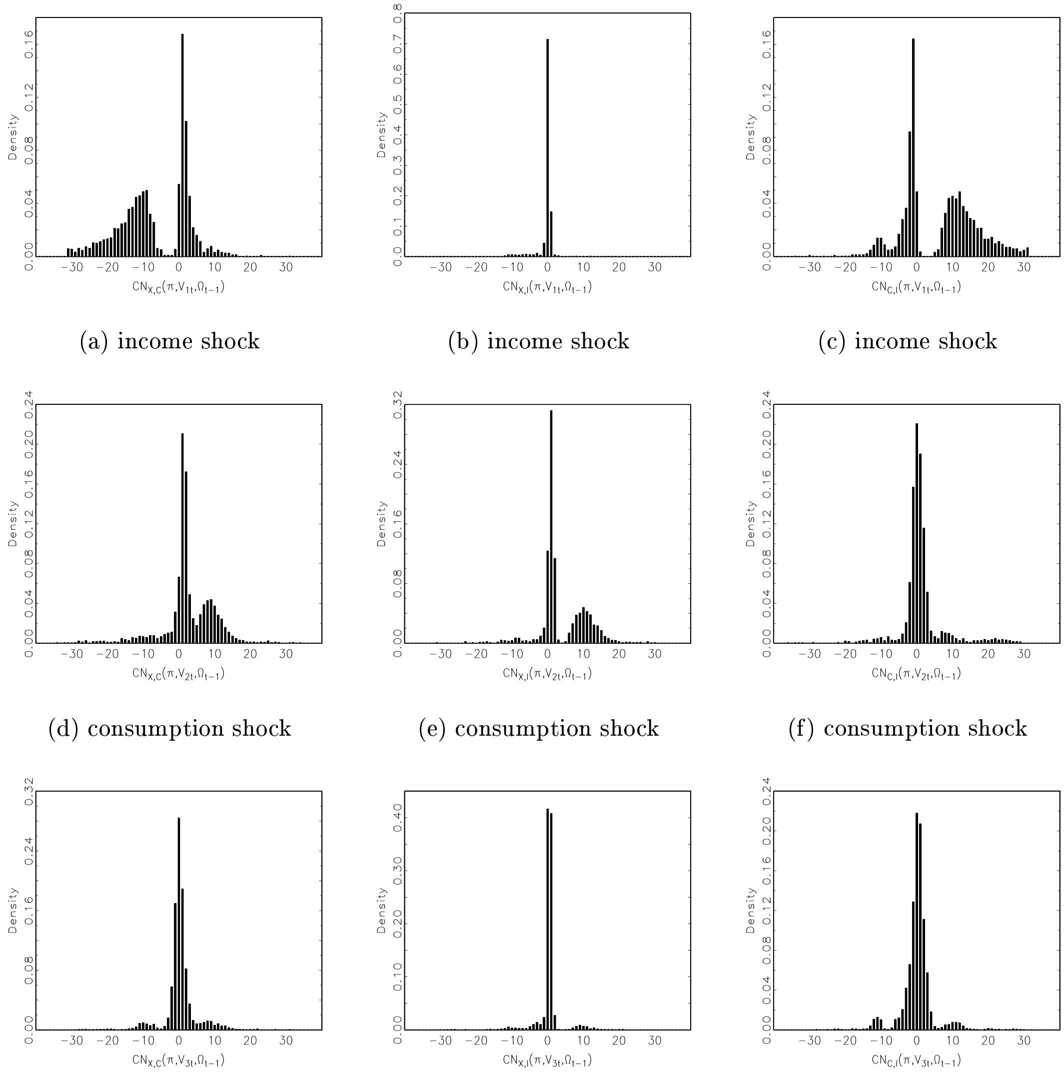
(h) investment shock



(i) investment shock

Note: Distribution of absorption times for STVECM for income, consumption and investment with common nonlinear component.

Figure 16: Common absorption measure in STVECM, $\pi = 0.50$



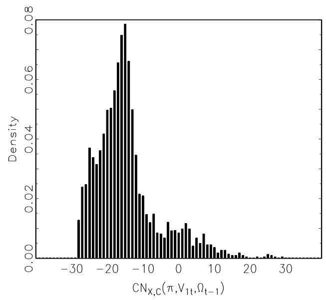
(a) income shock (b) income shock (c) income shock

(d) consumption shock (e) consumption shock (f) consumption shock

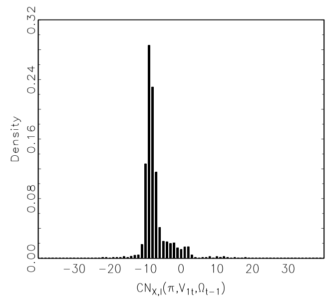
(g) investment shock (h) investment shock (i) investment shock

Note: Distribution of common absorption measure for STVECM for income, consumption and investment with common nonlinear component.

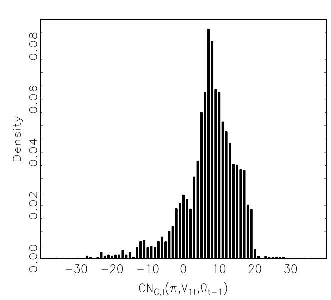
Figure 17: Common absorption measure in STVECM, $\pi = 0.10$



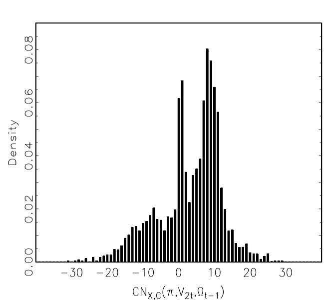
(a) income shock



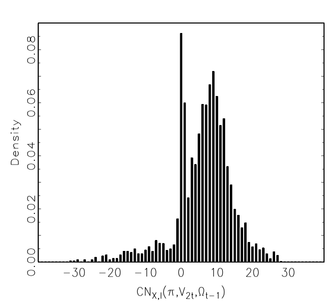
(b) income shock



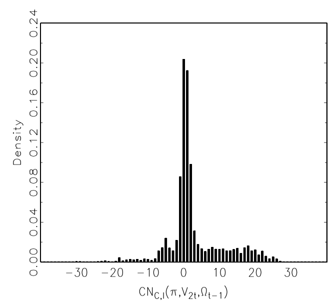
(c) income shock



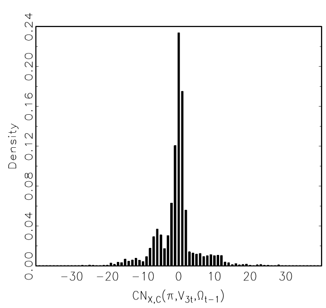
(d) consumption shock



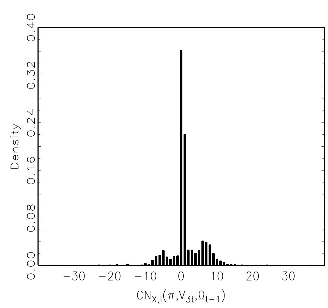
(e) consumption shock



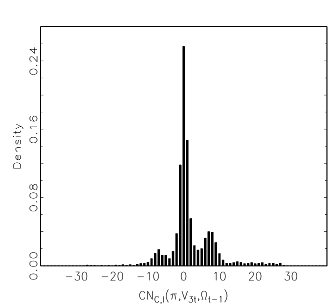
(f) consumption shock



(g) investment shock



(h) investment shock



(i) investment shock

Note: Distribution of common absorption measure for STVECM for income, consumption and investment with common nonlinear component.